Reliability-Based Optimal Design of Steel Box Structures. II: Ship Structure Applications

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Abstract: Traditional design of ship structures relies on a combination of experience, sound judgment, and deterministic approaches and typically ignores the potential for design improvement and other benefits offered through the use of reliability methods and structural optimization strategies. Part I of this article outlines the underlying theories involved in incorporating reliability methods and structural optimization strategies into the initial design of ship structures, whereas Part II (this paper) discusses their application to two case studies, namely, (1) a simple ship structure and (2) a more complex ship structure in an attempt to achieve weight reduction in the face of constraints on ultimate strength and buckling capacity. Using the approach outlined in the companion paper, a weight reduction of 5.6% was realized in the case of the simple vessel, whereas a 2.0% reduction was achieved in the case of the more complex vessel. A reduction in weight reduction has the potential to minimize the lifecycle cost, especially when including construction and operational and maintenance cost. These results highlight the potential benefits of reliability methods and structural optimization strategies, and encourage their implementation during the initial ship structural design phase. **DOI:** 10.1061/AJRUA6.0000830.

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Introduction and Motivation

Traditional design of ship structures has relied on a combination of engineering experience, sound judgment, and deterministic approaches, which effectively ignores many of the uncertainties inherent in structural design loads and capacities. These strategies have failed to incorporate advances in the areas of reliability methods and structural optimization (Kamat 1991). Part I of this two-part article reviews the theory involved with applying reliability methods and structural optimization to the initial design of ship hull structures, whereas Part II outlines the application of this theory to two ship structures: (1) a simple hull cross section and (2) a more complex ship hull titled “Energy Concentration.” In each case study, the objective of the analysis is to minimize weight, while ensuring that deterministic- and reliability-based constraints on ultimate moment and buckling capacities are satisfied. This demonstration will closely follow the format presented in the companion paper (Part I). The results of each case study are shown to validate the accuracy of the strength models with previously documented analytical results.

Simple Ship Structure

Selection of Initial Design

The initial design, taken from Mansour et al. (1997), is characterized by the principal dimensions shown in Fig. 1. Extra stiffeners are added to illustrate the concept of optimizing secondary stiffeners. The structure is constructed from steel, with a Young’s modulus of 206,000 MPa, a density of $7.85 \times 10^{-9}\ \mathrm{N \cdot s^2/mm^4}$, and a Poisson’s ratio of 0.30. The yield strength of the bottom and deck is 217.3 MPa, whereas that of the side shells is 276.5 MPa.

The Caldwell, modified Caldwell, Paik, and elastic strength models (Ayyub et al. 2015) were used to compute the ultimate strength of the initial design. Table 1 shows the results. It is noted from the strength analysis that the elastic strength model produces the lowest moment capacity, whereas the Caldwell model, which employs a totally plastic approach, produces the highest moment capacity. Table 2 shows the ultimate buckling capacities. A knockdown factor of 0.92 was used in the elastic strength model to account for buckling. The initial weight (per unit length/g) of the structure is $0.24179 \times 10^{-3}\ \mathrm{N \cdot s^2/mm^2}$. This is currently an acceptable design and will be optimized using the methodology presented by Ayyub et al. (2015).

Deterministic-Based Optimization of Initial Configuration

Definition of Design Variables

Some structural parameters, including plate thicknesses and stiffeners whose scantlings can be modified from those of the original design, are chosen as the design variables (Fig. 2 and Table 3). The current dimensions of the structural parameters constitute the initial design. To reflect practical realities, some parameters may be grouped and adjusted simultaneously in the design process. For example, a group may consist of a plate and three primary stiffeners. This group will be considered to have five design variables, i.e., plate thickness, web height, web thickness, flange width, and flange thickness, which are adjusted simultaneously in the

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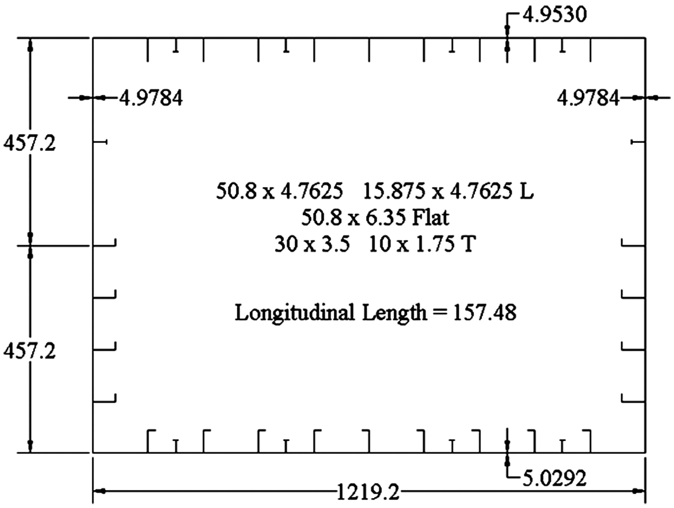

Fig. 1. Principle dimensions for initial design of simple ship structure in millimeters

Table 1. Ultimate Bending Moments for the Simple Ship Structure

Ultimate bending moment capacity model	Sagging (N · mm)	Hogging (N · mm)
Caldwell	2.4695×10^9	2.4695×10^9
Modified Caldwell	2.4340×10^9	2.4391×10^9
Paik	2.3061×10^9	2.3081×10^9
Elastic	1.9376×10^9	1.9376×10^9

Table 2. Ultimate Buckling Capacities for the Simple Ship Structure

Location	Ultimate (MPa)
Bottom	206.45
Deck	205.81
Port	191.83
Starboard	191.83

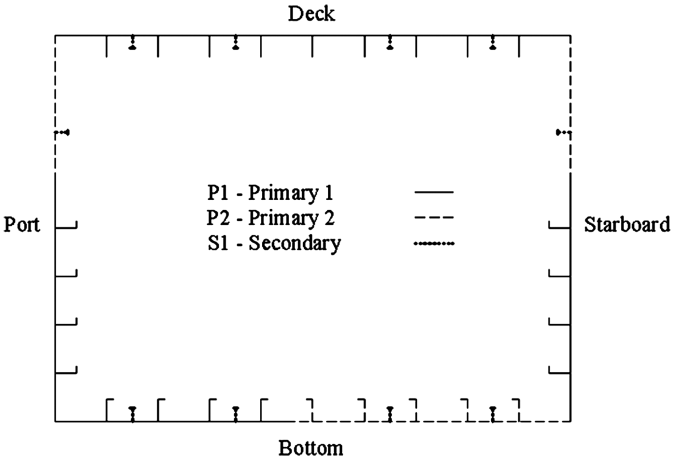

Fig. 2. Groups of design variables used for simple ship structure

optimization process. The final design after optimization in this instance will consist of three stiffeners, all with the same dimension. Fig. 2 shows the grouping of the design variables for this problem. Table 3 lists the upper and lower bounds on the 41 design variables. Once the design variables and objective/constraints are determined, the strategy developed by Ayyub et al. (2015) is employed to perform the optimization procedure.

Table 3. Deterministic Design Variables for the Simple Ship Structure

Component name	Design variables	Initial design (mm)	Design variable lower limit (mm)	Design variable upper limit (mm)
Bottom P1	Plate thickness	5.029	4.023	6.035
	Web length	50.800	40.640	60.960
	Web thickness	4.763	3.810	5.716
	Flange length	15.875	12.700	19.050
	Flange thickness	4.763	3.810	5.716
Bottom S1	Web length	30.000	24.000	36.000
	Web thickness	3.500	2.800	4.200
	Flange length	10.000	8.000	12.000
	Flange thickness	1.750	1.400	2.100
Bottom P2	Plate thickness	5.029	4.023	6.035
	Web length	50.800	40.640	60.960
	Web thickness	4.763	3.810	5.716
	Flange length	15.875	12.700	19.050
	Flange thickness	4.763	3.810	5.716
Deck P1	Plate thickness	4.953	3.962	5.944
	Web length	50.800	40.640	60.960
	Web thickness	6.350	5.080	7.620
Deck S1	Web length	30.000	24.000	36.000
	Web thickness	3.500	2.800	4.200
	Flange length	10.000	8.000	12.000
	Flange thickness	1.750	1.400	2.100
Port P1	Plate thickness	4.978	3.982	5.974
	Web length	50.800	40.60	60.90
	Web thickness	4.763	3.810	5.716
	Flange length	15.875	12.70	19.00
	Flange thickness	4.763	3.810	5.716
Port S1	Web length	30.000	24.00	36.00
	Web thickness	3.500	2.800	4.200
	Flange length	10.000	8.000	12.00
	Flange thickness	1.750	1.400	2.100
Port P2	Plate thickness	4.978	3.982	5.974
Starboard P1	Plate thickness	4.978	3.982	5.974
	Web length	50.800	40.60	60.90
	Web thickness	4.763	3.810	5.716
	Flange length	15.875	12.70	19.00
	Flange thickness	4.763	3.810	5.716
Starboard S1	Web length	30.000	24.00	36.00
	Web thickness	3.500	2.800	4.200
	Flange length	10.000	8.000	12.00
	Flange thickness	1.750	1.400	2.100
Starboard P2	Plate thickness	4.978	3.982	5.974

Table 4. Deterministic Objective and Constraints for the Simple Ship Structure

Optimization variable	Name and type
Objective	Minimize weight
Constraint 1	Moment capacity in sagging ≥1.00 initial
Constraint 2	Moment capacity in hogging ≥1.00 initial
Constraint 3	Ultimate buckling capacity of the deck ≥1.00 initial
Constraint 4	Ultimate buckling capacity of the bottom ≥1.00 initial
Constraint 5	Ultimate buckling capacity of the port ≥1.00 initial
Constraint 6	Ultimate buckling capacity of the starboard ≥1.00 initial

Definition of Objectives and Constraints for Deterministic Optimization

The optimization example consists of a single objective and six constraints, as defined in Table 4. The objective of the optimization

Table 5. Deterministic Optimization Objective/Constraints for the Simple Ship Structure

Optimization variable	Name and type	Initial	Final
Objective	Weight (per unit length/g) ($\mathrm{N \cdot s^2/mm^2}$)	0.2418×10^{-3}	0.2335×10^{-3}
Constraint 1	Sagging moment capacity (Paik, $\mathrm{N \cdot mm}$)	2.3061×10^9	2.3371×10^9
Constraint 2	Hogging moment capacity (Paik, $\mathrm{N \cdot mm}$)	2.3081×10^9	2.3301×10^9
Constraint 3	Ultimate buckling, deck (MPa)	205.81	207.72
Constraint 4	Ultimate buckling, bottom (MPa)	206.45	208.43
Constraint 5	Ultimate buckling, port (MPa)	191.83	191.83
Constraint 6	Ultimate buckling, starboard (MPa)	191.83	191.83

is to reduce the weight of the simple ship structure, while improving selected moment capacities and ultimate buckling strengths. Paik's elastic-plastic model is employed in this optimization analysis because it represents a nonextreme prediction of moment capacity.

Deterministically Optimal Configuration

Tables 5 and 6 present the results of the deterministic optimization. Table 5 presents a summary of the initial and optimal objective/constraint values, whereas Table 6 presents a summary of the initial and optimized design values. All strength/buckling strength constraints are satisfied and sometimes exceeded, whereas weight is reduced by 3.4%. Some design variables undergo significant change, whereas others remain close to their initial design values. The most sensitive design variables vital to the optimization analysis are those that undergo significant change, whereas nonsensitive slightly altered design variables are not as influential on the optimization analysis. An option may be to streamline the nonsensitive parameters in subsequent investigations.

Table 6. Deterministically Optimized Design Variables for the Simple Ship Structure

Component name	Design variable	Initial design (mm)	Optimal design (mm)
Bottom P1	Plate thickness	5.029	5.827
	Web length	50.800	51.722
	Web thickness	4.763	4.732
	Flange length	15.875	15.912
	Flange thickness	4.763	4.776
Bottom S1	Web length	30.000	29.909
	Web thickness	3.500	3.489
	Flange length	10.000	9.995
	Flange thickness	1.750	1.749
Bottom P2	Plate thickness	5.029	5.390
	Web length	50.800	50.710
	Web thickness	4.763	4.721
	Flange length	15.875	15.852
	Flange thickness	4.763	4.756
Deck P1	Plate thickness	4.953	5.944
	Web length	50.800	45.089
	Web thickness	6.350	5.204
Deck S1	Web length	30.000	29.179
	Web thickness	3.500	3.404
	Flange length	10.00	9.955
	Flange thickness	1.750	1.742
Port P1	Plate thickness	4.978	3.983
	Web length	50.800	40.648
	Web thickness	4.763	3.811
	Flange length	15.875	12.701
	Flange thickness	4.763	3.810
Port S1	Web length	30.000	24.001
	Web thickness	3.500	2.800
	Flange length	10.000	9.584
	Flange thickness	1.750	1.677
Port P2	Plate thickness	4.978	4.978
Starboard P1	Plate thickness	4.978	3.983
	Web length	50.800	40.648
	Web thickness	4.763	3.811
	Flange length	15.875	12.701
	Flange thickness	4.763	3.810
Starboard S1	Web length	30.000	24.001
	Web thickness	3.500	2.800
	Flange length	10.000	9.584
	Flange thickness	1.750	1.677
Starboard P2	Plate thickness	4.978	4.978

Because only the Paik model was employed in the optimization, a check is performed to ensure that the other strength models are not violated by the optimal design. A regular strength analysis using the suggested optimal design is performed, and Table 7 presents the results. Because all strength results have been either satisfied or improved, the goals of the deterministic optimization have been met successfully.

Reliability-Based Design Optimization

Definition of Initial Design

The deterministically optimal design results are used as the initial design for the reliability-based optimization, as described in the methodology presented by Ayyub et al. (2015).

Definition of Design Variables

Table 8 presents the initial design and upper/lower bounds of the design variables selected for this example. On the basis of the results of the deterministic optimization analysis, it is known that some design variables do not contribute significantly to the design. These variables have therefore been omitted, thus streamlining the list of variables to 23 for the reliability-based optimization. Again, the initial design in this case is taken as that predicted by deterministic optimization.

Definition of Random Variables

Uncertainties in material-, structural-, and load-based parameters are introduced to carry out a reliability-based analysis. Tables 9 and 10 define the random variables associated with these uncertainties.

Definition of Objectives and Constraints for Reliability-Based Optimization

The optimization problem consists of a single objective and eight constraints, as defined in Table 11. The goal is to ensure weight reduction, while at least maintaining the optimal deterministic moment, the optimal deterministic buckling capacities, and the reliability indices of moment capacities in the presence of uncertainties in loads and material properties.

Reliability-Based Optimal Configuration

Tables 12 and 13 summarize the results for the reliability-based optimization. Although Table 12 presents the initial and optimized objective/constraints, Table 13 displays the initial and optimized

Table 7. Ultimate Bending Moments for the Simple Ship Structure after Deterministic Optimization

Ultimate bending moment capacity model	Initial design		Final design	
	Sagging ($N \cdot mm$)	Hogging ($N \cdot mm$)	Sagging ($N \cdot mm$)	Hogging ($N \cdot mm$)
Caldwell	2.4695×10^9	2.4695×10^9	2.4693×10^9	2.4693×10^9
Modified Caldwell	2.4340×10^9	2.4391×10^9	2.4349×10^9	2.4438×10^9
Paik	2.3061×10^9	2.3081×10^9	2.3371×10^9	2.3301×10^9
Elastic	1.9376×10^9	1.9376×10^9	2.0289×10^9	2.0289×10^9

Table 8. Reliability-Based Optimization Design Variables for the Simple Ship Structure

Component name	Design variable	Initial design (mm)	Design variable lower limit (mm)	Design variable upper limit (mm)
Bottom P1	Plate thickness	5.827	4.662	6.993
Bottom P2	Plate thickness	5.390	4.312	6.467
Deck P1	Plate thickness	5.944	4.755	7.132
	Web length	45.089	36.071	54.107
	Web thickness	5.204	4.164	6.245
Port P1	Plate thickness	3.983	3.186	4.779
	Web length	40.648	32.510	48.778
	Web thickness	3.811	3.049	4.573
	Flange length	12.701	10.161	15.241
	Flange thickness	3.810	3.048	4.572
Port S1	Web length	24.001	19.201	28.801
	Web thickness	2.800	2.240	3.360
	Flange length	9.584	7.667	11.501
	Flange thickness	1.677	1.342	2.013
Starboard P1	Plate thickness	3.983	3.186	4.779
	Web length	40.648	32.518	48.778
	Web thickness	3.811	3.049	4.573
	Flange length	12.701	10.161	15.241
	Flange thickness	3.810	3.048	4.572
Starboard S1	Web length	24.001	19.201	28.801
	Web thickness	2.800	2.240	3.360
	Flange length	9.584	7.667	11.501
	Flange thickness	1.677	1.342	2.013

Table 9. Structure- and Load-Related Random Variables for the Simple Ship Structure

Random variable name	Mean	Coefficient of variation (COV)	Distribution
Modeling uncertainty for sagging moment	0.950	0.10	Normal
Modeling uncertainty for hogging moment	0.975	0.10	Normal
Modeling uncertainty for moment on port	1.025	0.10	Normal
Modeling uncertainty for moment on starboard	1.075	0.10	Normal
Wave load	0.900×10^9	0.10	Gumbel
Modeling uncertainty for wave load	1.020	0.01	Normal

Table 10. Random Variables Associated with Material for the Simple Ship Structure

Material identifier	Random variable name	Mean	COV	Distribution
Material 1	Young's modulus (MPa)	206,000	0.1	Lognormal
	Yield strength (MPa)	217.3	0.1	Lognormal
	Poisson's ratio	0.3	0.1	Lognormal
	Density ($N \cdot s^2/mm^4$)	7.85×10^{-9}	0.1	Lognormal
Material 2	Young's modulus (MPa)	206,000	0.1	Lognormal
	Yield strength (MPa)	276.5	0.1	Lognormal
	Poisson's ratio	0.3	0.1	Lognormal
	Density ($N \cdot s^2/mm^4$)	7.85×10^{-9}	0.1	Lognormal

Table 11. Reliability-Based Optimization Objectives and Constraints for the Simple Ship Structure

Optimization variable	Name and type
Objective	Minimize weight
Constraint 1	Moment capacity in sagging ≥1.00 initial
Constraint 2	Moment capacity in hogging ≥1.00 initial
Constraint 3	Ultimate buckling capacity of the deck ≥1.00 initial
Constraint 4	Ultimate buckling capacity of the bottom ≥1.00 initial
Constraint 5	Ultimate buckling capacity of the port ≥1.00 initial
Constraint 6	Ultimate buckling capacity of the starboard ≥1.00 initial
Constraint 7	Reliability index for safety margin of sagging moment ≥1.00 initial
Constraint 8	Reliability index for safety margin of hogging moment ≥1.00 initial

design variables. The optimal configuration satisfies all deterministic and reliability-based optimization goals, namely, weight reduction, subject to constraints on select moments, ultimate buckling capacities, and reliability indices. A weight reduction of 3.4% was previously achieved using purely deterministic optimization, whereas an additional 2.2% reduction was found through the reliability-based optimization for a combined weight savings of 5.6%.

Complex Ship Structure

Selection of Initial Design

An oil tanker was taken from Rutherford et al. (1990) and titled "Energy Concentration," and Table 14 gives the principal particulars. Fig. 3 shows the principal dimensions, and Table 15 presents the stiffener dimensions. The vessel is composed of two types of steel, both with an elastic modulus of $208,000\ N/mm^2$, a Poisson's ratio of 0.30, and a density of $7.85 \times 10^{-9}\ N \cdot s^2/mm^4$. The yield strength of the two metals is 235 MPa [mild steel (MS)] and 315 MPa [high tensile steel (HTS)].

The aged structure has corrosion to a depth of 1 mm on the plates and longitudinal webs and 2 mm on the stiffener flanges (Rutherford et al. 1990). This aged structure is used as a starting point for the analysis because several results published by Rutherford et al. may be used to validate the strength models.

Strength calculations were carried out on the initial design using the Caldwell, modified Caldwell, Paik, and elastic strength models (Ayyub et al. 2015). Table 16 presents the results. Table 17 shows the ultimate buckling capacities. A knockdown factor of 0.92 was

Table 12. Objective/Constraints before and after Reliability-Based Optimization for the Simple Ship Structure

Optimization variable	Name and type	Initial	Optimized
Objective	Weight (per unit length/g) $(\mathrm{N \cdot s^2/mm^2})$	0.2335×10^{-3}	0.2284×10^{-3}
Constraint 1	Sagging moment capacity $(\mathrm{N \cdot mm})$	2.3371×10^{9}	2.3606×10^{9}
Constraint 2	Hogging moment capacity $(\mathrm{N \cdot mm})$	2.3301×10^{9}	2.3571×10^{9}
Constraint 3	Ultimate buckling, deck (MPa)	207.72	208.84
Constraint 4	Ultimate buckling, bottom (MPa)	208.43	209.39
Constraint 5	Ultimate buckling, port (MPa)	191.83	191.83
Constraint 6	Ultimate buckling, starboard (MPa)	191.83	191.83
Constraint 5	Reliability index for safety margin of sagging moment	4.193	4.227
Constraint 6	Reliability index for safety margin of hogging moment	4.206	4.233

Table 13. Optimized Reliability-Based Design Variables for the Simple Ship Structure

Component name	Design variable	Initial design (mm)	Optimal design (mm)
Bottom P1	Plate thickness	5.827	5.760
Bottom P2	Plate thickness	5.390	5.862
Deck P1	Plate thickness	5.944	6.361
	Web length	45.089	45.975
	Web thickness	5.204	5.125
Port P1	Plate thickness	3.983	3.186
	Web length	40.648	32.518
	Web thickness	3.811	3.049
	Flange length	12.701	11.386
	Flange thickness	3.810	3.416
Port S1	Web length	24.001	23.127
	Web thickness	2.800	2.698
	Flange length	9.584	9.503
	Flange thickness	1.677	1.663
Starboard P1	Plate thickness	3.983	3.186
	Web length	40.648	32.523
	Web thickness	3.811	3.049
	Flange length	12.701	11.382
	Flange thickness	3.810	3.413
Starboard S1	Web length	24.001	23.108
	Web thickness	2.800	2.696
	Flange length	9.584	9.500
	Flange thickness	1.677	1.662

Table 14. Principal Particulars for the "Energy Concentration"

Name	Value
Overall length	326.75 m
Length between perpendiculars	313.0 m
Breadth, molded	48.19 m
Depth, molded	25.2 m
Draft, summer extreme	19.597 m
Gross tonnage	98,894 t
Deadweight	216,269 t

used in the elastic strength model to account for buckling. The initial weight (per unit length/g) of the structure is $0.5837 \times 10^{-01} (\mathrm{N \cdot s^2/mm^2})$.

Rutherford et al. (1990) published three hogging failure results for this corroded structure. Rutherford's first capacity of $1.7265 \times 10^{13}\ \mathrm{N \cdot mm}$ (no lateral pressure) is almost identical to the elastic result in this paper. Rutherford then applied a lateral pressure to the faces of the plates found on the bottom/side shells and reported a capacity of $1.7860 \times 10^{13}\ \mathrm{N \cdot mm}$ [$1.8522 \times 10^{13}\ \mathrm{N \cdot mm}$ using finite element analysis (FEA)], which is in the vicinity of the results in this paper on the basis of the Paik model. These close comparisons of results promote confidence in the strength calculations of this paper.

Deterministic-Based Optimization of Initial Configuration

Definition of Design Variables

This example is a very large problem, with 469 design variables. Appendix S1 gives a list of typical design variables with upper and lower bounds (±20%). Once the design variables have been defined, optimization objectives and constraints are determined.

Definition of Objectives and Constraints for Deterministic Optimization

Table 18 defines the single objective problem with nine constraints. The optimization objective/goal is to reduce the weight of the simple ship structure, while improving selected moment capacities and ultimate buckling strengths. The elastic-plastic Paik model is again employed in this deterministic optimization because it represents a nonextreme prediction of moment capacity. Once the design variables and optimization objectives/constraints have been defined, the Smart-Opt tool is employed to perform the optimization procedure.

Deterministically Optimal Configuration

Table 19 and Appendix S1 present the results of the deterministic optimization. Table 19 summarizes the objective and constraint results before and after optimization, whereas Appendix S1 gives the full output file for the optimization analysis. This complex problem met or exceeded all optimization goals, namely, weight reduction, subject to several constraints on the moment and buckling capacities. A weight reduction of 1.7% is achieved, whereas a slight increase was realized for all moment capacities and ultimate buckling strengths.

Some design variables are at the upper or lower limit and thus play a large role in the optimization process. Other design variables are less sensitive to optimization because they do not change much from their initial design values. Those design variables that do not contribute much to optimization may be streamlined in subsequent optimization analyses.

Because only the Paik model was employed in optimization, a check is performed to ensure that the other strength models are not violated by the optimal design. A regular strength analysis using the suggested optimal design is performed, and Table 20 presents the results. All of the strength results are satisfactory or improved; therefore, the goals of the deterministic optimization have been successfully met.

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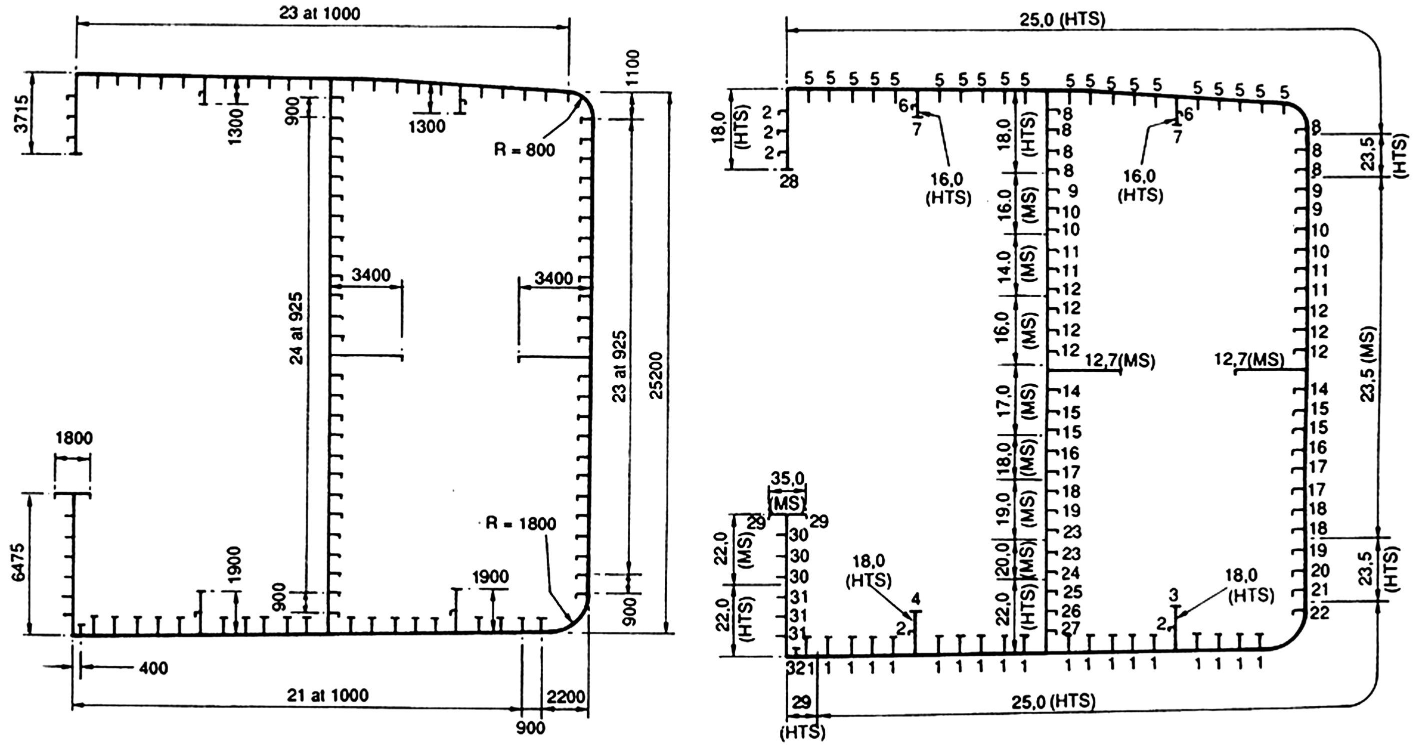

Fig. 3. Principal dimensions in millimeters for "Energy Concentration" (data from Rutherford et al. 1990)

Table 15. Stiffener Dimensions for the "Energy Concentration"

Stiffener number	Web (mm)	Flange (mm)	Plate used (mm)	Steel
1	797 × 15	20 × 33	825 × 22.49	HTS
2	297 × 11.5	100 × 16	350 × 14.33	HTS
3	370 × 16	—	630 × 26.23	HTS
4	425 × 25	—	630 × 26.23	HTS
5	480 × 32	—	480 × 32	HTS
6	297 × 11.5	100 × 16	450 × 24.41	HTS
7	370 × 16	—	450 × 24.41	HTS
8	447 × 11.5	125 × 22	500 × 15.78	HTS
9	549 × 11.5	125 × 22	600 × 15.1	MS
10	597 × 11.5	125 × 22	650 × 14.79	MS
11	597 × 11.5	125 × 22	650 × 15.37	MS
12	647 × 11.5	125 × 22	700 × 15.09	MS
13	350 × 25.4	—	350 × 25.4	MS
14	647 × 12.7	150 × 25	700 × 17.1	MS
15	697 × 12.7	150 × 25	700 × 18	MS
16	747 × 12.7	150 × 25	800 × 16.54	MS
17	747 × 12.7	180 × 25	800 × 17.48	MS
18	797 × 14	180 × 25	850 × 18.42	MS
19	847 × 14	180 × 25	900 × 18.18	MS
20	847 × 14	180 × 32	900 × 19.58	MS
21	847 × 15	180 × 25	900 × 19.12	HTS
22	847 × 15	180 × 32	900 × 20.52	HTS
23	897 × 15	200 × 25	950 × 19.43	MS
24	945 × 16	200 × 25	950 × 21.18	MS
25	897 × 15	200 × 25	950 × 19.43	HTS
26	797 × 15	180 × 25	950 × 17.32	HTS
27	347 × 11.5	125 × 22	1,000 × 7.91	HTS
28	397 × 25	—	397 × 25	HTS
29	300 × 35	—	300 × 35	MS
30	230 × 12.7	—	230 × 12.7	MS
31	230 × 12.7	—	230 × 12.7	HTS
32	397 × 11.5	100 × 25	450 × 28.47	HTS

Reliability-Based Design Optimization: Case II

Definition of Initial Design

The methodology, presented in the guidelines developed by Ayyub et al. (2015), endeavors to initially optimize a structure using deterministic optimization techniques and then to use these deterministic results as a starting point for reliability-based optimization. Therefore, the initial design dimensions for this reliability-based optimization are on the basis of the deterministic optimization results.

Definition of Design Variables

Appendix S2 presents the initial design variables, along with their upper/lower bounds. The number of design variables has been streamlined from the deterministic optimization results to reflect the fact that some variables did not contribute significantly to the design. The deterministic optimization analysis used 469 design variables, of which only 57 were considered high contributors to the optimized result. Therefore, only these 57 design variables will be used in the reliability-based optimization. Recall that the deterministically predicted optimal design is used as the initial design for reliability-based optimization.

Definition of Random Variables

Uncertainty in material-, structural- and load are introduced to perform a reliability analysis. Tables 21 and 22 define the random variables associated with these uncertainties.

Definition of Objectives and Constraints for Reliability-Based Optimization

Table 23 defines the objective and constraints for this reliability-based optimization. The goal is to reduce the structural weight, while maintaining the optimal moment and ultimate buckling capacities predicted previously by the deterministic optimization

Table 16. Ultimate Bending Moments for the "Energy Concentration"

Ultimate bending moment capacity model	Sagging (N · mm)	Hogging (N · mm)
Caldwell	2.0013×10^{13}	2.0013×10^{13}
Modified Caldwell	1.8406×10^{13}	1.9475×10^{13}
Paik	1.7462×10^{13}	1.8111×10^{13}
Elastic	1.7231×10^{13}	1.7231×10^{13}

Table 17. Ultimate Buckling Capacities for the "Energy Concentration"

Location	Ultimate (MPa)
Bottom	274.04
Deck	246.58
Port	199.52
Vertical 1	189.38
Vertical 2	174.82
Vertical 3	189.38
Starboard	199.52

Table 18. Deterministic Objective and Constraints for the "Energy Concentration"

Optimization variable	Name and type
Objective	Minimize weight
Constraint 1	Moment capacity in sagging ≥1.00 initial
Constraint 2	Moment capacity in hogging ≥1.00 initial
Constraint 3	Ultimate buckling capacity of the deck ≥1.00 initial
Constraint 4	Ultimate buckling capacity of the bottom ≥1.00 initial
Constraint 5	Ultimate buckling capacity of the port ≥1.00 initial
Constraint 6	Ultimate buckling capacity of Vertical 1 ≥ 1.00 initial
Constraint 7	Ultimate buckling capacity of Vertical 2 ≥ 1.00 initial
Constraint 8	Ultimate buckling capacity of Vertical 3 ≥ 1.00 initial
Constraint 9	Ultimate buckling capacity of the starboard ≥1.00 initial

and improving reliability indices for all moment capacities in the presence of uncertainties in load, strength, and material.

Reliability-Based Optimal Configuration

Table 24 and Appendix S2 present the results from the reliability-based optimization. Table 23 summarizes the change in the objective and constraints, whereas Appendix S2 contains the full output file from the optimization process. The optimal configuration

Table 21. Random Variables Associated with Structural and Load Uncertainties for the "Energy Concentration"

Random variable	Initial	COV	Distribution
Modeling uncertainty for sagging moment	1.000	0.125	Normal
Modeling uncertainty for hogging moment	1.000	0.05	Normal
Modeling uncertainty for moment on port	0.900	0.15	Normal
Modeling uncertainty for moment on starboard	1.150	0.03	Normal
Stillwater load	2.3×10^{12}	0.10	Normal
Modeling uncertainty for stillwater load	1.000	0.10	Normal
Wave load	3.3×10^{12}	0.10	Gumbel
Modeling uncertainty for wave load	1.000	0.10	Normal

Table 22. Random Variables Associated with Material for the "Energy Concentration"

Material identifier	Random variable	Initial	COV	Distribution
Material 1	Young's modulus (MPa)	208,000	0.05	Lognormal
	Yield strength (MPa)	235	0.05	Lognormal
	Poisson's ratio	0.30	0.05	Lognormal
	Density $(\mathrm{N \cdot s^2/mm^4})$	7.85×10^{-9}	0.05	Lognormal
Material 2	Young's modulus (MPa)	208,000	0.05	Lognormal
	Yield strength (MPa)	315	0.05	Lognormal
	Poisson's ratio	0.30	0.05	Lognormal
	Density $(\mathrm{N \cdot s^2/mm^4})$	7.85×10^{-9}	0.05	Lognormal

Table 19. Optimal Deterministic Design Configuration for the "Energy Concentration"

Optimization variable	Name and type	Initial	Final
Objective	Weight $(\mathrm{N \cdot s^2/mm^2})$	5.837×10^{-2}	5.739×10^{-2}
Constraint 1	Moment capacity in sagging (N · mm)	1.746×10^{13}	1.775×10^{13}
Constraint 2	Moment capacity in hogging (N · mm)	1.811×10^{13}	1.843×10^{13}
Constraint 3	Ultimate buckling capacity of the deck (MPa)	274.04	273.82
Constraint 4	Ultimate buckling capacity of the bottom (MPa)	246.58	249.05
Constraint 5	Ultimate buckling capacity of the port (MPa)	199.52	200.12
Constraint 6	Ultimate buckling capacity of Vertical 1 (MPa)	189.38	188.84
Constraint 7	Ultimate buckling capacity of Vertical 2 (MPa)	174.82	175.79
Constraint 8	Ultimate buckling capacity of Vertical 3 (MPa)	189.38	188.85
Constraint 9	Ultimate buckling capacity of the starboard (MPa)	199.52	200.11

Table 20. Ultimate Bending Moments for the "Energy Concentration"

Ultimate bending moment capacity model	Initial design		Optimal design	
	Sagging (N · mm)	Hogging (N · mm)	Sagging (N · mm)	Hogging (N · mm)
Caldwell	2.0013×10^{13}	2.0013×10^{13}	2.0028×10^{13}	2.0028×10^{13}
Modified Caldwell	1.8406×10^{13}	1.9475×10^{13}	1.8729×10^{13}	1.9519×10^{13}
Paik	1.7462×10^{13}	1.8111×10^{13}	1.7747×10^{13}	1.8425×10^{13}
Elastic	1.7231×10^{13}	1.7231×10^{13}	1.7970×10^{13}	1.7970×10^{13}

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Table 23. Reliability-Based Objective and Constraints for the "Energy Concentration"

Optimization variable	Name and type
Objective	Minimize weight
Constraint 1	Reliability index for safety margin of sagging moment ≥1.00 initial
Constraint 2	Reliability index for safety margin of hogging moment ≥1.00 initial
Constraint 3	Moment capacity in sagging ≥1.00 initial
Constraint 4	Moment capacity in hogging ≥1.00 initial
Constraint 5	Ultimate buckling capacity of the deck ≥1.00 initial
Constraint 6	Ultimate buckling capacity of the bottom ≥1.00 initial
Constraint 7	Ultimate buckling capacity of the port ≥1.00 initial
Constraint 8	Ultimate buckling capacity of Vertical 1 ≥ 1.00 initial
Constraint 9	Ultimate buckling capacity of Vertical 2 ≥ 1.00 initial
Constraint 10	Ultimate buckling capacity of Vertical 3 ≥ 1.00 initial
Constraint 11	Ultimate buckling capacity of the starboard ≥1.00 initial

Table 24. Optimal Reliability-Based Design Configuration for the "Energy Concentration"

Optimization variable	Initial	Final
Objective ($N \cdot s^2/mm^2$)	5.739×10^{-2}	5.722×10^{-2}
Constraint 1	5.394	5.388
Constraint 2	5.295	5.295
Constraint 3 ($N \cdot mm$)	1.7747×10^{13}	1.7812×10^{13}
Constraint 4 ($N \cdot mm$)	1.8425×10^{13}	1.8424×10^{13}
Constraint 5 (MPa)	273.82	273.82
Constraint 6 (MPa)	249.05	251.59
Constraint 7 (MPa)	200.12	200.31
Constraint 8 (MPa)	188.83	188.83
Constraint 9 (MPa)	175.79	175.79
Constraint 10 (MPa)	188.85	188.84
Constraint 11 (MPa)	200.11	200.21

satisfied all deterministic- and reliability-based optimization goals, namely, weight reduction, subject to constraints on moment, ultimate buckling capacities, and reliability indices. The final optimal design weight of $0.05722\ N \cdot s^2/mm^2$ represents a 2.0% weight reduction. The deterministic optimization reduced the weight by 1.7%, whereas the reliability-based optimization further reduced the weight by 0.3%.

Conclusions

This article has detailed the application of an innovative deterministic- and reliability-based optimal design strategy to two ship structures in an attempt to achieve weight reduction, while imposing a number of constraints on the moment and buckling capacities. The associated theory was presented in an accompanying paper (Part I). The methodology was applied to two case studies: (1) a simple ship structure and (2) a more complex vessel. Deterministic optimization of the simple structure was found to reduce its weight by 3.4%. A further weight reduction of 2.2% was found by performing a reliability-based optimization process, giving a total weight reduction of 5.6%. A more complex ship structure, titled "Energy Concentration," was then investigated. After validation of the strength calculation with previously documented results, deterministic optimization was performed using a total of 469 design variables, which reduced the vessel's weight by 1.7%. The most influential design variables were then used in the reliability-based optimization analysis, which further reduced the weight by 0.30%, giving a total weight loss of 2.0% was for this complex ship structure. These results emphasize the potential benefits offered through the application of reliability methods and structural optimization techniques, and encourage their implementation during initial design.

Supplemental Data

Appendixes S1 and S2 are available online in the ASCE Library (www.ascelibrary.org).

References

Ayyub, B. M., Akpan, U. O., Koko, T. E., and Dunbar, T. (2015). "Reliability-based optimal design of steel box structures. I: Theory." *ASCE-ASME J. Risk Uncertainty Eng. Syst., Part A: Civ. Eng.*, in press.

Kamat, M. P. (1991). *Structural optimization: Status and promise*, American Institute of Aeronautics and Astronautics, Reston, VA.

Mansour, A., Wirsching, P., Lucket, P., and Plumpton, A. (1997). "Assessment of reliability of ship structures." *SSC-398*, Ship Structure Committee.

Rutherford, S. E., and Caldwell, J. B. (1990). "Ultimate strength of ships: A case study." *SNAME Trans.*, 98, 441–471.