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RELIABILITY METHODS FOR SHIP STRUCTURES

THE AUTHORS

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ABSTRACT

The ever increasing use of high-strength materials and advanced technologies in surface ship structural design requires a very careful and systematic analysis to insure that levels of safety are maintained. This is much more easily said than done. The application of new technologies will not allow extrapolation of existing design criteria. Due to the uncertainties involved with future loading conditions, material properties, quality of workmanship in construction, and the limitations in numerical methods of analysis, the absolute safety of a structure cannot be established. It would therefore seem appropriate to use methods of analysis which would attempt to account for the various uncertainties and which allow the designer to limit the risk of unacceptable consequences. Estimation of this risk, even if used only to compare design alternatives, can be a useful and economical tool.

Structural reliability has its roots in the fields of civil and aerospace engineering and has made great strides forward in the last decade. Many methods have been proposed to evaluate the risk of structural failure. These methods include: first order second moment (FOSM), advanced second moment (ASM), and Monte Carlo simulation using both conditional expectation and antithetic variates techniques for variance reduction. All of these methods consider the type of the problem, the various parameters involved, and the uncertainties associated with these parameters. In estimating the risk, the uncertainties are modeled as random variables with mean values, variance, and probability density and distribution functions. Each method uses this information in a different manner, involving some assumptions and limitations.

In this paper we evaluate the available methods as to their suitability for estimating the risk of structural failure in ships. The merits and shortcomings of each method are discussed and each is then used to solve a simple example problem. The most effective method is chosen for more advanced work in this field.

INTRODUCTION

The structural weight of naval surface combatants constitutes approximately 35% of the total lightship displacement, making hull structures the heaviest of all ship subsystems. Any improvements in vessel capability through growth of the mission related payload will necessitate an equivalent reduction in weight in some other subsystem. Because of the proportionally low cost of the hull subsystem, improvements can be made without drastically increasing total vessel cost as shown in Figure 1.

How can structural weight be reduced? The use of new materials and technologies can provide a means of making the vessel lighter. But what about vessel strength? Current design criteria will become invalid or unrealistic as new materials and technologies are introduced. This is because the design criteria are typically codified in the form of simple equations or charts which are meant for a particular application. These usually contain some empirically derived factors of safety which may not be evident to the user. The use of these criteria can possibly lead to overdesigning the structure or, worse, underdesigning it without proper estimation of safety.

Consequently, improved design criteria and analysis methods need to be developed. These methods should be capable of handling the new technologies and materials as well as existing ones. Since the loading of a ship structure is mainly the sea, a truly random system, the most appropriate new method should be one which takes into account the randomness of both loading and structural properties to estimate the risk of unacceptable response. Many reliability or risk estimation methods have been proposed [1 to 19] which consider these parameters. Uncertainties are modeled in terms of the mean, the variance, and the probability density and distribution functions. The limitations and assumptions involved in each method are a result of how that method uses part or all of the statistical information available.

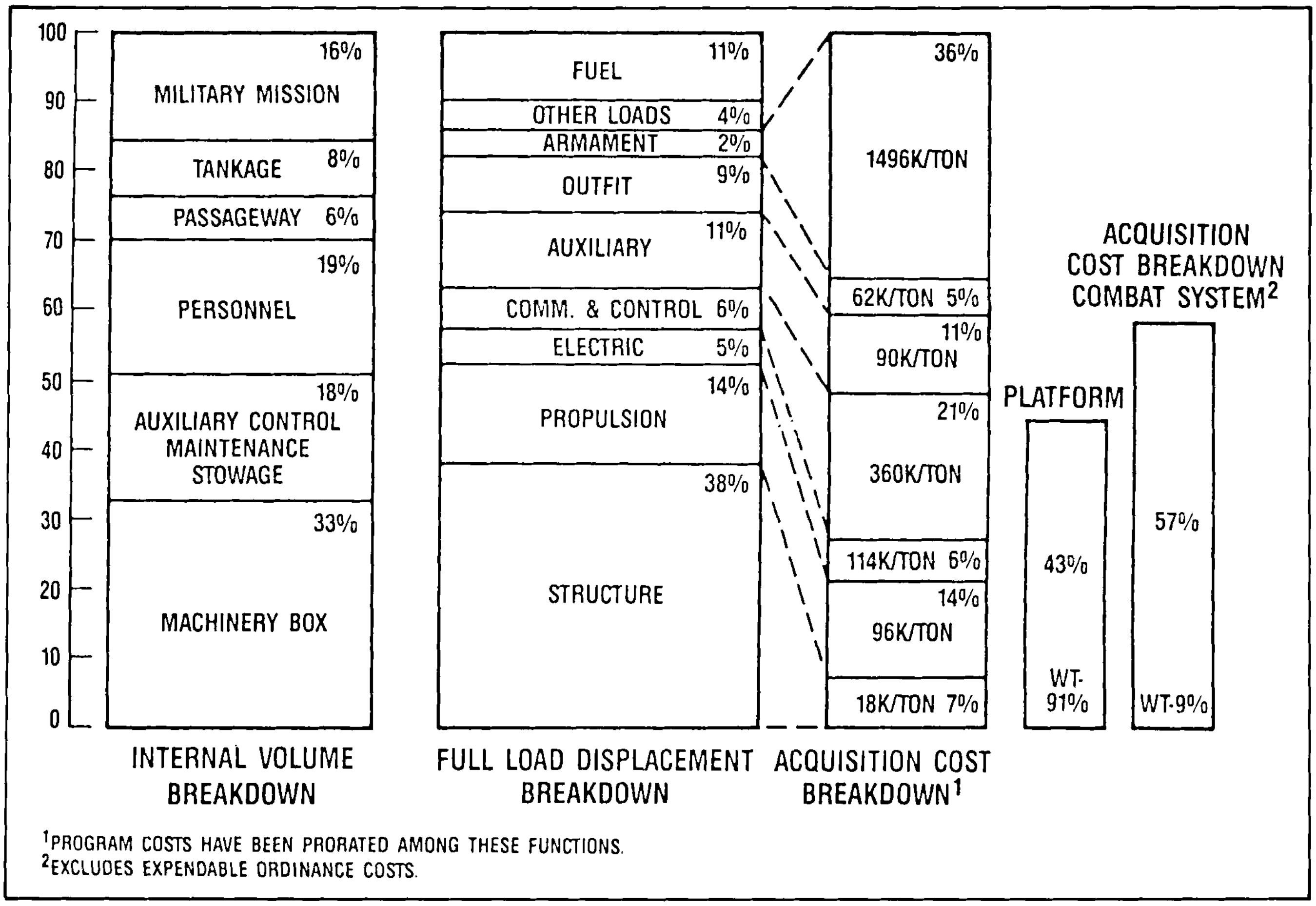

Figure 1. Functional breakdown of notional surface combatant.

The aim of this paper is to evaluate the practical available reliability methods and determine the most suitable for use in ship structures.

STRUCTURAL RELIABILITY

The definition of "reliability" according to Webster is "suitable or fit to be relied upon; worthy of dependence or reliance . . . suggests consistent dependability of judgment, character, performance, or results." Structural reliability is, then, how well some structural system will perform with regard to a given expectation. This can most easily be shown using the laws of probability. The reliability of a structure is the probability that the structure will survive for a given period of time and/or under specified loading conditions. For a structural system this can be stated as:

$$\text{Probability of survival} = P_s = P(\text{Strength} \geq \text{Load})$$
or
$$\text{Probability of failure} = P_f = 1 - P_s = P(\text{Strength} < \text{Load})$$

Ideally, structures are designed so that the strength will always exceed the load, but realistically there is always some finite chance of failure. For generations, the "safe" design of a structure was accomplished by estimating the maximum load the structure could expect to experience in its lifetime, multiplying that by some constant (factor of safety), and *requiring* the strength to equal that value.

A typical expression would be:

$$\text{Max Load}_{\text{Life}} \times \text{Factor of Safety} = \text{Design Strength}$$

The factor of safety is empirically arrived at to hopefully account for all of the uncertainties involved in the design and construction of the structure. Typically, this factor was tempered by experience with past successful structures and acceptable standards of the day. Many shipbuilding and land construction codes are still based on this approach.

In this country, the idea of treating both the load and the strength as random variables was first proposed by Freudenthal [1, 2] and later improved upon by others [3 to 9]. This idea allows one to then treat the problem of reliability using statistical methods. In order to perform a reliability analysis, a mathematical model is first derived which relates the load and resistance/strength parameters, called basic variables. This relationship is expressed in the form of a limit-state equation, foı instance:

$$Z = g(X_1, X_2, \ldots, X_n) \quad (1)$$

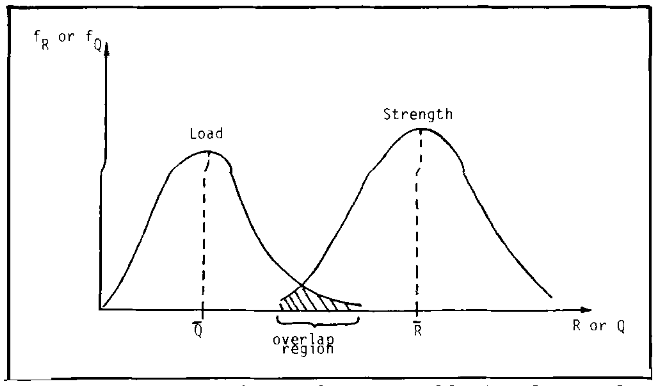

Figure 2. Probability density functions of load and strength.

where the X_i's are the basic variables; the load and strength random variables.

Equation (1) defines the "failure" surface, such that failure occurs for g ( ) < 0. Here failure can be defined by the collapse, fracture, buckling, or just loss of serviceability of the structure. Then the probability of failure can be expressed as:

$$P_f = \int \dots \int f_x (x_1, x_2, \dots x_n)\, dx_1, dx_2, \dots dx_n \quad (2)$$

where f_x is the joint probability density function of X_1, X_2 ..., and the integration is performed on the region where g ( ) < 0. For a typical problem determining the probability density function (PDF) of a single basic variable may be difficult, let alone finding a joint PDF. Consequently, one of the assumptions typically made to make equation (2) more tractable is that the basic variables are statistically independent.

Consider the case mentioned earlier, where the limit-state equation can be defined by just two basic variables in a linear combination.

$$g(x) = R - Q$$

where R and Q are the resistance and load effects.

For a limit-state equation in this form, the failure event becomes R - Q < 0 and the probability of failure is

$$P_f = P(R < Q) = \int F_R(x)\, f_Q(x)\, dx \quad (3)$$

where F_R is the cumulative distribution function of R and f_Q is the probability density function of Q.

Assuming that the load and strength are statistically independent random variables, they could be shown on a frequency diagram as in Figure 2. The probability of failure is represented by (however, not equal to) the overlap region [14]. This region is affected by the relative position of the distributions, that is the mean values of the strength and load. Reducing the mean value of the strength increases the overlap and thus the probability of failure (Figure 3). Changing the shape of the distributions by changing the variance about the mean value will also affect the probability of failure (Figure 4). It would, therefore, stand to reason that a measure of the reliability could be calculated from the

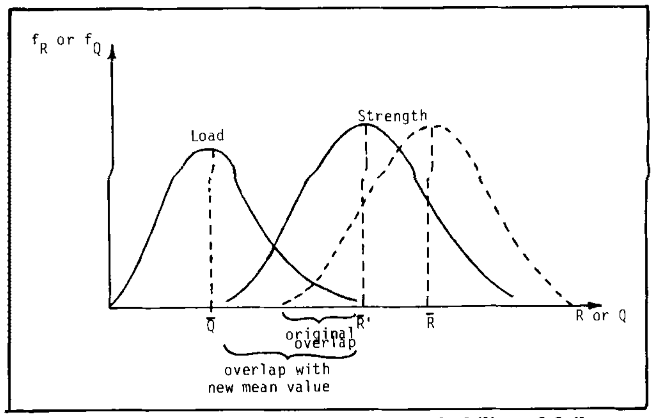

Figure 3. Effect of mean values on probability of failure.

mean and variance (the first two moments) of the random variables. This idea is carried forward in some of the methods which are discussed next.

The concept of structural reliability has been applied to ships principally by Mansour and Fallkner [6, 10, 11, 12, 13]. Their work in the last decade has been instrumental in helping the classification societies and navies of their respective countries begin investigations into modifying existing design criteria to include a probabilistic type of analysis. One of the methods they proposed [11] will be discussed later.

RELIABILITY METHODS

The various reliability methods can be classified conveniently according to the manner in which they deal with the statistical properties of the basic variables. This classification is common among most engineering disciplines [8, 14].

Level 1 or First Moment Methods

These methods typically employ only an estimated or nominal value in the analysis of the reliability of a structural event. The measure of reliability usually is a factor of safety in a form such as:

$$m = R/Q$$

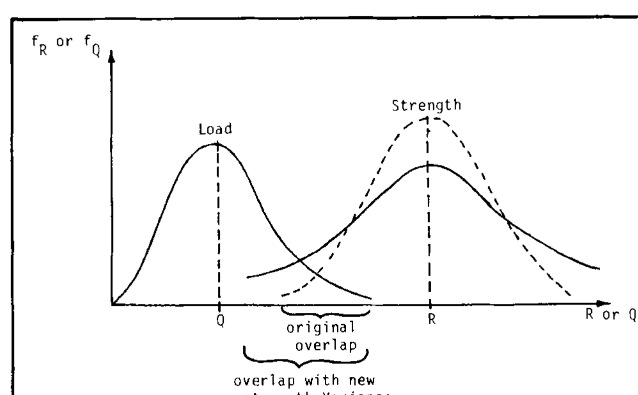

Figure 4. Effect of variance on probability of failure.

where m is the factor of safety; R is the nominal resistance; Q is the working load.

As mentioned earlier these factors of safety are conventionally arrived at based upon past experience and the intuition of the designer. The obvious disadvantage of this method is that it does not account for the uncertainties in the load and strength parameters. The failure probability is also dependent upon the shape of the density function of each basic variable as shown in Figure 1. This method does not include the different shapes (or uncertainties) in the analysis. Admittedly, when the mean values are separated far enough (if the safety factor is large enough) the variance of the basic variables becomes much less important. Unfortunately, this leads to a structure which might be typically expensive and inefficient.

Level 2 or Second Moment Methods

These methods make use of the knowledge of the first moments of the random variable, i.e., the mean and variance. The resulting margin of safety is still a scalar quantity, and these methods are therefore considered semiprobabilistic. That is to say that they combine statistics and determinism. These methods also vary in their sophistication in estimating the probabilistic nature of the basic variables. Two methods will be discussed here.

First Order Second Moment

This is one of the oldest methods, dating from the 1950s. The approximate mean and variance of Z in equation (1) can be estimated by expanding g(x) in a Taylor series about the mean value of the X_i's and truncate the series at the linear terms [5, 15, 16]. This will provide the first order approximate mean and variance of Z:

$$Z = g(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_3) \tag{4}$$

$$\sigma_{\bar{z}}^2 = \left[\sum_{i=1}^{m} \sum_{y=1}^{n} \left(\frac{\delta_g}{\delta X_i}\right) \left(\frac{\delta_g}{\delta X_j}\right) \mathrm{COV}(X_i, X_j) \right] \tag{5a}$$

For statistically independent basic variables, the variance is given by:

$$\sigma_z^2 \cong \left[\sum_{i=1}^{n} \sigma_{xi}^2 \left(\frac{\delta g}{\delta X_i}\right)^2 \right] \tag{5b}$$

where $\bar{z}$ is the mean value of Z; σ_z is the standard deviation of Z.

The partial derivatives of g are evaluated at the mean values of all of the parameters. A second order mean and variance can be calculated by including the next higher level terms in the Taylor series expansion. This is commonly done for the mean value but not for the variance because the procedure is much more involved. A measure of safety can be estimated in terms of the reliability index β^* from the knowledge of the mean and variance of Z

$$\beta^* = \frac{\bar{Z}}{\sigma_{\bar{Z}}} \tag{6}$$

β^* is, in fact, just the reciprocal of the coefficient of variation of Z. If Z is assumed to be normally distributed then it can be readily shown that the probability of failure, P_f, is

$$P_f = 1 - \Phi(\beta^*) \tag{7}$$

where Φ is the cumulative distribution function of the standard normal variate which is tabulated in many books on probability and statistics [15, 16].

The weaknesses of this method lie in the linearization of the expansion of equation (1). When g( ) is nonlinear, neglecting the higher order terms may, in fact, introduce significant errors. These errors are a result of choosing the mean value from equation (4) as the linearization point. It is possible, depending on the function g( ), for this mean point to not lie on the failure surface. Additionally, the reliability of index β^* will not be the same when different, though mechanically equivalent, formulations of the same problem are used, e.g., stress versus bending moment formulation [9, 14]. This is to say that equations (4, 5, 6, 7) will only give correct results when the function g( ) is linear and Z is normally distributed.

Advanced Second Moment

As a result of dissatisfaction with the first order second moment method, several researchers [5, 17] proposed improvements to that method. Here the Taylor series expansion of g( ) is linearized at some point on the failure surface called the design or checking point, e.g. (X_1^*, X_2^* .. X_3^*). This point is established by transforming the basic variables X_i to a set of reduced, uncorrelated variables with zero mean and unit variance X_i^*. The transformation is as follows:

$$X_i^* = \frac{X_i - \bar{x}_i}{\sigma_{x_i}} \tag{8}$$

The safe state and the failure state can now be shown, separated by the failure surface, in the space of the reduced coordinates in Figure 5. The equation describing the failure surface can now be written in terms of the reduced coordinates as

$$g(\sigma_{x_1} X_1^* + \bar{x}_1, \sigma_{x_2} X_2^* + \bar{x}_2, \ldots, \sigma_{x_n} X_n^* + \bar{x}_n) = 0 \tag{9}$$

One can see, by looking at Figure 5, that, as the failure surface moves closer to or farther from the origin, the size of the safe region increases or decreases accordingly. Therefore, the position of the failure surface with respect to the origin of the reduced coordinates could be used as a measure of the reliability or safety. Hasofer and Lind [17] have shown that β is the minimum distance between the failure surface and the origin. It

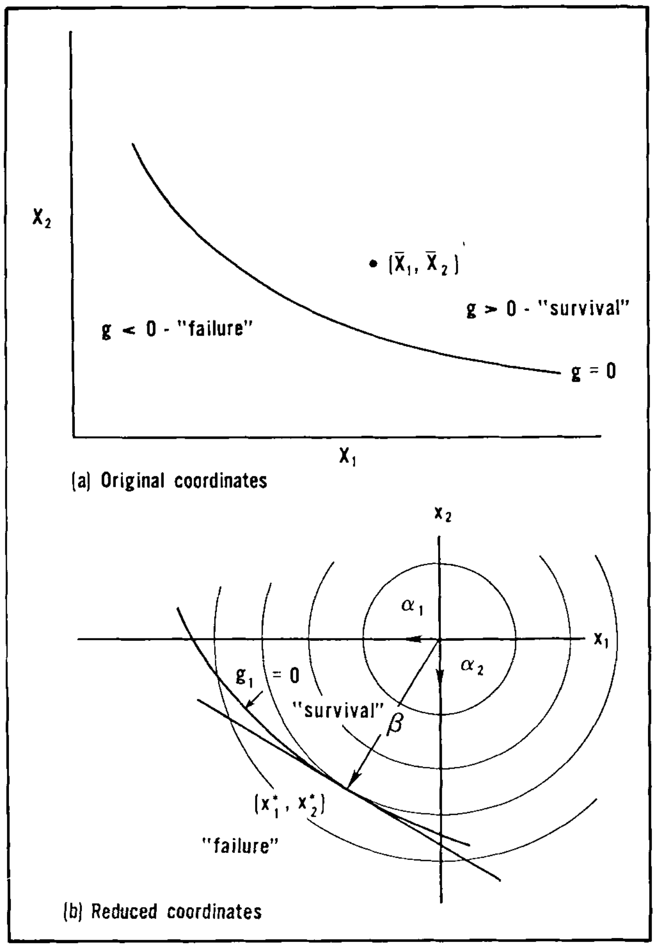

Figure 5. Formulation of safety analysis in both original and reduced coordinates.

has been further shown [18] that the point on the failure surface at the minimum distance from the origin is the most probable failure point. The object of the advanced second moment method is, then, to iterate until converging upon a minimum value of β by appropriately choosing the checking points on the failure surface and performing a first order analysis. The following system of equations represent this iteration [9]:

$$\alpha_i = \frac{\left(\frac{\delta g}{\delta X_i}\right) \sigma_{x_i}}{\left[\sum_{i=1}^{m} \left(\frac{\delta g}{\delta X_i}\right)^2 \sigma_{x_i}^2\right]^{1/2}} \tag{10}$$

$$X_i^* = \bar{x}_i - \alpha_i \beta \sigma_{x_i} \tag{11}$$

$$g(X_1^* X_2^* ,\ldots, X_3^*) = 0 \tag{12}$$

where the partial derivatives $\delta g/\delta x_i$ are evaluated at the X_i^*; α_i is the direction cosine of the basic variable, X_i. The relationship between the probability of failure P_f and β remain the same as in equation (7) as long as all of the basic variables are normally-distributed.

The case where the distributions of the basic variables X_i are non-normal may also be handled with this method. This procedure is to evaluate β using equivalent normal distributions [9, 19]. These are normal distributions with the same cumulative distribution function and probability density functions at the checking point $(X_1^*, X_2^* ,\ldots X_3^*)$ as the actual distributions for the basic variables. The mean value and standard deviation of the equivalent distribution at the checking point on the failure surface are given by

$$\sigma_{x_i}^n = \frac{\phi\,\{\Phi^{-1}\,[F_i\,(X_i^*)]\}}{f_i\,(x_i^*)} \tag{13}$$

$$\bar{x}_i^n = x_i^* - \Phi^{-1}\,[F_i\,(x_i^*)]\,\sigma_{x_i}^n \tag{14}$$

where F_i and f_i are the non-normal distribution and density functions of X_i; $\phi(\ )$ is the density function of the standard normal variate. These two equations (13, 14) would precede equations (10, 11, 12) in the iterative procedure described above.

The principal difficulty with the advanced second moment method lies in the complicated system of equations which must be developed and solved. As the number of basic variables increases, the solution process becomes long and the number of iterations required for a solution increases. In addition, it is no simple task to automate the process for a general form of limit-state equation, principally because of the partial derivatives in equation (10). The process is capable of estimating solutions for nonlinear equations as well as equations with correlated basic variables [19]. However, the accuracy of the solution and convergence of the algorithm depends on the behavior of the limit-state equation in the vicinity of the origin. If there are several local minimum distances to the origin, the solution process may not converge on the global minimum. Finally, the probability of failure is calculated from β using equation (7). Again this will only be an approximation because equation (7) assumes normally-distributed variables.

Level 3 or Probabilistic Methods

Level 3 methods are those in which calculations are made to determine the "exact" probability of failure of a structural element, making full use of a full probabilistic description of the basic variables. This can be done exactly if the joint PDF of the basic variables in equation (2) is known. In many cases even if the joint PDF is known the solution to equation (2) must be numerically approximated. An alternative means would be to use Monte Carlo methods to simulate the random variables and calculate the probability of failure from the limit-state equation.

"Exact" Method

Mansour [6, 10, 11] and Faulkner [11] have applied this "exact" approach to the problem of the longitudinal strength of ships. The word exact is in quotes because even though a correct form of equation (2) is developed for the problem, the solution to the equation requires numerical approximation. In this case a limit-state equation of the following form was assumed

$$R - Zn = g(x)$$

where R is the strength of the ship for the particular failure mode investigated; Zn is the total extreme bending moment. The strength is assumed to be normally distributed and the bending moment to be made up of a deterministic still water component and a wave component which follows a Weibul distribution. If statistical independence between load and strength is assumed, as commonly done in engineering, the probability of failure is given as

$$P_f = 1 - \int_0^{\infty} \Phi_z(x)\, f_R(x)\, dx \qquad (15)$$

where f_R is probability density function of the load. Including the equations for the cumulative distribution function of a standard normal distribution Φ_z and the PDF of a Weibul distribution for the load, equation (15) becomes

$$P_f = 1 - \frac{1}{\sigma\sqrt{2\pi}} \int_{m_o}^{\infty} \{1 - EXP[-(Z-m_o/k)^{\ell}]\}^N \cdot EXP\left[-1/2\left(\frac{Z-m_o}{\sigma}\right)^2\right] dz \qquad (16)$$

where N is the number of encounters or records used to find mean value of the wave bending moment; m_o is the value of the deterministic still water bending moment; μ and σ are the mean and standard deviations of the strength; k and ℓ are the Weibul characteristic value and distribution parameters.

A close-form solution of equation (16) in this form is not possible. Consequently, Mansour [6] presented the following approximation to the "exact" solution for a long-term (ships lifetime) analysis.

$$P_f = [1 - \Phi\left(\frac{\mu-m_o}{\sigma}\right)] + EXP\left[\frac{-(\mu-m_o)}{\sigma} + \frac{\sigma^2}{2\lambda^2}\right] \cdot \Phi\left(\frac{\mu-m_o}{\sigma} - \frac{\sigma}{\lambda}\right) \qquad (17)$$

Here the Weibul characteristics value $k = \lambda$, the expected value of the extreme wave bending moment and the shape parameter $\ell = 1$.

Within the field of naval architecture there has been a tendency not to pursue this formulation in favor of a semiprobabilistic method [8, 12, 13, 20, 21]. This is due to the limited amount of data available to determine the precise form of the distribution of the basic variables and because the type of solution σ given in equation (17) is not readily attainable for a general case or a case including coupling of the failure modes.

Simulation Methods

Monte Carlo simulation techniques can be used to estimate the probabilistic characteristics of the functional relationship Z in equation (1). Monte Carlo, or direct simulation, consists of drawing samples of the basic variables according to their statistical properties and then feeding them into the limit-state equation. If it is known that failure occurs when g ( ) < 0 then an estimate of the mean probability of failure can be found by

$$\bar{p}_f = \frac{N_f}{N} \qquad (18)$$

where N_f is the number of simulation cycles where g ( ) < 0; N is the total number of simulation cycles. Obviously, as $N \rightarrow \infty$, then $\bar{p}_f \rightarrow P_f$, the true mean of the population. The accuracy of equation (18) can be evaluated in terms of its variance. For a small probability of failure and/or a small number of simulation cycles, the variance of p_f can be quite large. Consequently, it may take a large number of simulation cycles to achieve a specified accuracy with an unknown probability of failure.

To overcome this weakness investigators have proposed techniques to increase the efficiency of simulation methods by reducing the variance of the estimated output [9]. This type of investigation has led to several variance reduction techniques (VRTs). Ayyub and Haldar [9] have shown that, for typical structural problems, combining two of these techniques, conditional expectation and antithetic variates, provides the most efficient results. A short description of the two techniques follows:

a) Conditional expectation VRT. Utilizing the concept of conditional means and variances [15, 16] allows for the reduction of the variance of the simulated estimate of the probability of failure. For instance, if there is a function such that the probability of failure could be written as:

$$P_f = P(R\text{-}Q < 0) \qquad (18)$$

It is obvious that the probability of failure depends on the value of R for a given value of Q. This can be written as:

$$P_f = P(R < Q \mid Q = q) \qquad (19)$$

If the cumulative distribution function of R is known, then the solution of equation (19) becomes:

$$P_f = F_R(Q = q) \qquad (20)$$

The advantage of this technique in simulation methods is that the basic variable which is being conditioned on does not need to be generated at each simulation cycle, and, therefore, any uncertainties associated with that variable are removed. Consequently, one simply chooses to condition on the variable with the largest variance. The steps involved in the process are further explained later with a practical example.

b) Antithetic variates VRT. Suppose that for two consecutive simulation cycles the estimated values of the limit state equation are $Z_i^{(1)}$ and $Z_i^{(2)}$. These two estimates can be combined to form another estimator.

$$Z_i = 1/2\,[Z_i^{(1)} + Z_i^{(2)}] \qquad (21)$$

then the expected value of Z_i will be given by:

$$E[Z_i] = 1/2\,[E(Z_i^{(1)}) + E(Z_i^{(2)})] = Z$$

which says that Z_i is an unbiased estimator of the population mean. It can be shown also that the variance of Z_i is:

$$Var(Z_i) = 1/4\,[Var(Z_i^{(1)}) + Var(Z_i^{(2)}) + 2 * COV(Z_i^{(1)}, Z_i^{(2)})] \quad (22)$$

Now, if negative correlation between $Z_i^{(1)}$ and $Z_i^{(2)}$ can be induced, the COV term in equation (22) will become negative which will, in turn, reduce the variance of the estimator Z_i.

Within a simulation algorithm this can most easily be accomplished when generating the random variables. If U is a random variable uniformly distributed between 0 and 1, then 1-U is another random variable which is negatively correlated with U and is also uniformly distributed between 0 and 1. Each of these variables is then used to generate a basic random variable X_i such that a pair of negatively correlated basic variables result. This is done for each basic variable in the limit-state equation. The limit-state equations are solved for $Z_i^{(1)}$ and $Z_i^{(2)}$. The average of these estimators are found using equation (21) and the estimate of the population mean value $\overline{Z}$ is given by

$$\overline{Z} = \frac{\sum_{i=1}^{n} Z_i}{N} \quad (23)$$

The advantages of simulation methods are most apparent in the analysis of complex probability problems which have mathematically intractable analytical models. Simulation methods can deal with problems involving coupling of failure modes, correlation between basic variables, and complex limit-states. However, the results of the simulation should be viewed in the same light as results of laboratory experiments. This is to say that no matter how intricate the simulation model, the results will only be as good as the assumptions involved in creating the model and the reliability of the data input.

EXAMPLE PROBLEM

In order to compare the methods just discussed, an example problem is solved using each method. The problem chosen for analysis is to determine the probability of ductile yielding of a vessel's deck under extreme bending moments. Any of the other possible modes of failure could have been chosen, i.e., fatigue, plastic collapse, buckling, but the availability of data on this problem facilitated comparison of methods. The vessel chosen for the analysis is a naval frigate, the same one used by Mansour and Faulkner [11]. The principal dimensions are given in Table 1 and the midship section is shown in Figure 6.

TABLE 1. Vessel Characteristics

Length Between Perpendiculars	360 ft	(110 m)
Beam (moulded)	41.0 ft	(12.5 m)
Depth	28.9 ft	(8.78 m)
Draft	12.0 ft	(3.66 m)
Displacement	2,800 tons	(2,845 tonnes)
Section Modulus (at deck)	5,700 in² ft	(1.12 m³)

The problem is essentially a simple beam in bending and can be written as:

$$M_u = CY \quad (24)$$

where M_u is the ultimate bending moment; C is the section modulus of the vessel; and Y is the tensile yield stress of the vessel material.

In order to see the effect of different, but mechanically equivalent, formulations on each method two limit-state equations will be used. The first limit-state equation is in a very simple linear form as used by Mansour and Faulkner [11]:

$$Z = R - Q \quad (25)$$

where R is the resistance, given in tons/in² and is equal to Y in equation (24); Q is the total load in tons/in² and is equal to M_u/C in equation (24).

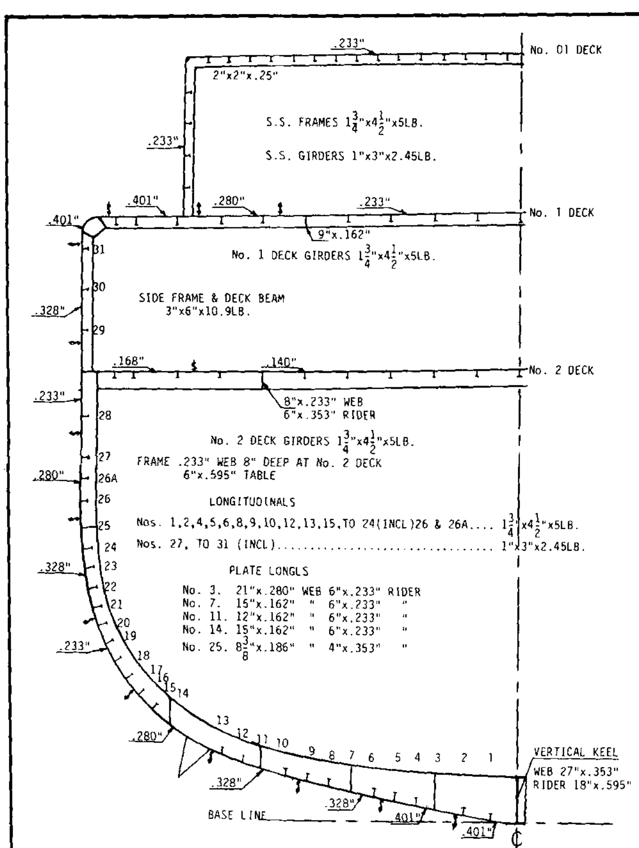

Figure 6. Midship section of frigate of 360 ft LBP [11].

TABLE 2. Probabilistic Characteristics of Basic Variables

Basic Variable	Mean	COV	Distribution
Linear Formulation			
R	22.2 tons/in^2	.071	Normal
Q	2,696 tons/in^2	.539	Weibul (k = 1)
Nonlinear Formulation			
Y	22.2 tons/in^2	.061	Normal
C	5,700 in^2 ft	.0379	Log-Normal
M_o	7,080 ft-tons	—	Deterministic
M_w	8,290 ft-tons	1.0	Weibul (k = 1)

Next a more complicated nonlinear form of equation (24) is used. This form separates the wave and still-water bending moments, M_w and M_o respectively; and Z is expressed in units of bending moment:

$$Z = YC - M_o - M_w \tag{26}$$

The basic variables for each form are shown in Table 2 along with their respective statistical properties.

First Moment Method

As described earlier this method only makes use of the mean of nominal values of the basic variables. Since the two forms of the limit-state equation are mechanically equivalent one would expect the factor of safety based on mean values to be the same for both forms. This, in fact, holds true as shown below:

$$F.S. = R/Q = (22.2 \text{ tons/in}^2) / \left(\frac{7{,}080 + 8{,}290}{5{,}700}\right) = 8.23$$

or

$$F.S. = YC / (M_o + M_w) = (22.2 \times 5{,}700) / (7{,}080 + 8{,}290) = 8.23$$

(Note: The values used above are mean values; nominal values were not available.)

These results, along with the results from the other methods, are shown in Table 3.

The weakness of this method in demonstrated by the value 8.23. What does it mean? Is this factor of safety unusually large? Factors of safety can really only be used to compare similar systems under similar loadings. In the case of ships, what could be an adequate factor of safety for a 3,000-ton frigate might be totally inadequate for a 360,000-DWT tanker.

Second Moment Method

These methods are considered semiprobabilistic for the reasons stated earlier. Because they do consider the basic variables to have a degree of uncertainty associated with them, an extensive study must be carried out to determine that uncertainty before these methods can be used.

The strength uncertainties were evaluated in References [6, 11, 22]. In those studies the total coefficient of variation of the strength term in equation (25) was evaluated using equation (5b). For the nonlinear case in this investigation, we will separate those uncertainties associated with the material properties and those associated with the configurations, structural geometries, and construction. The former is applied to Y in equation (26) and the latter to the C in the same equation. The distribution of Y has been shown in the literature [11, 22] to be approximated well by a normal distribution. For the section modulus, the central limit theorem suggests that

TABLE 3. Example Problem Results

	First Moment Method	First Order Second Moment	Advanced Second Moment	Mansour and Faulkner 11	Direct Simulation	Conditional Exp. & Antithetic Variates
Linear Equation z = R - Q	F.S. = 8.23	$\beta^* = 9.09$ $P_f \cong 10^{-20}$	Normal $\beta = 9.09$ $P_f \cong 10^{-20}$ Non-Normal $= 4.75$ $P_f \cong .97\text{x}10^{-6}$	$\beta^* \cong 4.465$ $P_f = 4\text{x}10^{-6}$	**	Normal $P_f \cong 10^{-20}$ Non-Normal $P_f = .98\text{x}10^{-6}$ COV = .0192
Non-Linear Equation z = YC-Mo-Mw	F.S = 8.23	$\beta^* = 9.03$ $P_f \cong 10^{-20}$	Normal $\beta = 9.48$ $P_f \cong 10^{-20}$ Non-Normal $\beta = 4.75$ $P_f \cong .976\text{x}10^{-6}$	***	**	Normal $P_f \cong 10^{-20}$ Non-Normal $P_f = .9801\text{x}10^{-6}$ COV = .0174

**N_f = 0 for these cases

***Solution format from Ref. [11] does not fit nonlinear limit-state equation form for multiplicative models such as C, a log-normal distribution should be adopted.

The statistics and distributions of the load variables M_o and M_w are estimated by the approach used by Mansour [10] and adopted by many since [11, 13, 20, 22]. This technique involves utilizing either strip-theory ship motions computer programs or extensive model testing to generate bending moment response amplitude operators (RAO). then using the principle of superposition (a linear assumption) to obtain a bending moment spectrum and thus an RMS value of bending moment. This allows the calculation of the expected value of the extreme bending moment for long periods or for the entire vessel life. The approach is limited by the number of linearizing assumptions made and the imperfect knowledge of the vessel's actual lifetime voyage pattern.

First Order Second Moment

For the simple linear case given by equation (25), a solution can be quickly obtained by rearranging equations (4 to 7) into the following [12]:

$$\beta^* = \frac{\bar{R} - \bar{Q}}{\sqrt{\sigma_R^2 + \sigma_Q^2}} = \frac{\theta - 1}{\sqrt{\theta^2 V_R^2 + V_Q^2}} \qquad (27)$$

where $\bar{R}$, $\bar{Q}$ are the mean values of the strength and the load; σ_R, σ_Q are the standard deviations of the strength and the load; θ is equal to $\bar{R}/\bar{Q}$; V_R, V_Q are the coefficients of variation of the strength and the load. Thus equation (25) yields:

$$\beta^* = \frac{\frac{22.1}{2.696} - 1}{\sqrt{(8.23)^2\,(.071)^2 + (.539)^2}} = 9.09$$

$$P_f = 1 - \Phi(\beta^*) \approx 10^{-20}$$

For the nonlinear case equations (4) through (7) must be followed more directly:

$$\bar{Z} = (22.2\ \frac{\text{tons}}{\text{in}^2})\ (5{,}700\ \text{in}^2\ \text{ft}) - (7{,}080\ \text{ft-tons}) - (8{,}290\ \text{ft-tons})$$

$$= 111{,}170\ \text{ft-tons}$$

$$\sigma_Z^2 = [\sigma_y^2\,(\bar{C})^2 + \sigma_c^2\,(\bar{Y})^2 + \sigma^2_{\bar{m}_o} + \sigma^2_{\bar{m}_w}]$$

$$= [(22.2 \times .061)^2\,(5{,}700)^2 + (5{,}700 \times .0374)^2\,(22.2)^2 + 0 + (8{,}290 \times 1)^2]$$

$$\sigma_z = 12{,}300.67\ \text{ft-tons}$$

$$\beta^* = \frac{111{,}170}{12{,}300.67} = 9.03$$

$$P_f \cong 1 - \Phi(\beta^*) = 10^{-20}$$

Advanced Second Moment

In order to test the effects of including the distribution types on the solution, two cases were examined for each of the equations (25) and (26). The first case is to assume all variables are normally distributed, as is the case in the first order method. The second case is to include the form of the distribution as shown in Table 2.

A computer program was written to facilitate the solution of each equation type. The programs are not difficult to code, but there are enough differences between the solution algorithms to necessitate separate programs. The solutions of the four trials are shown in Table 3. As expected the solution of the case for normal distributions is identical to the one using the first order second moment method (FOSM) for a linear limit-state equation. For the nonlinear equation the normal results again reflect the similarity in results with FOSM.

The non-normal results for both limit-state equations show marked differences with the first order method. Obviously, this is due to including the knowledge of the distribution type.

Direct Simulation

Again, to compare the effects of distribution upon the probability of failure two cases were investigated for each type of limit-state equation. In all cases it is obvious from earlier results that the probability of failure is small, consequently the number of simulation cycles will be exceptionally large. The large number of simulation cycles required in all cases made finding solutions by this method unrealistic. As a result no solution was obtained using this method.

Conditional Expectation and Antithetic Variates VRT

Solving either equation (25) or (26) is accomplished using the same fairly simple computer program. The primary steps involved are:

Step 1) Identify the basic variable with the most variability in the limit-state equation
Step 2) Condition the variable in step 1 with respect to all the remaining variables in the limit-state equation.
Step 3) Generate a uniformly distributed random deviate for each of the conditional variables.
Step 4) Generate a second uniformly distributed random deviate which is negatively correlated to the one from step 3.
Step 5) Using the inverse transform method produce a random variable for each deviate from step 4.
Step 6) Calculate the probability of failure using the probabilistic characteristics of the variable identified in step 1 for each set of random variables.
Step 7) Find the average probability of failure for the two P_f's in step 6.
Step 8) Repeat steps 3 to 7 N times.
Step 9) Calculate the statistics of the N number of probabilities of failure thus generated.

The results are shown in Table 3 for 2,000 simulation cycles. Figures 7 and 8 show how the simulation scheme converges on a solution with increasing N.

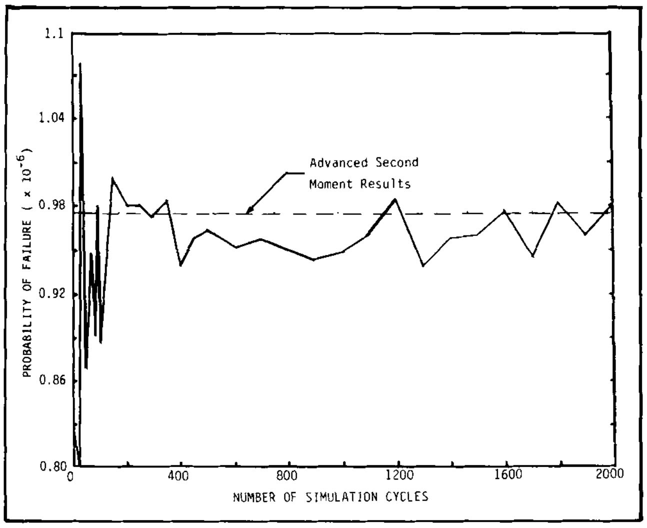

Figure 7. Probability of failure vs. number of simulations using variance reduction techniques.

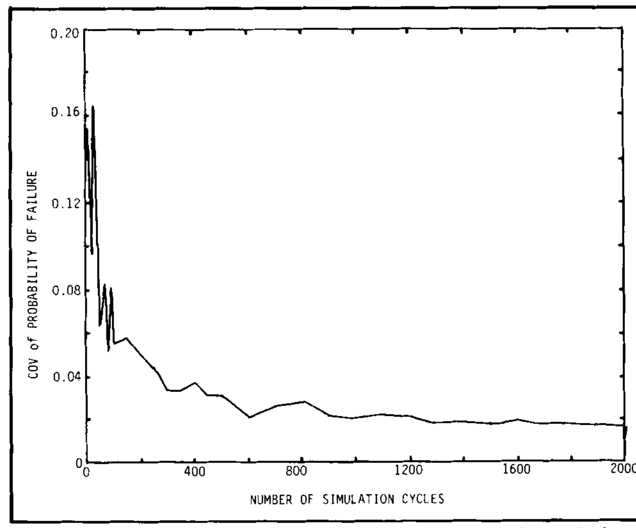

Figure 8. COV of probability of failure vs. number of simulations using variance reduction techniques.

DISCUSSION

The example problem highlights some of the important characteristics of each of the reliability methods discussed.

The conventional factor of safety method is not really a reliability method. The results do not indicate anything more than a means of ranking exactly similar systems under identical conditions. Such a tool may be useful in testing equipment on an assembly line but is not really beneficial in ship construction. The principal problems with the approach are that 1) there is no way to relate factor of safety to an actual probability of failure; 2) there is no way to include uncertainties in load or resistance; and 3) the method can just as easily lead to an unsafe design when attempting to use it on some unconventional structure.

The first order second moment method (FOSM) is a useful tool in cases where the basic variables are linearly related and can be considered normally distributed. As was shown in Table 3, the method breaks down rather badly when either of those conditions is not met.

Because of the potentially different results when using the two different, but equal, limit-state equations one must be careful in using the β^* value for comparing different systems. It is very important to not only specify the value of β^* but also the form of the limit-state equation used.

The advanced second moment method overcomes many of the problems with the FOSM method and is an efficient routine. For the example problems investigated, the method quickly came up with solutions which were later verified by the simulation methods. The method is capable of handling very complex problems, but, as the level of complexity increases, the uncertainty in the solution increases. In the case of highly nonlinear limit-state equations, as pointed out earlier, it is very possible to converge on a local minimum solution rather than on the system minimum. That difficulty, along with the fact that it is extremely difficult to develop a general program for the solution of any form of limit-state equation, somewhat dims the usefulness of this approach.

The fully probabilistic approach by Mansour and Faulkner is somewhat limited in scope. Any change in the form of the limit-state equation or change in the assumed distribution types will require changes in equations (15) to (17). So, while the method is fine for the special cases investigated, a considerable amount of effort would be required to make a more general and useful solution system. Even if this was achieved the process would not be capable of dealing with problems where there is coupling of failure modes.

The results of the direct simulation method clearly show why a number of investigators in structural reliability consider that "Monte Carlo methods should be avoided if at all possible" [14]. Because of the small size of the probability of failure the method was unable to come up with a solution in a large number of simulation cycles ($\approx$ 500,000 simulation cycles). This enormous use of computer resources cannot be justified when other less expensive methods are available.

Simulation using variance reduction techniques is much more efficient. The results in Table 3 are based on 2,000 simulation cycles. Figures 7 and 8 show how the method converges on a solution. It is interesting to note that in as few as 50 simulation cycles the coefficient of variation of the solution is less than 10% and the solution is correct to $.1 \times 10^6$. Notice also in Table 3 the similarity of the solutions with the advanced second moment method for both normal and non-normal basic variables. One advantage of the simulation with VRT is that the same computer program can be used for both types of limit-state equations and normal or non-normal basic variables. The limitation on simulation methods is the accuracy of the algorithm used to generate the random variables. As the probability of failure becomes smaller it is increasingly important for the simulation algorithm to correctly generate deviates in the tails of the distributions.

CONCLUSIONS

The purpose of the investigation on which this paper reports was to determine which reliability method is the most suitable for use with ship structures. It should be obvious from Table 3 that the choice comes down to either the advanced second moment method or simulation method using VRT. The strong points of each have already been discussed. Based on the discussion presented the authors believe the simulation method using VRT to be the most suitable.

The major factor behind this decision is that the method is easily capable of handling complex problems, such as cases involving load combinations or coupling of failure mechanisms. Admittedly, as the complexity of the problem increases, the amount of computer time required will also increase. Fifteen or twenty years ago the large amount of computer time required would make such an analysis cost prohibitive. Today, any number of smaller computer systems could adequately handle such a program, only the execution time would increase. The authors see this vast change in computer technology as an invitation to use simulation methods on problems which have heretofore been considered unmanageable.

It is the intent of the authors to use the investigation in this report as groundwork for developing a simulation algorithm for investigating the ultimate strength of ship structures. Eventually, a system which will allow the designer to quickly investigate alternative configurations and materials and their effects on the whole structure will be developed.

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