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A METHOD FOR ESTABLISHING SHIP DESIGN WAVE BENDING MOMENT AND ITS COMPARISON WITH CLASSIFICATION SOCIETIES' RULES

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Abstract—A procedure has been developed for the long-term distribution of the short-term extreme values of ship wave bending moments. The procedure has been applied in a systematic manner to investigate the effect of the principal ship characteristics on the design wave bending moment. The results have been compared to Classification Societies' Rules and demonstrate a rational way to improve them. The same procedure can also be applied to other ship responses or the responses of any structure excited by seaways.

1. INTRODUCTION

DURING the last 25 years the prediction of the performance of a ship in waves, and more recently of other floating or fixed ocean structures, has been the subject of intensive experimental and theoretical investigations. Initially the behaviour of the ship in regular waves was examined, but knowledge of this behaviour has almost no practical value. Then it became possible to establish the short-term behaviour of the ship in a seaway of known spectrum. These results are more useful, especially for comparative purposes, but again do not lead to the determination of *design values*. To obtain design values one has to use statistical methods for the prediction of the long-term behaviour of the ship in the ocean. Such a method has been developed and is presented herein, using statistics of extremes for the short-term behaviour.

The application of this method requires the knowledge of the ship response in regular waves and the probability distribution of the sea states in a region of the ocean. Further, by applying the method in a systematic manner with respect to the principal ship characteristics it has become possible to investigate the effect of these characteristics on the design value for the vertical wave bending moment amidships and compare the results with the corresponding values of the Classification Societies.

Obviously, the same method can be used for the determination of design valves for other responses of a ship or any other ocean structure.

2. SOME STATISTICAL TOOLS

For the sake of brevity, only the definitions and some statistical tools from the theory of time series and statistics of extremes necessary for this work will be described. The reader can find in the references a complete treatment of the subject.

Thus, for a time series $f(t)$ which is normally distributed and ergodic (e.g. OCHI and BOTTON, 1973) as is assumed to be the case with short term samples of seaways and corresponding linear ship responses, one can define the following:

The *mean value* μ_f of $f(t)$ is given by:

$$\mu_f = \lim_{T\to\infty} \frac{1}{T}\int_0^T f(t)\,dt \quad , \tag{1}$$

and if this value is different from zero*, one can redefine a new process $f'(t) = f(t) - \mu_f$ for which the following formulae hold (PRICE and BISHOP, 1974):

The variance σ^2_f of $f(t)$ is given by:

$$\sigma^2_f = \lim_{T\to\infty} \frac{1}{T} \int_0^T f(t)^2 \mathrm{d}t \quad , \tag{2}$$

and equals the *mean square value* of $f(t)$.

The square root of the variance is called the *standard deviation* σ_f of $f(t)$ or the *root mean square value* (*r.m.s.*).

The *autocorrelation function* $\Phi_f(\tau)$ of $f(t)$ is given by:

$$\Phi_f(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_0^T f(t) f(t+\tau) \mathrm{d}t, \; 0 \leq \tau \leq \infty. \tag{3}$$

The *spectral density function* $\Phi_{ff}(\omega)$ of $f(t)$ is given by:

$$\Phi_{ff}(\omega) = \frac{2}{\pi} \int_0^\infty \Phi_f(\tau) \cos(\omega\tau) \mathrm{d}\tau, \; 0 \leq \omega \leq \infty. \tag{4}$$

Then one can easily derive that:

$$m_0 = \Phi_f(0) = \sigma^2_f = \int_0^\infty \Phi_{ff}(\omega) \mathrm{d}\omega, \tag{5}$$

where m_0 is the *zeroth moment* of the spectral density function and equals the area under it. In general, the nth moment of the spectral density function is given by:

$$m_n = \int_0^\infty \Phi_{ff}(\omega)\, \omega^n \mathrm{d}\omega. \tag{6}$$

The *mean period* between successive zero upcrossings (or downcrossings) is given by:

$$T_m = 2\pi \sqrt{\frac{m_0}{m_2}} \quad . \tag{7}$$

A measure of the root mean square width of the spectral density function of $f(t)$ is the *broadness factor* ε given by:

*This is exactly the case for longitudinal wave bending moments (MURDEY, 1972). However, the observed zero shifts are not accounted for by the linear theory used in the analytical calculations.

$$\varepsilon = \left(1 - \frac{m_2^2}{m_0 m_4}\right)^{1/2} \quad . \tag{8}$$

For an *ideally narrow band* process $f(t)$, i.e. $\varepsilon = 0$, it can be shown (CRANDALL AND MARK, 1963) that its envelope is symmetrical about its mean and that it has a Rayleigh probability density function. If one assumes that the global maxima or minima of $f(t)$ between zero level crossings lie very close to the envelope, then the approximate probability density function of the *apparent amplitudes** α of $f(t)$ is also Rayleigh, that is:

$$p(\alpha) = \frac{2\alpha}{R} \exp\left[-\alpha^2/R\right] \quad , \tag{9}$$

where α is the apparent amplitude of $f(t)$ and $R = 2\sigma_f^2 = 2m_0$.

The average $1/n$ *highest values* of a Rayleigh distributed variable α can be easily computed from (8) (LONGUET-HIGGINS, 1952) and for the average 1/3 highest values, *the significant values*, of the variable the following relation holds:

$$\alpha^{1/3} = 1.416\,(\overline{\alpha^2})^{1/2}, \tag{10}$$

where $\overline{\alpha^2}$ is the mean square value of α.

With regard now to the statistics of extremes (GUMBEL, 1966) the following definitions and results will be needed in the sequel:

We consider a random variable X with $F(x)$ as its cumulative probability function and $f(x) = \mathrm{d}F(x)/\mathrm{d}x$ its probability density function. We consider further the random variable X_n defined as:

$$X_n = \max\{X^1, X^2, \ldots, X^n\}, \tag{11}$$

where $\{X^1, X^2, \ldots X^n\}$ is a sample of n independent observations of X.

The cumulative probability function of X_n is given by:

$$\Phi_n(x_n) = [F(x_n)]^n, \tag{12}$$

and the probability density function of X_n is given by

$$f_n(x_n) = n f(x_n)\,[F(x_n)]^{n-1} \quad . \tag{13}$$

Since the probability of a value equal to or larger than x is $1 - F(x)$, and thus $n[1-F(x)]$ values in a sample of size n are expected to be equal to or larger than x, we define the *characteristic largest value* $u_n(n) = u_n$, for $n \geq 2$, as:

$$F(u_n) = 1 - 1/n \quad . \tag{14}$$

We further define the *extremal intensity function* $\alpha_n(n) = \alpha_n$, as:

*Defined as the distance between the zero mean level and the global maximum or minimum between zero crossings, excluding ripples.

$$\alpha_n = n f(u_n) \quad . \tag{15}$$

According to Gumbel, the suitable asymptotic approximation to the exact cumulative proability function given by relation (12) for an initial distribution of exponential type* is given by:

$$\Phi(x) = exp\,[-\,e^{-y}] \quad , \tag{16}$$

where $y = \alpha_n(x - u_n)$.

3. EXTREME VERTICAL WAVE BENDING MOMENTS FOR A SHIP

The ultimate purpose of this work is to develop a method for the derivation of design wave bending moments to be used in ship design and/or in the formulation of the Classification Societies' Rules. Due to its practical importance the subject matter has been treated by many investigators using long-term prediction methods (e.g. NORDENSTRØM, 1973; FUKUDA, 1967). These methods can be modified and presumably improved by using in addition statistics of extremes concepts (e.g. LINDEMAN, 1971; OCHI and MOTTER, 1969).

Vertical wave bending moments are in general considered to be a linear ship response and are treated as such by the analytical prediction methods.† It can even be assumed (MANIAS and NUMATA, 1968) that it is a reasonable approximation to use the linearity assumption and the linear superposition principle for the prediction of the vertical wave bending moments in extreme seaways. In this method the analytically determined vertical wave bending moments do not differentiate between hogging and sagging values. It is also usually assumed that the apparent amplitude of vertical wave bending moments is Rayleigh distributed. However, vertical wave bending moments are not close to a very narrow-band process and analysis of very long wave bending moment records and records of other linear ship responses for a cargo ship model (LOUKAKIS, 1970) has shown that their crest-to-trough values are indeed Rayleigh distributed, but with an R parameter which depends on the broadness factor of the spectrum. These results are shown in Fig. 1 for the measured mean square values of several ship responses as a function of the broadness factor and it is suggested that a correction factor of $(1 - \varepsilon^2/2)$ gives better agreement with the experimental results, that is that the Rayleigh distribution parameter R should be equal to $8m_0$ $(1 - \varepsilon^2/2)$ or $2m_0(1 - \varepsilon^2/2)$ for the case of crest-to-trough values and apparent amplitudes respectively.

According to the above, the probability distribution of the apparent amplitudes of vertical wave bending moments in a seaway with given spectrum can be analytically calculated. Then the *short term distribution* of the vertical wave bending moment extreme values can be derived using the Gumbelian approach.

Thus, for the apparent amplitudes, we have:

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*Distributions of exponential type are those that satisfy the relation: $\frac{f(x)}{1 - F(x)} = -\frac{f'(x)}{f(x)}, x \to \infty$ which is the case for a Rayleigh distribution.

†Recently however an attempt has been made (JENSEN and PEDERSEN, 1978) to develop a non-linear theory for wave induced bending moments, which accounts for the experimentally observed differences in hogging and sagging values.

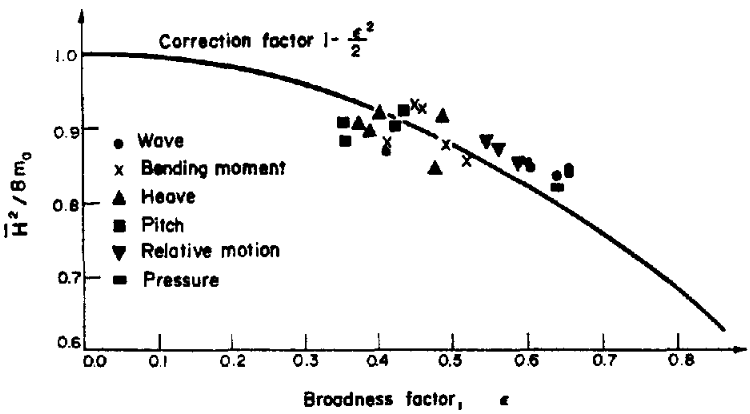

FIG. 1. Measured mean-square responses as a function of the broadness factor.

$$f(\alpha) = \frac{2\alpha}{R} \exp\left[-\alpha^2/R\right] \quad R = \text{constant}, \tag{17}$$

$$F(\alpha) = P[M_w \leq \alpha \mid R] = 1 - \exp[-\alpha^2/R],$$

where M_w is the apparent wave bending moment amplitude and $R = 2m_0$ or $R = 2m_0(1 - \epsilon^2/2)$ is the constant parameter.

The constant parameter R is a function of the ship characteristics and loading, her speed, the heading angle and the seaway spectrum.

We can now use (14) and (15) to obtain:

$$\begin{aligned} u_n &= (R \ln n)^{1/2} \quad , \\ \alpha_n &= 2(\ln n/R)^{1/2} \quad , \end{aligned} \tag{18}$$

where n is the number of wave bending moment apparent amplitudes in a sample record of given (short) duration.

Thus, the short-term cumulative probability function for the vertical wave bending moment extremes is given by (16).

To proceed further one should determine the value of n, which corresponds to a particular value of R. Obviously, in an experimental case n can be measured directly from the sample record. For the purposes of this investigation the value of n can be determined if we postulate a *proper sample duration* T_s. Then, since the spectral density function of the bending moment and its moments can be determined analytically, we get from (7) that

$$n = \frac{2T_s}{T_m} = \frac{T_s}{\pi}\sqrt{\frac{m_2}{m_0}} \quad . \tag{19}$$

At this stage we have completed the development of the analytical tools necessary for the evaluation of short-term extremes.

For the purposes of this investigation and for reasons to be explained, the ship was considered to sail at all times in head, fully-developed, long-crested seaways (PIERSON and MOSKOWITZ, 1964). In this case the spectrum of the seaway depends on one parameter only, e.g. the significant wave height $\bar{H}^{1/3}$. Then, if the probability density function $p(\bar{H}^{1/3})$ of $\bar{H}^{1/3}$ is known for a specific ship route, one can proceed to calculate the *long-term distribution* of the wave bending moment extremes as follows:

Given the ship characteristics, loading and speed, we calculate analytically for each $\bar{H}^{1/3}$ of interest the relationships $R = R(\bar{H}^{1/3})$ and $n = n(\bar{H}^{1/3})$. The later relationship requires the assumption of a 'proper sample duration'.

For each $\bar{H}^{1/3}$ we calculate the corresponding $u_n = u_n(\bar{H}^{1/3})$ $\alpha_n = \alpha_n(H^{1/3})$.

The long-term cumulative probability function of short-term wave bending moment extremes can be then evaluated, using the above and (16), as follows:

$$P[M_w \geq x_0] = \int_0^\infty [1 - \Phi(x_0 \mid \bar{H}^{1/3})] p(\bar{H}^{1/3}) \mathrm{d}(\bar{H}^{1/3}). \tag{20}$$

If one now assigns a specific value P_0 to $P[M_w \geq x_0]$ and solves (20) numerically for x_0 one obtains a *design wave bending moment* $M_w = x_0(P_0)$ corresponding to the assigned value P_0. In this manner the assigned value P_0 becomes the real design criterion as it represents the *probability of failure* for ships designed to withstand a *design vertical wave bending moment* $M_w(P_0)$.

Actually, (20) represents a simplification of the real problem. This is because heading angle, stage of development of the seaway and short-crestedness are not considered.

The above simplifications should be considered bearing in mind the real scope of the investigation. In this respect, any attempt to simulate the 'real' problem and to try to evaluate quantitatively what will be a 'definite' value for the design vertical wave moment is a lost cause. This is because many important parameters of the problem cannot be known in advance. Thus the actual loading condition of the ship, her exact speed, her heading angle and the actual directional wave spectrum at all times are indeterminate parameters.

One should strive, therefore, to obtain results useful for purposes of comparison between different ships or for the same ship in different loading conditions, and let the Classification Societies determine in a semi-empirical way the design vertical wave bending moments for their rules. Moreover, it is known (FUKUDA, 1967) that long-term wave bending moment calculations give quite comparable results for head seas and when all headings are taken into consideration. Finally, it is known (LOFT, 1969) that fully developed seas are more often encountered in the ocean.

Thus, the simplifications used in this investigation do not subtract substantially from the generality of the results when they are used for comparative purposes. In any case the suggested procedure can be readily modified to account for any of the aforementioned additional parameters.

The application of the methodology described previously requires firstly knowledge of the short-term distribution parameters $R = 2m_0$ and $n = (T_s/\pi)\sqrt{m_2/m_0}$. These parameters can be determined analytically. The analytical results used in the sequel were taken from the Seakeeping Standard Series (LOUKAKIS and CHRYSSOSTOMIDIS, 1975), where average weight distributions were used for the vertical wave bending moment calculations.*

Unfortunately, the second moment of the bending moment response spectrum is not included in the Seakeeping Standard Series and thus an approximation was used for the calculation of n. That is, it was assumed that the average period of the wave bending moment, which is a response that exists only in the frequency of encounter domain, is equal to the average period of the sea state considered also in the frequency of encounter domain. The latter can be obtained readily for fully developed seaways (GRIVAS, 1976). This assumption, which is a reasonable one, hardly influences the results since α_n and u_n are not very sensitive to changes in n as is also shown by the sensitivity analysis of the following paragraph.

4. APPLICATIONS OF THE METHOD AND RESULTS

The procedure described in the previous paragraph has been applied in a systematic way to investigate changes in *design wave bending moments* as a function of the principal characteristics of a ship and her speed, for seaways occurring in the North Atlantic region. In addition, the sensitivity of the procedure to changes in the values of the assumed probability of failure P_0 and the duration of the short-term sample T_s was investigated.

The numerical results have been compared with the corresponding values for the *wave bending moment* obtained using the formulae given by A.B.S. (1975).

The probability density function $p(\overline{H}^{1/3})$ used to demonstrate the application of the procedure is shown in Fig. 2. It has been obtained from (HOGBEN and LUMB, 1967) by manipulating the data for the regions 1, 2, 3 and 4 (North Atlantic—all seasons, all directions).

For all calculations the probability of failure was taken as $P_0 = 10^{-2.5}$ and the short-term sample duration as $T_s = 30$ min, except when performing the corresponding sensitivity analysis.

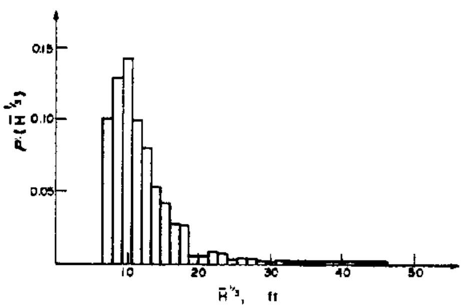

FIG. 2. Probability density function $p(\overline{H}^{1/3})$.

*One should mention that analytical wave bending moment calculations are only reasonably successful. Therefore the real value of the final results lies in the establishment of trends and in their use for comparative purposes.

The results obtained from the systematic changes in the principal ship characteristics are grouped in the following three 'Series' A, B and C. The effect of the assumed values of the probability of failure P_0 on the numerical results is investigated in Series D and the same effect of the short-term sample duration T_s is shown in Series E.

A series

Constant Parameters: $C_B = 0.85$, $L/B = 5.5$, Fr. No. $= 0.2$, $P_0 = 10^{-2.5}$
Variable Parameter: $B/T = 2, 3$ and 4.

The calculations described in the previous section were performed for various ship lengths. Since for a given ship length the beam is also fixed, the different values of B/T correspond to changes in draft. The purpose of this series is to investigate the effect of draft on design wave bending moment M_w, which is a parameter not included in the rules of the various Classification Societies.

The results of the calculations are presented in Fig. 3 in the form

$$M_w = f(L, B/T) \quad .$$

In the same figure the M_w value given by the A.B.S. rules (sagging) is also shown.

From these results we can conclude that draft seems to be an important parameter for the determination of M_w.

By comparing also the present results to the A.B.S. values it follows that, although the form of the curves is similar, the M_w value increases faster with the ship length in the A.B.S. formulation than according to the present calculations.

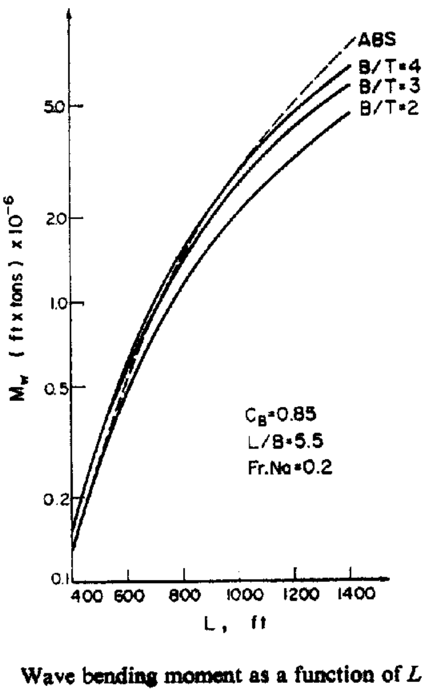

FIG. 3. Wave bending moment as a function of L and B/T.

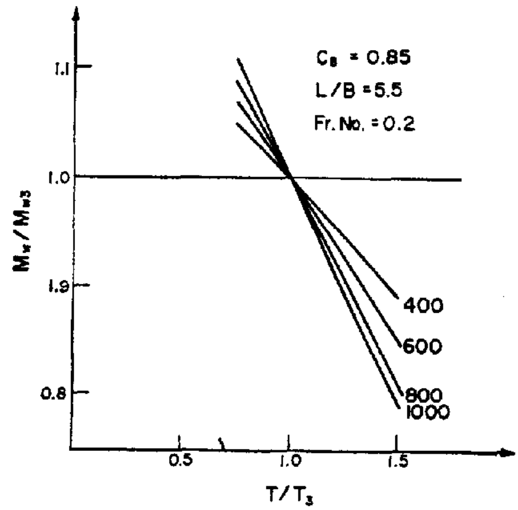

FIG. 4. Wave bending moment as a function of ship draft T for various ship lengths.

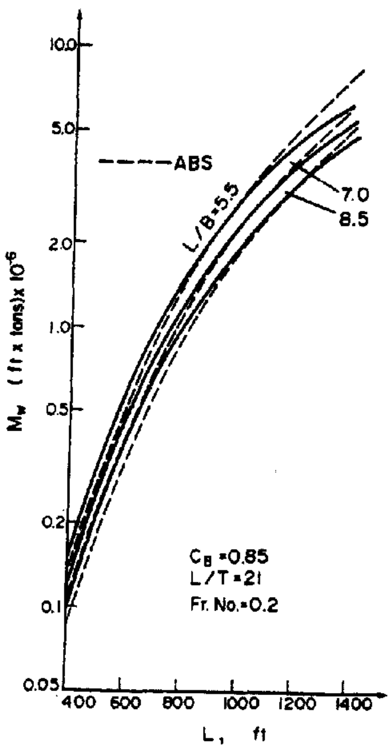

FIG. 5. Wave bending moment as a function of L and L/B.

The same results can be plotted in a different manner as shown in Fig. 4, where the $B/T = 3$ ship is used as the basis of comparison and the ratio M_w/M_{w3} is plotted vs T/T_3, with the subscript 3 signifying the base ship. From this figure we can conclude that draft is a more important parameter for longer ships.

B series

Constant Parameters: $C_B = 0.85$, $L/T = 21$, Fr. No. $= 0.2$, $P_0 = 10^{-2 \cdot 5}$

Variable Parameter: $B/T = 2.47$, 3 and 3.82.

The purpose of this series is to investigate the linearity of M_w with the beam of the ship (as in the rules of the Classification Societies), since for a given ship length the draft is constant and the different values of B/T correspond to changes in beam. The particular values of B/T used, correspond to values of $L/B = 8.5$, 7.0 and 5.5 for which the values of r.m.s. wave bending moment are tabulated in LOUKAKIS and CHRYSSOSTOMIDIS (1975).

The results are plotted in Fig. 5, together with the corresponding A.B.S. values. It can be seen that both the calculated results and the A.B.S. rules give similar dependence on beam. Again the A.B.S. values increase slightly faster with ship length than the calculated results.

The same results are plotted in Fig. 6 with the $L/B = 7$, $B/T = 3$ ship as the basis of comparison in the form M_w/M_{w7} versus B/B_7, with the subscript 7 signifying the base ship. These results indicate that the assumption of linear dependence of M_w on beam is reasonable, with the longer ships departing slightly from it. However, the results of the present calculations are for constant draft for a given ship length, which is not the case with the Classification Societies' rules.

C series

Constant Parameter: $L/B = 7$, $B/T = 3$, Fr. No. $= 0.2$, $P_0 = 10^{-2 \cdot 5}$

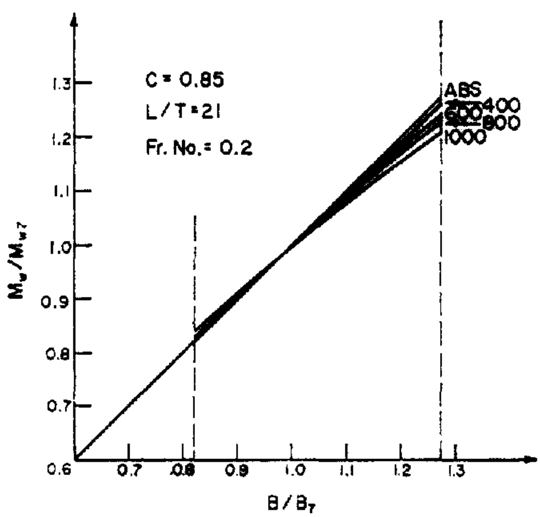

FIG. 6. Wave bending moment as a function of ship beam B for various ship lengths.

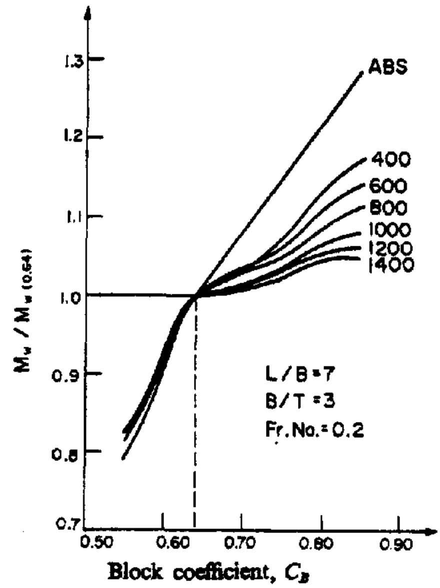

FIG. 7. Wave bending moment as a function of block coefficient C_B for various ship lengths.

Variable Parameter: $C_B = 0.55, 0.6, 0.65, 0.7, 0.75, 0.80$ and 0.85.

With this series the functional dependence of the design wave bending moment on the value of C_B is investigated and the results are compared with the corresponding calculations based on the A.B.S. rules and the rules of Lloyd's and Bureau Veritas.

The results are plotted in Fig. 7 in the form $M_w/M_{w\,(0\cdot 64)}$ versus C_B, because the A.B.S. rules consider that M_w is proportional to a term of the form $\alpha C_B + b$, where $C_B \geq 0.64$. The analytical calculations show that the influence of C_B on M_w is different depending on the length of the ship. The design wave bending moment increases faster with C_B for the smaller values of C_B than when C_B becomes large. Also, and for the larger values of C_B, M_w increases with C_B for the shorter ships and is almost independent of it for the very long ships.

In this case A.B.S. uses a different dependence of M_w on C_B than Lloyd's Register of Shipping Bureau Veritas or Det Norske Veritas. The results of the analytical calculations show that the A.B.S. rules assume a stronger dependence of M_w on C_B than the other Classification Societies, whose results are closer to the results of the analytical calculations.

D series

Constant Parameters: $C_B = 0.85$, $L/B = 5.5$, $B/T = 3$, Fr. No. $= 0.2$

Variable Parameter: $P_0 = 10^{-2}, 10^{-3}, 10^{-3\cdot 5}$ and 10^{-4}.

The sensitivity of the present procedure to the values P_0 of the probability of failure P is examined with this series. The results are shown in Fig. 8 in the form $M_w/M_{w,v=2} = f(v)$, where $v = -\log_{10}P_0$.

The results indicate an almost linear increase of the design wave bending moment with v, but with different slopes with regard to the ship length.

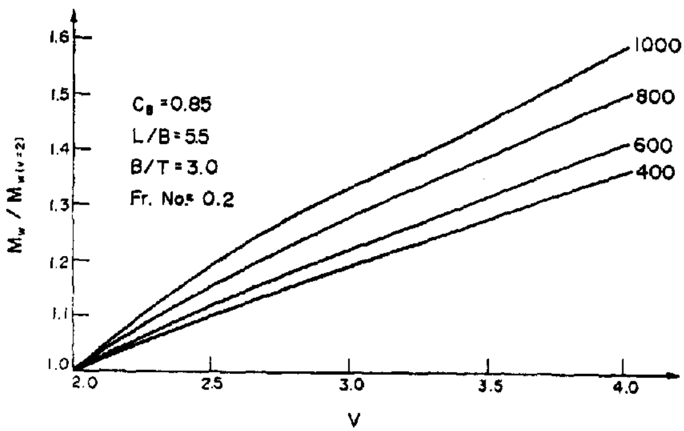

FIG. 8. Wave bending moment as a function of $\nu = -\log_{10} P_0$ for various ship lengths.

E series

Constant Parameters: $C_B = 0.85$, $L/B = 5.5$, $B/T = 3$, $L = 400$ft, $P_0 = 10^{-2.5}$
Variable Parameters: $T_s = 30$, 60, 90 and 120 min. Fr. No. = 0.15, 0.20 and 0.25.

The sensitivity of the procedure to the value of T_s is shown in Fig. 9. As has been stated earlier, the procedure is not very sensitive to the value of T_s or to the value of n.

In the same figure the well-known weak dependence of the wave bending moment on speed is also shown.

5. SYNOPSIS AND CONCLUSIONS

The procedure developed herein is not an asymptotic extreme distribution but instead a long-term distribution of the short-term extremes. It is based on analytical results for the short-term records and on statistics of extremes for the derivation of the asymptotic distribution of the short-term extremes. Subsequently, the long-term distribution of the short-term extremes is obtained in a straightforward manner using actual sea-state probability distributions. From this distribution a design wave bending moment value M_w is derived assuming a specific value P_0 for the probability of failure P.

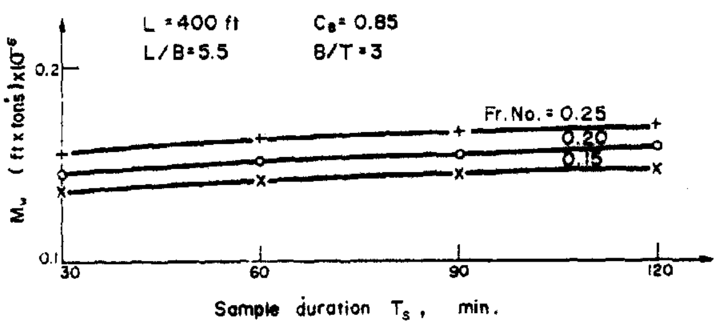

FIG. 9. Wave bending moment as a function of T_s for various Fr.Nos.

The physical meaning of the probability of failure P is that in a fleet of N identical ships, which are operating under identical conditions for a period of time T equal to the period for which the probability of occurrence of a seaway $p(\bar{H}^{1/3})$ has been measured, $n = P_0 N$ ships will experience a wave bending moment larger than M_w.

Obviously, the same procedure can be applied to any ship response or more generally to any response of a structure excited by seaways for which the short-term extreme distribution can be established.

The systematic application of the procedure in Series A, B and C can help in establishing the form of the dependence of design wave bending moment on the principal ship characteristics. In addition, from the D series we can conclude that, if Classification Society wishes to uniformly decrease the probability of failure by increasing the design wave bending moment, the ship length becomes an important parameter. That is, according to Fig. 8, long ships will require a larger percentage increment of their design wave bending moment than short similar ships.

Various comparisons of the results of the present procedure to the A.B.S. rules have been presented and they show similar trends and reasonable quantitative agreement. The similar trends are due to the fact that the Classification Societies have recently adopted the use of Seakeeping Theory coupled with statistical methods for the determination of design parameters (e.g. Det Norske Veritas, 1978). However, the Classification Societies use also *safety factors* based on their experience and other considerations and thus their requirements cannot be directly compared with the results of any analytical procedure. In this respect one can also note that the design values for wave bending moment given in analytical form by the various Classification Societies have differences, sometimes large. Nevertheless, if one takes into account the different values used for the yield stress and the different methods used for the determination of the still water bending moment, the values of the resulting required section modulus for the midship section according to the Rules of several

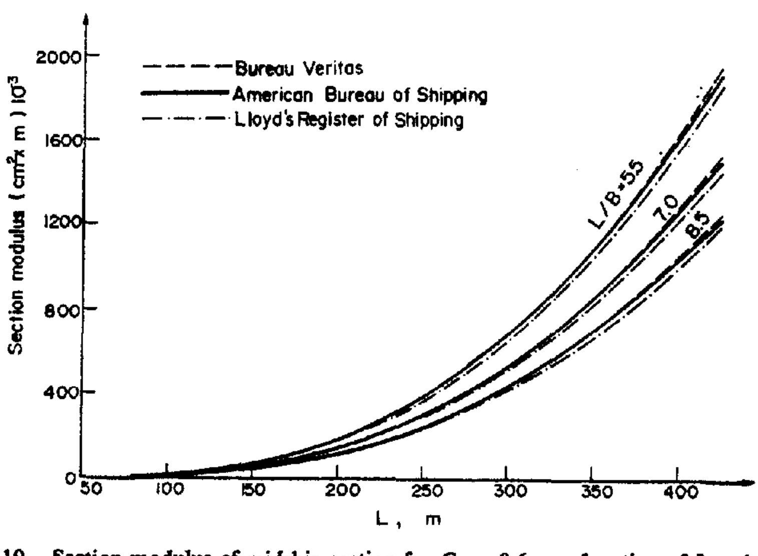

Fig. 10. Section modulus of midship section for $C_B = 0.6$ as a function of L and L/B.

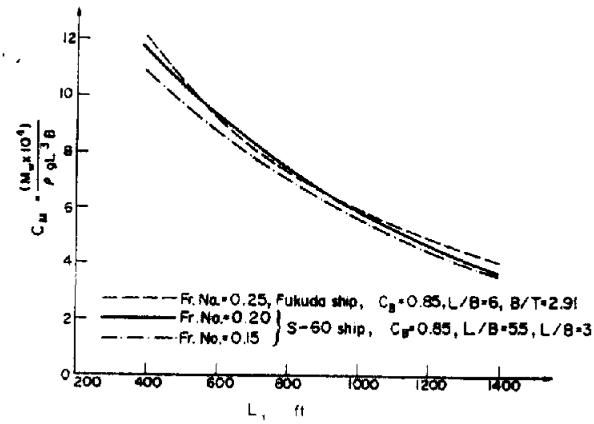

FIG. 11. Wave bending moment coefficient C_M as a function of ship length.

Classification Societies are very close, as shown in Fig. 10, diminish with increasing C_B and the results become almost identical for $C_B = 0.85$.

Finally, the results of the present procedure have been compared with similar results from FUKUDA (1967), where a long-term prediction method has been used with a probability level $Q = 10^{-8}$, Fig. 11. The two different methods show similar trends. However, a direct comparison of the numerical values is not possible since the two ships have somewhat different characteristics, different probability distributions for the sea states have been used and, most importantly, both methods use arbitrary values for the probability of failure P_0 or the corresponding probability level Q.

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