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CHAPTER 2

Ship Resistance

The theory of ship resistance has been elaborated by naval architects as a means of predicting ship performance from preliminary experiments with models. A full discussion of this theory or of the technique of testing the resistance of models or of full-scale ships by trial runs is beyond the scope of the present volume. However, since the data bearing on the effects of fouling and of protective coatings on the efficiency of ships during operation are expressed in the terms of this theory and were obtained by these techniques, it is necessary to present an elementary account of these matters. For a more complete treatment, standard works such as those of Taylor (*24*), Davidson (*7*), Saunders and Pitre (*18, 20, 21*) may be consulted.

The resistance offered by a ship to movement through water may be resolved into two principal components: *frictional resistance* and *residual resistance.* The frictional resistance arises from frictional forces set up by the flow of water along the surface of the hull, and is consequently influenced by fouling and the coatings of paint used for its prevention. The residual resistance is due to pressures developed in pushing the water aside, and arises from the form of the hull.

William Froude first recognized that the residual resistance of a model could be scaled up to give the residual resistance of the full-scale ship by use of the principle of similitude developed by Newton. The frictional resistance, however, follows laws of its own and can not be so treated. Froude consequently studied the frictional resistance of towed planks in order to determine empirically the relations between frictional resistance, length, surface area, and speed. Armed with this information, it is possible to estimate the frictional resistance of a model. This value is subtracted from the total resistance of the model to obtain its residual resistance. The residual resistance is then scaled up to give that of the full-sized ship. The frictional resistance, calculated for the full scale from the plank tests, is added to give the total resistance of the ship. This is the fundamental procedure in all model testing.

The total resistance of a ship to motion may be measured by trial runs over measured courses made both before and after fouling has occurred. The influence of fouling on the relation of speed to propulsive force can be measured in a direct and convincing way. This method is unavoidably expensive, since a full-sized ship must be kept available over a protracted period. It does not lend itself to the full analysis of the nature of the resistance unless supplemented by tests on "planks" which determine the frictional resistance separately.

Plank tests are conducted by towing long, thin plates in tanks. The resistance offered by such structures may be assumed to be due almost entirely to frictional forces and may be related directly to the roughness of the surface or to its fouled condition. This method of study is indirect in that the results can be applied to actual ships only with the aid of theoretical calculations supplemented by towing data on ship models or full-scale ships. Its relative simplicity and lower cost commends it, however, for detailed studies on the effects of surface roughness which may characterize painted, corroded, or fouled bottoms.

For the purposes of the paint technologist, effective information can be obtained without the complete solution of the resistance problem required by the naval architect. Reliable and simple procedures for estimating the relative frictional resistance of variously treated surfaces will be of value in guiding his technique, even though they do not supply data adequate for the needs of the ship designer.

The plank tests may be likened to the panel tests used in evaluating the protective action of coatings. Their value to the paint technician lies in the ease with which comparative evaluations can be made, not in the precision with which they foretell the performance of ships in sevice. The tests by trial run, on the other hand, like the service tests of paint coatings, give a direct measure of the phenomena in question.

THE TOTAL RESISTANCE OF SHIPS

The force required to propel a ship at any given speed may be measured by trial runs over a standard course in which the ship is self-propelled or is towed by another vessel. To obtain reliable results, an exacting technique must be followed in which a series of observations are made at each fixed speed, during which the vessel alternates its direction over the course in order to neutralize the effects of current. The trials should be run in quiet waters, since the state of the sea can not be

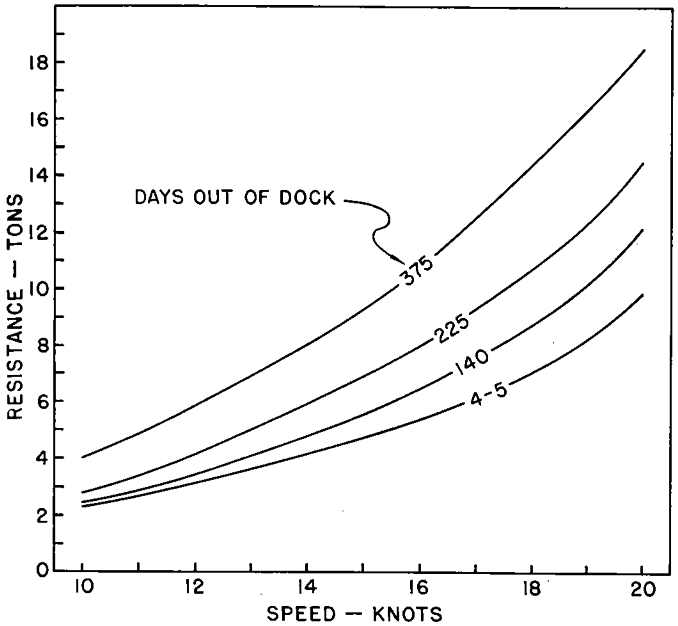

FIGURE 1. Resistance of destroyer *Yudachi* towed at different speeds after various periods at anchor. From data of Izubuchi (*13*).

allowed for. The force and direction of the wind must be measured and its effect calculated, to permit the results to be reduced to standard conditions.

If the ship is towed, the total resistance is given by the force exerted by the towline. The effective horsepower, *EHP*, is related to the total resistance, *R*, by the expression

$$EHP = 0.00307\ RV$$

where R is expressed in pounds, and the speed, V, in knots.

If the ship is self-propelled, the propulsive force is best obtained from measurements of the thrust of the propeller shaft.

The propulsive force is more usually estimated from the shaft horsepower. This is the power delivered by the shaft to the propeller (*20*). At a given speed, shaft horsepower is always greater than effective horsepower because of the inefficiencies inherent in propeller design and in the disturbed motion of the water at the stern of the ship. Effective horsepower is at best not more than 75 per cent of shaft horsepower, and more commonly is about 67 per cent (*15*). The propulsive efficiency of certain types of naval vessels may be even less than this. Fouling of the propellers may greatly decrease their efficiency, and thus may result in increases in the shaft horsepower required to maintain a given speed, which may be erroneously attributed to failure of the antifouling shipbottom paint. For this reason measurements of thrust are to be preferred to measurements of shaft horsepower. Thus in tests on the U.S.S. *Hamilton* as the result of fouling of the propellers, the increase in shaft horsepower was two or three times the increase in thrust (*18*).

The indicated horsepower of the engine differs still more than the shaft horsepower from the effective horsepower because of losses inherent in the efficiency of the engine.

Finally, the resistance may be reflected directly by the fuel consumed or its cost. These terms are of little use in the analysis of the physics of resistance, but give compelling evidence of the actual increase in cost of operating with a fouled bottom.

A most complete towing test showing the effect of fouling on hull resistance was made on the Japanese ex-destroyer *Yudachi* (*13*). This 234-foot vessel was docked, painted, and had the propeller removed in March, 1931. Immediately after undocking it was subjected to systematic towing tests which were repeated at intervals to show the effect of fouling.

The results of the tests on the *Yudachi* are shown in smoothed curves in Figure 1. They demonstrate the very great increase in resistance which developed while the ship remained at anchor. The resistance developed at a speed of 16 knots after various periods is shown in Figure 2 as a per cent of the initial resistance of the freshly painted hull. In 375 days the total resistance is exactly doubled. In Figure 3, the loss in speed with a towing force of 10 tons is plotted against the time at mooring. This force produced a speed of 20 knots with the freshly painted hull. After 375 days the speed had fallen to 15.4 knots, represented by a loss in speed of 4.6 knots.

The condition of the bottom of the *Yudachi* during the period of these tests is not reported.

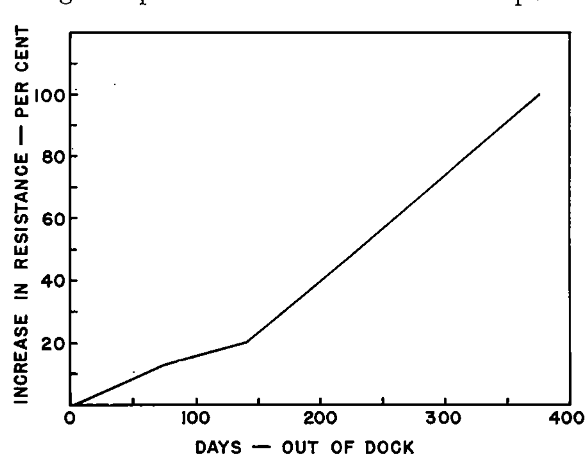

FIGURE 2. Percentage increase in resistance of destroyer *Yudachi* when towed at 16 knots after various periods out of dock. From data of Izubuchi (*13*).

The behavior of steel test panels, painted like the ship bottom and hung from the vessel, indicated that the paint system was not very satisfactory. After 140 days the paint had fallen off in several places, with the development of rust spots and fouling with *Bugula*. By the end of the test, barnacles and *Bugula* covered the entire surface, and 30 per cent of the area was rusted and devoid of paint. The weight of adhering matter was 5.2 and 2.28 kilograms per square meter on plates hung on the starboard and port side respectively. The results of the *Yudachi* tests may be associated with the development of rather severe fouling and corrosion.

The effect of fouling on the shaft horsepower re-

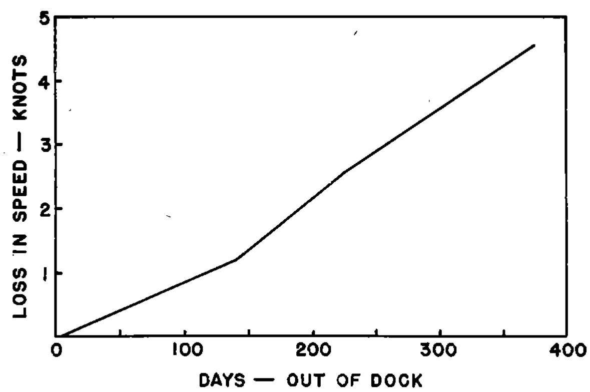

FIGURE 3. Loss in speed of destroyer *Yudachi* when towed with a force of 10 tons after various periods out of dock. Initial speed 20 knots. From the data of Izubuchi (*13*).

quired to develop various speeds in tests with the United States destroyer *Putnam* and the battleship *Tennessee* has been reported by Davis (*8*). The destroyer was undocked at Boston in October, spent the winter operating in New England waters, and at the end of March proceeded to Guantanamo where she remained until May before returning to northern waters. The battleship was undocked in October at Bremerton and operated during the following year between Puget Sound and Panama. These ships were subjected to trial runs periodically during the period following undocking, with the results shown in Figures 4 through 9. These figures are based on smoothed curves published by Taylor (*24*). The increase in resistance indicated by these tests is very similar to that shown by the *Yudachi*. In the case of the destroyer, the shaft horsepower required for a speed of 14 knots was practically doubled in eight months, as shown in Figure 5. At higher speeds the percentage increase in shaft horsepower was less, because of the relatively greater importance of wave-making resistance at high speed. The loss in speed amounted to more

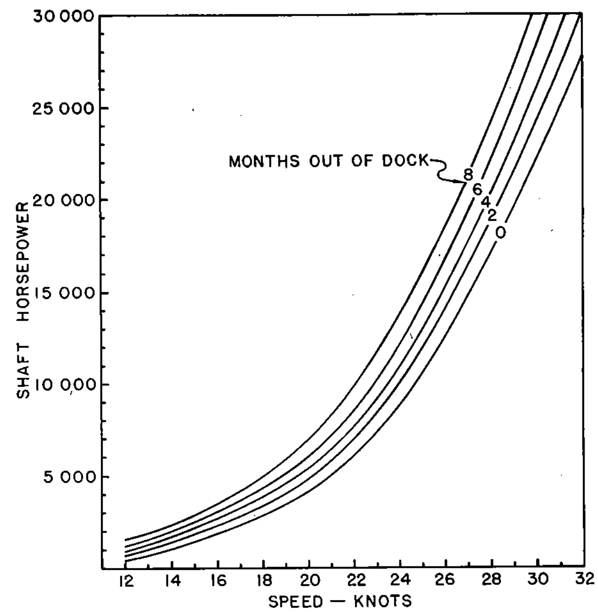

FIGURE 4. Shaft horsepower required to propel the destroyer *Putnam* at different speeds after various periods out of dock.

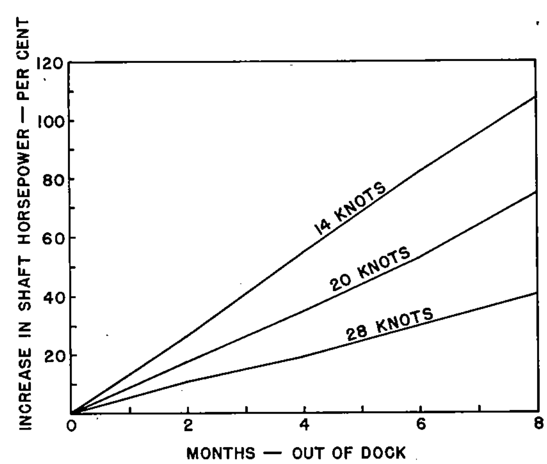

FIGURE 5. Percentage increase in shaft horsepower required to propel the destroyer *Putnam* at different speeds after various periods out of dock.

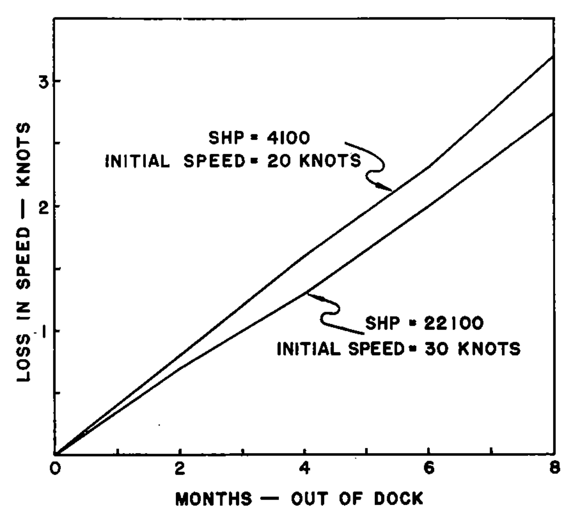

FIGURE 6. Loss of speed of destroyer *Putnam* at constant shaft horsepower after various periods out of dock.

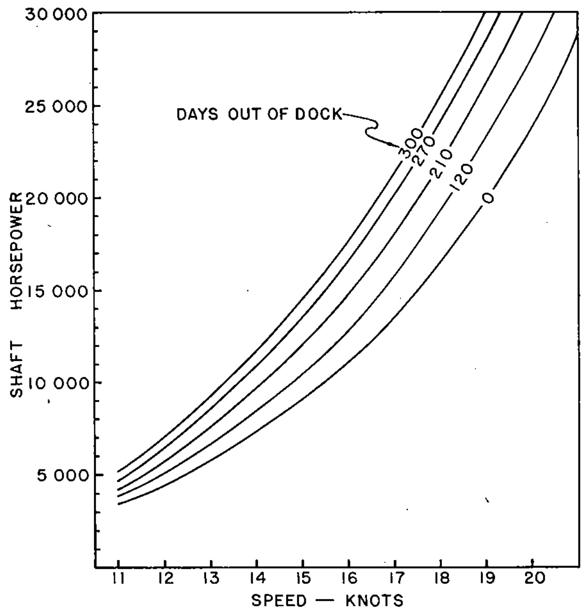

FIGURE 7. Shaft horsepower required to propel the battleship *Tennessee* at different speeds after various periods out of dock.

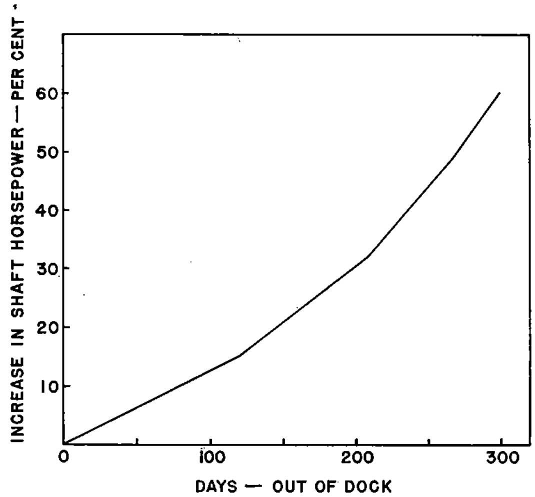

FIGURE 8. Percentage increase in shaft horsepower required to propel the battleship *Tennessee* at a speed of 15 knots after various periods out of dock.

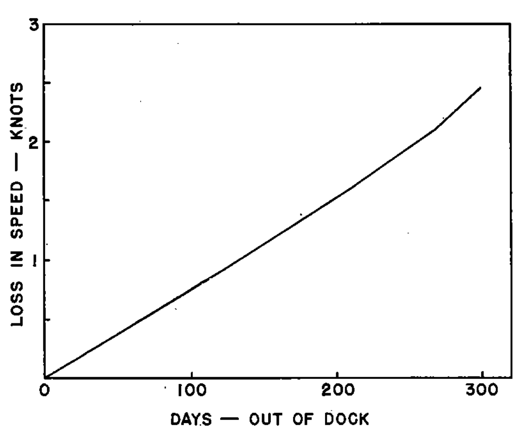

FIGURE 9. Loss in speed of battleship *Tennessee* at 23,500 shaft horsepower after various periods out of dock. The initial speed with clean bottom was 20 knots.

than 3 knots at a shaft horsepower which initially yielded 20 knots as shown in Figure 6. It was slightly less at higher speeds. The results with the battleship were somewhat less severe. In these tests and those on the *Yudachi* the general rate of increase in resistance was about ⅓ per cent per day. The condition of the bottom of these ships at the end of the period is not recorded.

Davis (*8*) has attempted to relate the development of excess shaft horsepower required to the development of fouling as controlled by the season and area of operation, as suggested in Figure 10.

While these quantitative tests support the many

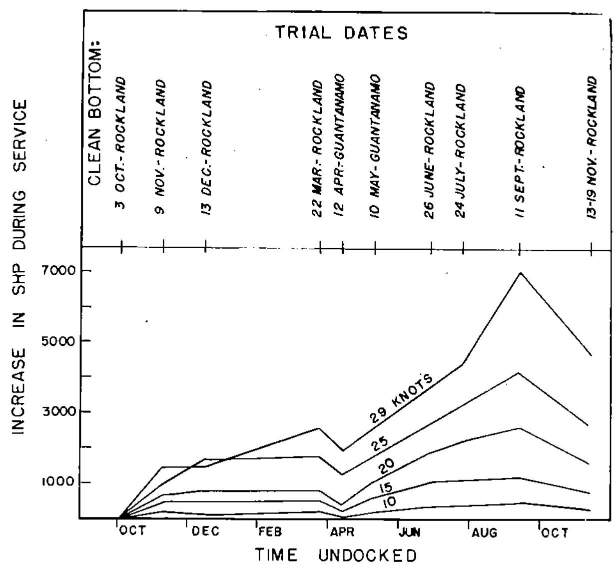

FIGURE 10. Increase in shaft horsepower required to propel the destroyer *Putnam* at various speeds in relation to season and area of operation. After Davis (*8*).

estimates of the severity of the effects of fouling on ship resistance which appear in the literature, it should be borne in mind that they probably represent the results of rather severe failure of the paint coatings. The paints used fifteen years ago were not to be depended on for more than six months. With the improved coatings now available, much less severe effects are to be expected. During the service in the recent war, fouling of the bottoms of active war vessels did not present a serious problem.

THE FRICTIONAL RESISTANCE OF SHIPS

Theoretical Formulation

According to the theory of ship resistance developed by William Froude, the total resistance, R_t, of a vessel moving at the surface of water is the sum of two components: (1) the frictional resistance, R_f, and (2) the residual resistance, R_r.

The frictional resistance is caused by tangential stresses due to the drag of the water moving parallel to the surface of the vessel.

The residual resistance is caused by the distribution of pressure which develops about the hull because of the waves and eddies occasioned by the ship's motion.

Froude (*9, 10*) found experimentally that the frictional resistance, R_f, of towed planks could be expressed by the relation

$$R_f = fSV^n \quad (1)$$

in which f is the coefficient of frictional resistance
S is the wetted surface in square feet
V is the velocity in knots
n is a number nearly equal to 2.

The values of both f and n depend upon the length of the plank and on the character of the surface, as shown in Table 1.

TABLE 1. William Froude's Plank Friction Experiments

Nature of Surface	*Length, L* 2 feet	8 feet	20 feet	50 feet
*Values for f**				
Varnish	0.0117	0.0121	0.0104	0.0097
Paraffin	0.0119	0.0100	0.0088	......
Calico	0.0281	0.0196	0.0184	0.0170
Fine Sand	0.0231	0.0166	0.0137	0.0104
Medium Sand	0.0257	0.0178	0.0152	0.0139
Coarse Sand	0.0314	0.0204	0.0168	......
Values for n				
Varnish	2.00	1.85	1.85	1.83
Paraffin	1.95	1.94	1.93	....
Calico	1.93	1.92	1.89	1.87
Fine Sand	2.00	2.00	2.00	2.06
Medium Sand	2.00	2.00	2.00	2.00
Coarse Sand	2.00	2.00	2.00	....

* The f values are for fresh water. For sea water multiply by 64/62.4.

As the result of towing experiments with planks, a plank ship of 77.3 feet W.L. and 0.525 foot beam, and actual ships with clean bottoms, Hiraga concluded that the frictional resistance of planks and ships exceeding 26 feet in length could be expressed by the similar equation

$$R_f = K_2 SV^{1.9} \quad (2)$$

in which the character of the surface affects only the value of the constant, K_2, which for a clean painted surface in sea water is 0.0104.

A number of attempts have been made to relate frictional resistance to the Reynolds number of the surface (*11, 19, 29*). This is a constant of fundamental importance in fluid mechanics whose value depends on the product VL/ν in which V is the velocity, L the length of the surface, and ν is the kinematic viscosity of the fluid medium. These equations take the form

$$R_f = C_f(\rho/2)SV^2 \quad (3)$$

where C_f, the coefficient of frictional resistance, has a value determined by the Reynolds number. The term $\rho/2$ permits the equation to be applied to water of any temperature and salinity, ρ being the mass density of the medium. A number of empirical equations have been proposed which express the relation between the coefficient of frictional resistance and the Reynolds number approximately, provided the Reynolds number is high enough to assure turbulent flow (*14, 22*). The Taylor Model Basin uses Gebers' formula which has the form

$$C_f = 0.02058\left(\frac{V \cdot L}{\nu}\right)^{-1/8}. \quad (4)$$

Recently Liljegren (*15*) has proposed a treatment which assumes that the frictional resistance of a plank may be divided into two components. For some distance behind the leading edge, energy is expended in accelerating the motion of the water. Further back the water flows past the surface at a constant velocity. The frictional resistance in the latter region may be expressed by a constant, C_2, which is independent of length or velocity. The excess resistance exerted behind the leading edge is expressed by a term, $C_1/LV^{3/4}$. The entire frictional resistance is consequently given by

$$R_f = \left(\frac{C_1}{LV^{3/4}} + C_2\right)SV^2. \quad (5)$$

These relationships are given only in enough detail to permit a presentation of the material to follow. For a fuller discussion, Taylor (*24*) or Davidson (*7*) may be consulted.

Relation of Frictional to Total Resistance

The condition of a ship's bottom, as determined by the character of the paint coating itself and the degree to which this coating permits corrosion or fouling, may be expected to have its effect primarily upon the frictional resistance. When the bottom is clean, the value of frictional resistance relative to the total resistance gives a basis for judging the importance of keeping the frictional resistance to a minimum.

The results of the towing tests on the Japanese destroyer *Yudachi* were broken down into frictional and residual resistance by Izubuchi (*13*). The frictional resistance was computed from the results of towing tests made with a plank 77.3 feet long and 0.525 feet thick as described by Hiraga (*12*). This was scaled up to apply to the 232-foot destroyer with the aid of formula (2) above. The result of the analysis is shown in Figure 11 from which

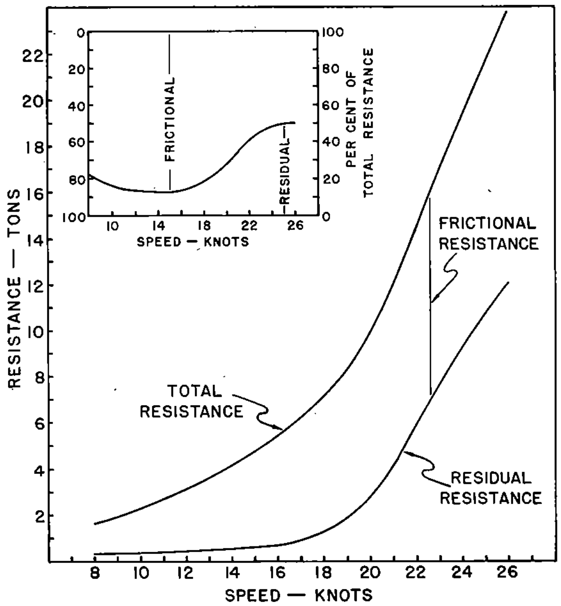

FIGURE 11. Analysis of the total resistance of the destroyer *Yudachi* into its components of frictional and residual resistance at various speeds. Inset. Percentage of total resistance due to frictional and residual resistance at different speeds. From data of Izubuchi (*13*).

it may be seen that at all speeds the residual resistance forms a relatively small portion of the total. In the inset of the figure the frictional resistance is expressed as a percentage of the total resistance at different speeds. At the comparatively low speed of 14 knots the frictional resistance amounts to as much as 87 per cent of the total. As speed increases, the relative importance of frictional resistance diminishes, but at the maximum speed of 27 knots it still amounts to as much as 50 per cent of the total.[1]

These results are concordant with estimates made from trial runs of the United States destroyer *Hamilton*, in which the percentage of the total resistance attributable to frictional resistance at several speeds were as follows.

Speed	Frictional Resistance
10 knots	67 per cent
20 knots	60 per cent
30 knots	41 per cent

[1] It should be noted that residual resistance usually does not increase steadily with speed; but increases rapidly at certain speeds and less rapidly at other intermediate speeds. This is because of the way in which the bow and stern wvaes "interfere" as speed increases. It is presumed to be the reason why the relative value of frictional resistance in the *Yudachi* tests does not decrease steadily from the lowest to highest speeds.

With fast ships at high speed the frictional resistance may account for an even smaller part, amounting to as little as 35 per cent of the total resistance.

Since frictional resistance is responsible for a relatively greater part of the total resistance in ships at low speed, it is important to keep this factor at a minimum in vessels such as cargo carriers which normally operate at relatively low speed-length ratios.

The fraction of the total resistance attributed to friction depends on the formula and on the basic data for the resistance of planks used in the computation. Thus Hiraga (*12*) found that the frictional resistance of the *Yudachi* given by his formula at speeds from 8 to 28 knots was 1.36 to 1.49 times that by Froude's and 1.58 to 1.63 times that by Gebers' formula. The degree to which the results depend on the basis of calculation is brought out in Table 2 in which the frictional resistance of a 400-foot vessel is estimated in a variety of ways. The estimations of frictional resistance based on the more recent formulations of Liljegren and Hiraga, and on the later determinations of plank resistance by Kempf and Hiraga, give the higher values. The methods of Liljegren and Hiraga are not generally accepted in this country, where the Gebers-United States Navy method and others which are closely comparable are preferred.

TABLE 2. Estimated Frictional Resistance of a 400-foot vessel assumed to have a wetted surface of 20,000 square feet and to develop a total resistance of 43,146 pounds at 16 knots and 212,333 pounds at 32 knots

Method of Estimation	*Frictional Resistance*		*Ratio of Frictional Resistance to Total Resistance*	
	16 knots	*32 knots*	*16 knots*	*32 knots*
	pounds		*per cent*	
Hiraga	40,082	149,591	92.9	70.5
Liljegren	38,840	143,360	90.0	67.5
Gebers-Kempf	33,000	125,000	76.5	58.8
Froude-Tideman	29,269	103,038	67.8	48.5
Gebers-U.S. Navy	25,800	94,100	59.8	44.3

Effect of Surface Roughness on Frictional Resistance

In estimating the resistance of a full scale ship from a towing test on a model, it is necessary to make allowance for the different texture of the surface of the model and of the actual ship bottom. In estimating the frictional resistance of the model, constants are employed appropriate to its smooth surface, which is usually varnished. In estimating that of the actual ship, the values of these constants are increased to take account

TABLE 3. Tideman's Constants for Frictional Resistance.*

For use in the equation $R_f = fSV^n$ where R_f is in pounds, S is in square feet and V is in knots. The values for varnished surface are from Froude. The constants are for sea water; for fresh water multiply by 62.4/64

Nature of Surface	*Length of Surface* 10	20	50	100	200	500
Values for f						
Varnish	0.011579	0.010524	....	....	....	....
Iron bottom						
Clean and painted	0.011240	0.010570	0.00991	0.00970	0.00944	0.00904
Copper or Zinc Sheathed						
Smooth, in good condition	0.010000	0.009900	0.00976	0.00966	0.00943	0.00926
Rough, in bad condition	0.014000	0.013500	0.01250	0.01200	0.01170	0.01136
Values for n						
Varnish	1.8250	1.8250	......	......	......	......
Iron bottom						
Clean and painted	1.8530	1.8434	1.8357	1.8290	1.8290	1.8290
Copper or Zinc Sheathed						
Smooth, in good condition	1.9175	1.9000	1.8300	1.8270	1.8270	1.8270
Rough, in bad condition	1.8700	1.8610	1.8430	1.8430	1.8430	1.8430

* As adopted by the International Congress of Model Basin Superintendents. Paris, 1935. For complete table see Davidson (7).

of its roughness, or a correction factor is employed to allow for its effect. It is also necessary to use constants applicable to the greater lengths of modern ships.

Froude's original studies on the frictional resistance of towed planks included observations on surfaces artificially roughened to various degrees. The values of the constants of equation (1) obtained with these surfaces are given in Table 1. Both constants, n and f, increase with the roughness of the surface. Neglecting the effect of n, which is important chiefly in defining the effect of velocity on the resistance, and focusing attention on the values of f, it may be noted that with 50-foot planks, the surface roughened with medium sand develops a resistance about 40 per cent greater than the smooth varnish surface. With shorter planks the difference is even greater.

An extended table of constants deduced from Froude's data was prepared by Tideman and served for many years as the basis of estimating the frictional resistance of ships from equation (1). Table 3 contains a selection of Tideman's constants and those of Froude which serve to illustrate the magnitude of the allowances which have been made for the actual roughness of clean ships' bottoms.

The United States Experimental Model Basin adopted coefficients of frictional resistance proposed by Gebers which are employed with equation (3) and which vary with the Reynolds number. A partial list of these values is given in Table 4. These values are for a smooth surface. In applying them to full-sized vessels it has been the practice to make an allowance for roughness by multiplying the ship's calculated frictional resistance by an appropriate factor. Its value is varied as may be considered desirable to suit vessels built with flush or lapped plating. The factor ranges from 1.14 for a 400-foot cargo vessel to 1.22 for a 900-foot battle cruiser (*20*).

TABLE 4. Geber's Coefficients of Frictional Resistance.*

For use in equation $R_f = C_f(\rho/2)SV^2$ where R_f is in pounds, C_f is dimensionless, ρ is in pounds per cubic foot divided by 32.2 feet per second, S is in square feet and V is in feet per second

Reynolds number	C_f
5×10^6	2.992×10^{-3}
1×10^7	2.744×10^{-3}
5×10^7	2.242×10^{-3}
1×10^8	2.060×10^{-3}
5×10^8	1.676×10^{-3}
1×10^9	1.544×10^{-3}
5×10^9	1.256×10^{-3}

* For complete table see Davidson (7).

Kempf (*14*) has developed a Roughness Coefficient, C_k, to express the effect of roughness on frictional resistance. The values of this coefficient were determined by towing tests with 252-foot pontoon variously roughened, and are given in Table 5, These values are to be added to smooth surface coefficients, given in Table 4, in applying equation (3); i.e.

$$R_f = (C_f + C_k)(\rho/2)SV^2.$$

TABLE 5. Kempf's Roughness Coefficients (C_k)

Surface	C_k
1. Plane, smooth surface of steel plates, with new paint but without rivets, butts, and straps. Average roughness about 0.012-inch.	0.10×10^{-3}
2. Same as 1, but with butts 0.79-inch high, spaced every 16.4 feet.	0.40×10^{-3}
3. Old copper-sheathed hull.	0.75×10^{-3}
4. New hull with new paint in normal condition with rivets, butts, and straps.	0.75×10^{-3}
5. Normal hull surface like 4, but after 22 years of service, newly painted but with roughening from rust.	0.75×10^{-3}
6. Plane surface with sand particles 0.0394-inch in diameter, covering 100 per cent of area. (about)	1.0×10^{-3}
7. Plane surface with barnacles 0.118 to 0.157-inch high, covering 25 per cent of area. (about)	3.0×10^{-3}

TABLE 6. Values of c_1 and c_2 in the Liljegren formula

Surface and conditions	c_1	c_2
Varnish, fresh water	0.0830	0.00625
Varnish, salt water	0.0851	0.00641
Steel, welded, salt water	0.0928	0.00665
Steel, lapped, salt water	......	0.00690
Ibid., U.S.S. *Saratoga*	......	0.00700

By comparing Gebers' coefficients for smooth surfaces given in Table 4 with the roughness coefficients in Table 5, it may be seen that the roughness coefficient adds significantly to the coefficient of frictional resistance.

Theoretically the roughness coefficient varies with the Reynolds number. Additional knowledge and experience may ultimately permit the roughness factor to be given in a form which takes account of this and other variables (*14*).

Values for C_k which agree well with Kempf's have been deduced from tests of the S.S. *Clairton* and of the United States destroyer *Hamilton* as follows (*7*):

	Reynolds number	C_k
S.S. *Clairton*	ca. 5.5×10^8	0.55×10^{-3}
U.S.S. *Hamilton*	ca. 1.2×10^9	0.42×10^{-3}

Liljegren (*15*) has utilized Kempf's data to evaluate the frictional coefficients of equation (4) for varnished and steel surfaces. This formula separates the resistance, C_2, due to moving through water at constant velocity from the excess resistance, C_1, arising from the acceleration of the water dragged by the surface. The values in Table 6 collected from Liljegren's book show that C_2 is 4 per cent greater for a welded steel surface than for varnish, while C_1 is 8 per cent greater.

While it is admitted that the whole matter of the effect of surface roughness is in a far from satisfactory state at the present time (*7*), the data which are available show that effects are produced by the conformation of the surface which are great enough to warrant serious study.

TABLE 7. Effect of Fouling on Frictional Resistance of Towed Steel Plates in McEntree's Experiments

Time of Immersion months	*Dry Weight of Fouling ounces per foot²*	f *clean*	f *fouled*	n *clean*	n *fouled*
1	0.8	0.0107	0.0114	1.869	1.994
2	0.4	0.0100	0.0128	1.918	1.928
3	0.6	0.0100	0.0167	1.937	2.029
4	2.8	0.0119	0.0239	1.855	2.002
5	2.8	0.0108	0.0255	1.874	2.003
6	3.6	0.0095	0.0252	1.938	1.988
7	4.0	0.0108	0.0275	1.880	2.000
8	3.2	0.0101	0.0267	1.912	2.000
9	2.0	0.0108	0.0275	1.869	1.967
10	3.6	0.0090	0.0285	1.848	2.015
11	3.2	0.0096	0.0273	1.914	2.055
12	3.2	0.0095	0.0292	1.924	2.035

Effect of Fouling on Frictional Resistance

The first comprehensive tests of the effect of fouling on the frictional resistance were made by McEntee (*16*). Steel plates 10 feet long and 2 feet wide were painted with anticorrosive paint and exposed in Chesapeake Bay, where they became fouled with "small barnacles." Their frictional resistance was determined periodically by towing at velocities ranging from 2 to 9 knots at the United States Experimental Model Basin. One plate was removed for testing each month and was subsequently cleaned, repainted, and tested again to obtain a measure of its unfouled resistance.

The tests showed that the resistance of the plates increased to four times the value for the clean plate in the course of twelve months. The values of the constants in Froude's formula, $R_f=fSV^n$, are presented in Table 7. They show that the value of f increases about threefold as a consequence of the fouling. The value of n in the equation increases from about 1.9 to about 2.0, as expected from Froude's experiments with roughened planks. The increase in frictional resistance, f, parallels roughly the determined weight of fouling per unit area.

Izubuchi (*13*) has estimated the coefficient of frictional resistance of the destroyer *Yudachi* from the trials made during a year-long period in which the resistance increased, presumably as the result of fouling and corrosion. The values of K_2 and n in the equation of Hiraga, $R_f=K_2SV^n$, obtained after various periods were the following:[2]

Days undocked	K_2	n
4–5 (clean)	0.00995	1.9
75	0.00635	2.1
140	0.00763	2.1
225	0.00881	2.1
375	0.01225	2.1

The value of K_2 decreases at first, presumably as a consequence of the increased value of the n exponent. Subsequently K_2 increases regularly with the time of exposure, and doubles during the last 300 days of the tests. Attempts to quantitate the fouling occurring on the *Yudachi* were unsatisfactory, though they showed that fouling on the ship was substantial.

Hiraga (*12*) records the effect of fouling on the resistance to towing of a brass plate coated with

[2] The values of K_2 are recalculated to apply when S is measured in square feet and resistance in pounds instead of the metric units employed by the author.

Veneziani composition. After 24 days' immersion, barnacles grew on the surface of this plate with the result that K_2 increased from 0.01046, characteristic of the clean surface, to 0.0130. During the towing test the resistance decreased until the plate had been towed 18,000 feet, after which it remained constant with $K_2 = 0.01262$, as shown in the upper curve of Figure 12. Thus the fouling with barnacles increased the resistance about 20 per cent. The initial fall in resistance during towing was attributed to the washing off of slime, as discussed in the following section.

Kempf (*14*) has measured the effect of fouling on the frictional resistance of a pontoon 252 feet long. From the results he estimated a roughness coefficient, C_k, to be applied in the formula

$$R_f = (C_f + C_k)\ (\rho/2)SV^2$$

as explained on page 27. The value of C_k was found to be about 3.0×10^{-3} for fouling with barnacles 0.118 to 0.157 inch high covering 25 per cent of the area. Estimates made from the trials of the destroyer and battleship, described on page 23, indicate that the increase in resistance of these ships while waterborne may be accounted for by roughness coefficients having the following values (*7*):

Destroyer—after 8 months	$C_k = 3.62\times10^{-3}$
Battleship—after 10 months	$C_k = 2.43\times10^{-3}$

These values are concordant with the roughness coefficient obtained by Kempf.

The order of magnitude of the effect of fouling predicted by Kempf's roughness coefficient on the frictional resistance of a ship may be obtained from the following comparison.

Unfouled ship

C_f for smooth surface—see Table 4 at Reynolds number 1×10^{-8}	2.0×10^{-3}
C_k for butted steel plates after Kempf	0.4×10^{-3}
$(C_f + C_k)$—unfouled ship	2.4×10^{-3}

Fouled ship

C_f for smooth surface	2.0×10^{-3}
C_k for barnacle fouling after Kempf	3.0×10^{-3}
$(C_f + C_k)$—fouled ship	5.0×10^{-3}

The frictional resistance of the fouled ship is thus $5.0/2.4 = 2.08$ times that of the unfouled vessel.

The three investigations of the effect of fouling on frictional resistance which have been summarized agree in indicating that fouling may more than double the frictional resistance of a moving submerged surface. The data are quite inadequate in regard to the quantitative effects of various degrees of fouling, or of the geometry of the roughened surface produced by various types of sessile organisms.

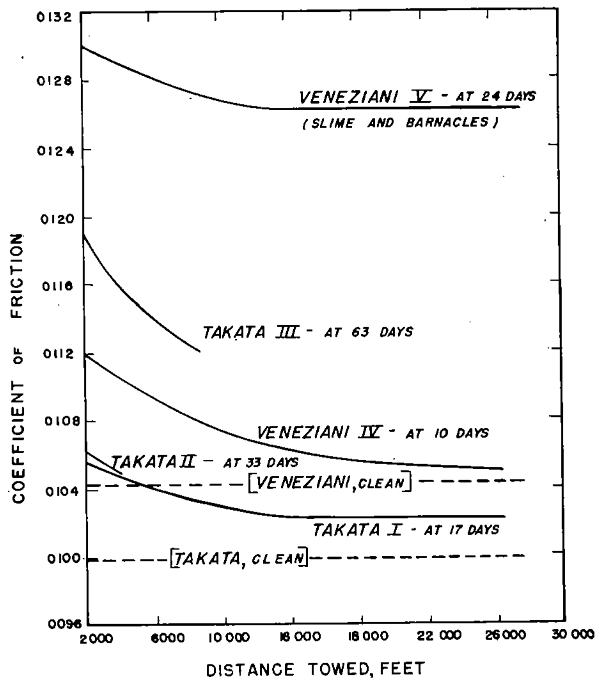

FIGURE 12. Coefficient of friction of towed brass plates coated with Veneziani and Takata antifouling paints. Each curve represents the results of a test made after the period of immersion indicated. The curves show the fall in resistance which occurs as the plate is towed during each day's test. After Hiraga (*12*).

The Effects of the Slime Film on Frictional Resistance

A number of observations indicate that the frictional resistance of a submerged surface may increase with time of immersion in the absence of macroscopic fouling. This effect is attributed to the slime film, formed by bacteria and diatoms, which rapidly develops on surfaces exposed in the sea. For example, in discussing the paper of McEntee Sir Archibald Denny stated that vessels lying in the brackish water of the fitting out basin on the river Leven increased their friction nearly ½ per cent per day for several months even when there was no apparent fouling (*16*).

Tests conducted at Langley Field with the object of determining the effect of various paint systems on frictional resistance give some quantitative information on this subject (*1, 3, 4, 5*). Painted plates, 10 feet by 2 feet in size, were exposed for periods up to one month in sea water and towed at intervals of a few days at speeds ranging from 12 to 24 feet per second. No evidence of a change in resistance was observed in the plates at the end of 24 hours' immersion. After 48 hours the

TABLE 8. Effect of Slime Film on Resistance to Towing of Plates Coated with Paint in Tests at Langley Field.

The plates were given a preliminary run to remove loosely adhering slime before testing.

Paint	*Exposure days*	*Velocity feet/second ±0.1*	*Resistance pounds ±0.3*	*Resistance per cent increase*
Moravian	0	22.2	58.5	0.8
	10	21.0	59.0	
15RC	0	22.8	55.2	
	10	22.1	57.7	4.5
15A	0	23.6	61.2	
	10	22.5	64.2	4.9

resistance of the plate coated with Moravian antifouling paint increased 1½ per cent, that with anticorrosive paint 15A showed a greater increase in resistance, while that coated with antifouling paint 15RC showed no change. After five days' exposure, 15RC also showed an increased resistance which amounted to 11 per cent on the tenth day, when the increase in resistance of Moravian had mounted to 13 per cent. The results obtained are attributed to the effects of the slime film which formed on the plates, since no macroscopic fouling was present except for a few barnacles which appeared on 15A after 25 days' exposure.

It was found that when towing a plate, some of the deposit of slime would peel off. On 15A the deposit washed off readily, but on 15RC enough slime remained to leave the paint surface with a muddy appearance. On the Moravian the slime formed a thin membrane that exfoliated at very low towing speeds. After 25 or 30 days' exposure there were two membranes of slime, an outer one which was washed off by towing and a thin inner one which persisted and gave a marked increase in the resistance.

In order to overcome the variation in resistance caused by the washing off of the slime film during a test, each plate was given a preliminary scrubbing run at 20 feet per second to remove as much of the loose film as would come off during the runs. The results obtained with these relatively stable films are given in Table 8.

TABLE 9. Effect of Fouling with Slime on the Resistance of Plates in Hiraga's Experiments

Plate Number	*Composition*	*Period of Immersion days*	*Distance Towed, feet: 0–5,000 K_2*	*Distance Towed, feet: 20,000–25,000 K_2*
—	Takata	0 (clean)	0.01000	....
I	Takata	17	0.01056	0.01018
II	Takata	33	0.01062†	....
III	Takata	63	0.01190†	....
—	Veneziana	0 (clean)	0.01046	....
IV	Veneziana	10	0.01119	0.01048
V*	Veneziana	24	0.01300†	0.01262†

* This plate was fouled with barnacles.
† Data from Hiraga's graph.

Towing tests with friction plates described by Hiraga (*12*) also gave an increased resistance which may be attributed to the formation of slime on the painted surface and its subsequent partial removal during towing. Hiraga exposed thin brass plates coated with Veneziani and Takata compositions in the sea for various periods and then tested their resistance in a towing tank. The plates were towed 5,000 feet each day. It was observed that the resistance was higher on the first day and decreased progressively with each day's towing, when after three or four days it reached a constant value, still in excess of the resistance of the cleaned plate. Hiraga's results were presented graphically as shown in Figure 12. The numerical values in Table 9 are extracted from his text supplemented by the data presented in the figure.

These tests, like those from Langley Field, indicate that the frictional resistance of the paint surface may increase as the result of the formation of slime film, but that after towing, the resistance is reduced to within a few per cent of the initial value for the clean surface. It may be presumed that with ships in service the slime film will be reduced by the motion of the ship through the water, and that its presence will not greatly affect the total resistance to motion.

It is of interest to observe that the magnitude of the effects vary with the particular paints on which the film forms. Some minor advantage might be achieved by the use of formulations which discourage slime formation or result in flocculent films which will be readily washed away.

Effect of Paint Surface on Frictional Resistance

Paint technologists are well aware that the antifouling compositions applied to larger ships differ greatly in the smoothness of the resulting surfaces, both as the result of the inherent properties of the paint and because of different methods of application. Spray application may result in a "pebbly" surface; some coatings tend to sag, and some may flow if the ship is set in motion before the paint film has had time to harden adequately, resulting in a surface such as that illustrated in Figure 13. Although such effects may be readily avoided, relatively little data exist to gauge their importance except for the measurements on artificially roughened planks discussed above.

The systematic towing tests with painted planks made at Langley Field and referred to in the discussion of the effect of slime formation, were designed to show the effects of the paint surface on frictional resistance. The results given in Table 8 show that the fresh surface of Moravian developed about 6 per cent more resistance at comparable speed than did the surface of the standard formula 15RC.

Hiraga (*12*) also reports the results of plank tests, shown in Table 9, which indicate that the Veneziani surface develops about 4 per cent more resistance when clean than the Takata coating.

No towing tests appear to have been made with the modern hot or cold plastic shipbottom paints in current use by the Navy, nor of the variety of special compositions, such as the bronze yacht paints, which are favored for small boats in which high speed is desired.

The possible advantage to be gained by polishing or lubricating the bottom was examined by McEntee (*16*) in tests conducted at the United States Experimental Model Basin. The tests showed no advantage of a coating of black lead, oil, or soap over the original shellac surface. The results obtained are given in Table 10.

Trials on ships with clean bottoms, made before fouling could become significant, have sometimes indicated the superiority of one coating over another. Thus the U.S.S. *Marblehead* (*28*) reported that a 6 per cent increase in horsepower was required to obtain a given speed, when coated with Moravian shipbottom paint, as compared to the results expected with 15RC, the standard formulation then in use. The effect was attributed to the roughness of the Moravian paint and is consistent with the results of the Langley Field tests with planes. An application of an experimental plastic paint developed at the Edgewood Arsenal caused a reduction in speed of the U.S.S. *Dent* (*27*) equivalent to that due to five months' fouling with standard coating. This effect again was attributed to roughness. Tests of this character are not very convincing in view of the large number of factors which are involved in determining the results of trial runs if they are inadequately controlled.

TABLE 10. Resistance of "Lubricated" Shellac Surfaces After McEntree (*16*)

f and n are the values in the formula $R_f = fSV^n$. $S = 82$ square feet.

Plane	*Surface*	*Net Resistance 7 knots pounds*	*Increase in Resistance at 7 knots per cent*	f	n
1	Shellac	28.1	..	.00878	1.883
2	Shellac	27.4	..	.00849	1.886
2	Black Lead over Shellac	27.9	2	.00866	1.886
1	Light Engine Oil over Shellac	28.3	5*	......	.....
1	Ivory Soap over Shellac	34.5	23	.01045	1.898
2	Heavy Cylinder Oil over Shellac	40.5	48	.00484†	2.380†

* At 6 knots.
† This low coefficient of resistance is combined with a high velocity exponent and probably would become greater at speeds lower than those at which experiments were made.

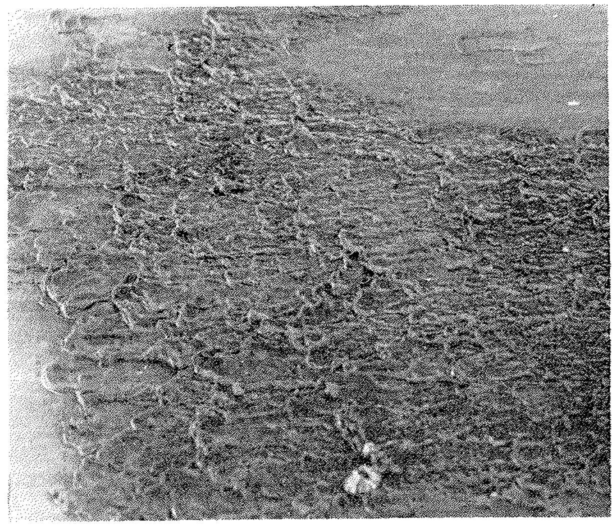

FIGURE 13. Roughened surface of cold plastic antifouling paint, resulting from cold flow due to operation before the film had hardened properly.

The purpose of antifouling coatings is to keep the frictional resistance as low as possible for a maximum period. The resistance of the clean surface is important only as long as fouling with slime or macroscopic organisms is prevented. The final value of the paint system should be judged by the integration of resistance during the waterborne period. Only two series of trials appear to have been made which compare the virtues of various paint systems by systematic measurements of resistance during the undocked period.

The four members of Destroyer Division 27 were each coated with a different antifouling paint system and were subjected to careful speed trials at subsequent intervals. The first series of trials was terminated after about six months because of the unexpected failure of the paint systems. The vessels were repainted and subjected to a second series of trials which were successfully continued for 70 weeks (*25*). To check the conclusions from these trials, a second series of tests was made on Destroyer Division 28 (*26*).

The results of these tests are of interest in showing 1) the effect of the different coatings on the performance of the ships while they are in a clean

TABLE 11. Comparison of Results of Full-Scale Tests with Freshly Painted Bottoms and Results Predicted from Model Studies for Clean Bottom Conditions

The numbers indicate the average percentage difference from the prediction in RPM required in trial for a range in speed of 12–22 knots.

	Division 27		
Coating	*First Series*	*Second Series*	*Division 28*
Navy Standard (15RC)	—	−0.75	+1.4
Mare Island Hot Plastic	+2.9	+0.30	+1.4
Moravian Imported	+0.2	+0.75	+3.1
NRL Plastic	—	—	+3.4
Edgewood Plastic	+2.4	+2.10	—
Norfolk 15 PA	+1.3	—	—

condition, and 2) the relative value of the coatings in preventing the increase in resistance which would result from fouling or corrosion during service.

The effect of the fresh paint coatings on the performance of the ships can be brought out only by comparing the actual performance of the ships during trials immediately after undocking with the results predicted from model studies. Such a comparison is made in Table 11 for the three series of tests. These results demonstrate how closely the performance may be predicted from model studies, and suggest that the characteristics of the various paint systems produce very little difference. Such differences as do appear can not be attributed to the paint itself with any assurance, since the influence of variations in smoothness of the ship's plating and the influence of propeller characteristics are not excluded from the comparison.

The relative value of the different coatings in maintaining the initial low resistance during a prolonged period of service is demonstrated clearly by the data presented in Table 12, based on the trials of Division 28.

It is evident that in the long run the *Southard*, coated with Mare Island Hot Plastic, did much better than the others. The *Chandler*, painted with the standard Navy formulation, equalled the *Southard* in performance during the first four months of the tests, but subsequently developed increasing resistance, presumably as the paint failed. The *Hovey* and *Long*, coated with Moravian and an experimental imitation of this plastic, both developed greatly increased resistance between the second and fourth month of service.

TABLE 12. Results of Trials of Destroyer Division 28 Designed to Compare the Change in RPM Required to Maintain Given Speed during Undocking with Various Paint Applications
Ships undocked 6 May 1938

Ship	*Southard*	*Chandler*	*Hovey*	*Long*
Paint	*Mare Island Hot Plastic*	*Navy Standard (15RC)*	*Moravian Imported*	*N.R.L. Plastic*
Trials	*Per cent increase in RPM*			
6–7 June	1.4	1.4	3.1	3.4
6–7 September	4.9	5.2	12.2	13.8
28–29 November	3.0	6.9	10.8	12.3
3–7 March	3.5	7.9	10.4	13.8
5–6 June	4.4	11.1	12.2	13.2
5–6 September	7.5	14.0	14.2	14.4

The tests on Destroyer Division 28, made in 1938, show a great improvement in the paint coatings over those in use in 1922–1923 when the trials of the destroyer *Putnam* and the battleship *Tennessee* were run. The shaft horsepower required by these ships to maintain a given speed was increased practically 100 per cent as the result of increased frictional resistance during less than one year of service. Tests of the destroyer *McCormick* undocked on October 6, 1936, after painting with Mare Island Plastic Paint, showed an average increase in shaft horsepower of 42 per cent required to maintain a given speed after 450 days of service (*6*). The tests of the U.S.S. *Southard* in 1938 indicated an increased power requirement of 38 per cent with Mare Island Plastic after 16 months' service, as compared with 70 per cent required by the *Chandler*, which was coated with the then standard 15RC antifouling paint.

How much improvement has subsequently been achieved is undetermined. Prior to the war the Rules for Engineering Competition allowed for a 3 per cent increase in fuel consumption per month waterborne. It is reported that during the war in the Pacific it was found unnecessary to make any allowance for increased fuel consumption due to fouling. Whether this was due to the improvement in underwater coatings, or to the greater activity of the ships in wartime, can not be stated with assurance. It is evident, however, that the very large losses in ease of propulsion which may result from fouling of the bottom have been substantially reduced through advances in paint technology.

The Effect of Fouling on Propellers

According to modern theory, the blade of a propeller may be likened to an airfoil which develops "lift" (thrust) as a result of the pattern of flow about the blade. Actually the decrease in pressure at the back of the blade can be demonstrated to be greater than the increase in pressure at its face (*23*). It is consequently to be expected that any condition, such as roughening of the surface by fouling, which disturbs the flow pattern will have a marked effect on the development of propulsive force.

Bengough and Shepheard (*2*) have described the case of the H.M.S. *Fowey* which failed to develop the anticipated speed on its initial trials.

When subsequently docked, the propellers were found to be almost completely covered with calcareous tube worms. On the bosses the hard tubes were about $1\frac{1}{4}$ inches long. Toward the tips of the blades the fouling had been washed off during the trials. The condition of the bottom was good except for patches of worms about 2 inches thick where holidays had been left in the antifouling paint. (See Figure 14.) After cleaning, the trials were repeated and the anticipated speed was realized. While it is probable that the improvement was due to cleaning the propellers, the effects of

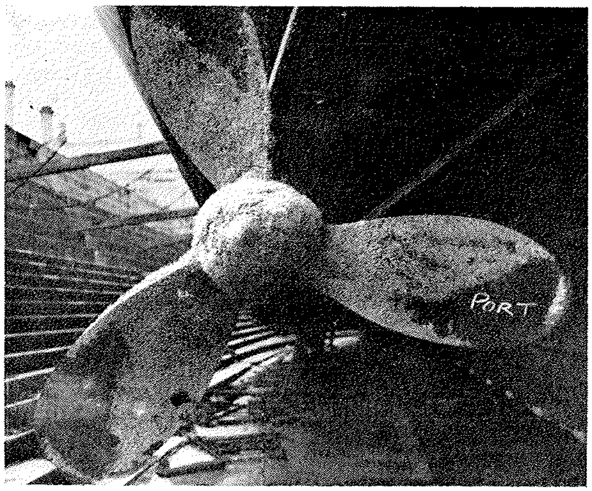

FIGURE 14. Fouling of propeller of H.M.S. *Fowey*. After Bengough and Shepheard (*2*).

the patches of fouling on the bottom can not be completely ruled out.

Speed trials of the destroyer *McCormick* indicate that about two-thirds of the increased fuel consumption due to fouling is due to its effect on the propellers. After 226 days out of dock the average fuel consumption required to maintain a given speed had increased to 115.8 per cent of the consumption with clean bottom. After cleaning the propellers, the fuel consumption dropped to 105.5 per cent. Thus in seven months the propellers alone were responsible for a 10 per cent increase in fuel consumption (*6*).

More satisfactory evidence comes from experiments on model propellers, artificially roughened. In experiments at the United States Navy Model Basin, McEntee (*17*) determined the efficiency of four similar propellers, one of which was smooth, the others in the rough condition of the original casting. The results are shown in Figure 15, and indicate that a loss of efficiency amounting to about 10 per cent results from the roughness of the cast surface. In another test a model propeller

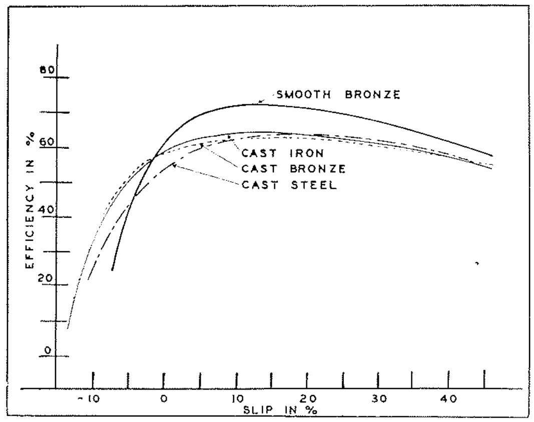

FIGURE 15. Effect of surface roughness on the efficiency of four similar mode propellers. After McEntee (*17*).

was painted and roughened by stippling while the coating was wet. The results, shown in Figure 16, indicate a loss in efficiency of about 20 per cent as a result of the stippling. Finally, tests were made on a propeller covered with ground cork which caused the efficiency to drop from over 70 to about 35 per cent.

Taylor (*24*) concludes that most ships operating with propellers in moderately good condition suffer an avoidable waste of power in the order of 10 per cent above that obtainable with new, accurately finished bronze propellers. It may be supposed that roughness of a grosser sort occasioned by fouling will produce much greater losses in efficiency, and will readily explain such results as those recorded for the H.M.S. *Fowey*.

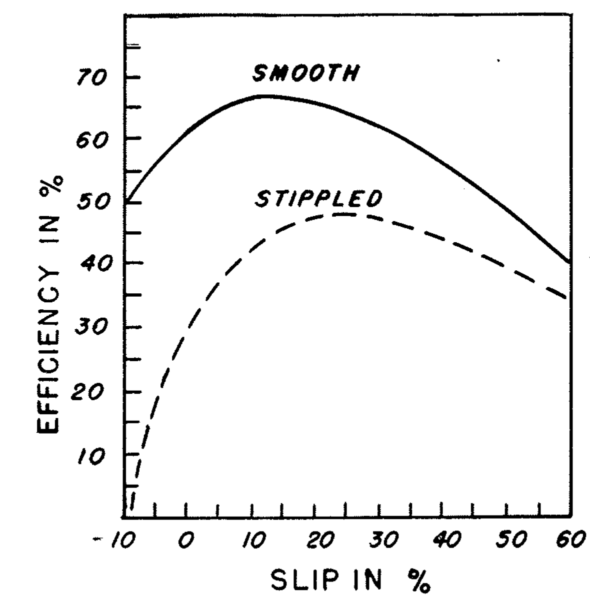

FIGURE 16. Comparison of the efficiency of a model propeller in the smooth condition and after roughening by stippling a wet paint coating. After McEntee (*17*).

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