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Logistics scheduling to minimize inventory and transport costs

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ABSTRACT

We study a logistics scheduling problem where a manufacturer receives raw materials from a supplier, manufactures products in a factory, and delivers the finished products to a customer. The supplier, factory and customer are located at three different sites. The objective is to minimize the sum of work-in-process inventory cost and transport cost, which includes both supply and delivery costs. For the special case of the problem where all the jobs have identical processing times, we show that the inventory cost function can be unified into a common expression for various batching schemes. Based on this characteristic and other optimal properties, we develop an $O(n)$ algorithm to solve this case. For the general problem, we examine several special cases, identify their optimal properties, and develop polynomial-time algorithms to solve them optimally.

1. Introduction

The production logistics activity of enterprises is typically composed of three stages, namely supply, production and distribution. In recent years, much of the literature has studied logistics scheduling that integrates production scheduling and job delivery to customers. For example, the reader is referred to Lee and Chen (2001), Chang and Lee (2004), Chen and Vairaktarakis (2005), Hall and Potts (2005), Pundoor and Chen (2005, 2009), Chen and Pundoor (2006), and Wang and Cheng (2007, 2009a). This line of research focuses on optimizing the total distribution cost and customer service level. On the other hand, for scheduling research that takes supply and production into consideration, Selvarajah and Steiner (2005) presented a polynomial-time algorithm to minimize the sum of total inventory holding cost and product batch delivery cost. Qi (2005) considered a logistics scheduling model that deals with material supply and job scheduling at the same time, where the objective is to minimize the sum of work-in-process (WIP) inventory cost and raw material supply cost.

There are research results on logistics scheduling that deals with all the three stages of supply, production and delivery. Hall and Potts (2003) considered multiple-production-stage scheduling with batch delivery in an arborescent supply chain. They analyzed the complexity of the problems and developed some dynamic programming algorithms. Wang and Cheng (2009b) considered a machine scheduling problem with supply and delivery of materials and products, where the warehouse, the factory and the customer are located at three different sites. The objective is to minimize the makespan. They did not take the WIP inventory cost into consideration.

In this paper we formulate a logistics scheduling model that considers production scheduling, raw material supply and product delivery at the same time. We assume that the supplier, manufacturer and customer are located at three different sites. Transportation service is provided by a third party, so transport vehicles are available at any time. The manufacturer needs to pay the third party for its service to transport materials to the factory, and deliver products to the customer. A job may be transported from

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the supplier's warehouse to the manufacturer's factory at any time just before it starts processing, and a product is available for delivery to the customer as soon as it finishes processing in the factory. On the other hand, since the cost of processing all the jobs in a planning period is normally fixed and independent of the production schedule used, we only consider the cost of holding intermediate inventory, which is in terms of the WIP inventory level of the factory. The problem under study is to find an optimal joint schedule for material supply, production scheduling, and job delivery so that the sum of WIP inventory cost and transport cost is minimized. This logistics scheduling problem models the practical situation where a single dominant firm controls both upstream and downstream stages in a supply chain. So the firm could, in the short run, simply optimize its own operational decisions regardless of the impact of such decisions on the other stages of the chain (see, e.g., Erengüc et al., 1999).

The rest of the paper is organized as follows. In Section 2 we formally describe our model and present the notation. In Section 3 we develop an optimal algorithm for the special case of the problem where all the jobs have identical processing times. In Section 4 we examine several special cases of the general problem, identify their optimal properties, and develop polynomial-time algorithms to solve these cases optimally. In the last section we conclude the paper and suggest topics for future research.

2. Description and notation

Suppose that the manufacturer receives n orders (jobs), $N = \{J_1, J_2, \ldots, J_n\}$, from a customer. Here, the customer may represent a distribution center that serves some customers that are close to one another in a geographical area. The n jobs are processed by a single machine (facility) in the factory. Each job J_i has a processing time p_i. The jobs as raw materials before processing in the factory have to be transported from the supplier's warehouse. We suppose that a vehicle can load at most K_s jobs on a supply trip from the warehouse to the factory, and the transport cost of a supply trip is $\mu_s + x_s\gamma_s$, where μ_s is a fixed cost for each supply trip, γ_s is the cost per loaded job and x_s is the number of the loaded jobs in the supply trip. All the jobs on a supply trip constitute a supply batch. The jobs as products after processing in the factory need to be delivered to the customer. We suppose that a vehicle can load at most K_d jobs on a delivery trip from the factory to the customer, and the transport cost of a delivery trip is $\mu_d + y_d\gamma_d$, where μ_d is a fixed cost for each delivery trip, γ_d is the cost per loaded job and y_d is the number of the loaded jobs in the delivery trip. All the jobs on a delivery trip constitute a delivery batch.

By controlling the sizes of supply and delivery batches, the arrival times of supply batches, the departure times of delivery batches, and selecting a suitable sequence to process the jobs in the factory, we seek to minimize the sum of WIP inventory cost and transport cost.

The following notation will be used throughout the paper:

B_k^s: the kth supply batch;
t_k^s: the arrival time at the factory of supply batch B_k^s;
B_h^d: the hth delivery batch;
t_h^d: the departure time from the factory of delivery batch B_h^d;
$\varphi = [B_1^s, B_2^s, \ldots, B_u^s]$: a supply scheme that transports all the jobs from the warehouse to the factory, where u is the number of supply batches in a supply scheme;
$x_k = |B_k^s|$: the number of jobs in B_k^s for $k = 1, 2, \ldots, u$;
$X = (x_1, x_2, \ldots, x_u)$: a vector denoting the numbers of jobs in the supply batches;
$\psi = [B_1^d, B_2^d, \ldots, B_v^d]$: a delivery scheme that transports all the jobs from the factory to the customer, where v is the number of delivery batches in a delivery scheme;
$y_h = |B_h^d|$: the number of jobs in B_h^d for $h = 1, 2, \ldots, v$;
$Y = (y_1, y_2, \ldots, y_v)$: a vector denoting the numbers of jobs in the delivery batches;
$\lceil x \rceil$: the smallest integer that is no less than x;
$\lfloor x \rfloor$: the largest integer that is no larger than x.

The WIP inventory cost in the factory should in theory be a function of the sum of the time that each job spends in the factory. We assume that the WIP inventory cost associated with a job J_i is $c_i(k,h) = \alpha(t_h^d - t_k^s)$, if $J_i \in B_k^s$ and $J_i \in B_h^d$, where $\alpha(>0)$ is the inventory cost of each job per time unit. So the objective function is given by

$$F(\varphi, \psi) = (u\mu_s + n\gamma_s) + (v\mu_d + n\gamma_d) + \sum_{J_i \in B_k^s, B_h^d} c_i(k,h) \qquad (1)$$

where the three terms on the RHS represent the supply cost, the delivery cost, and the total WIP inventory cost, respectively.

For the problem under study, there exists an optimal solution in which the following properties hold obviously.

Observation 1: Once a supply batch arrives at the factory, a job in the batch begins processing.

Observation 2: Once all the jobs of a delivery batch have finished processing, the delivery batch should depart from the factory.

Observation 3: There should be no idle time between the first and the last processed jobs in the factory.

Observation 4: The jobs belonging to the same supply batch should be processed consecutively, and a delivery batch should consist of consecutively processed jobs.

In the following discussion, we only consider solutions that possess the properties stated in Observations 1–4.

3. Identical job processing times

In this section we first give a unified expression of the objective function for the case where the jobs have identical job processing times. We then prove some optimal properties, and develop an $O(n)$ algorithm for this case. This case models the practical situation of a monopolistic manufacturer that specializes in making a

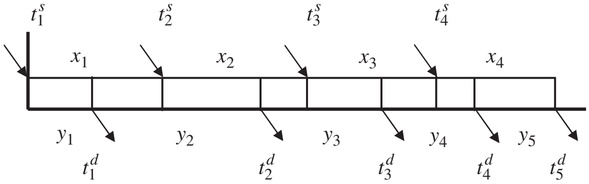

Fig. 1. An example.

single product, or where a manufacturer dedicates the factory to producing a single model of its products over an extended period of time as a result of successful bidding for a long-term, high-volume contract to supply the model to a major customer.

Based on Observations 1–4, when the jobs have identical processing times, in order to obtain an optimal solution, the main tasks are obviously to determine the numbers of supply and delivery batches, and the numbers of jobs in each supply and delivery batch. Since job sequence has no effect on the objective function in this case, we assume in the following that the job sequence is $(J_1, J_2, \ldots, J_n)$.

From (1) and Observations 1–4, we see that the WIP inventory cost function of any instance with identical job processing times can be obtained by multiplying a constant to that of the instance with unit job processing times. So, without loss of generality, we assume that the jobs have unit processing times in the sequel.

For example, there are n jobs with unit processing times, and the supply scheme and delivery scheme are given as $\varphi = [B_1^s, B_2^s, B_3^s, B_4^s]$ and $\psi = [B_1^d, B_2^d, B_3^d, B_4^d, B_5^d]$, respectively. The numbers of jobs in individual batches are shown in Fig. 1. For the first delivery batch, the arrival time of its jobs is $t_1^s = 0$ and its departure time is $t_1^d = y_1$, so each job stays in the factory for time y_1. Then, the total WIP inventory cost of jobs in the first delivery batch is αy_1^2. The WIP inventory cost function $f(X, Y)$ is given by

$$\begin{aligned} f(X,Y) = \alpha\{&y_1^2 + (x_1 - y_1)(y_1 + y_2) + [(y_1 + y_2) - x_1]^2 \\ &+ [(x_1 + x_2) - (y_1 + y_2)][(y_1 + y_2 + y_3) - x_1] \\ &+ [(y_1 + y_2 + y_3) - (x_1 + x_2)]^2 + [(x_1 + x_2 + x_3) \\ &- (y_1 + y_2 + y_3)][(y_1 + y_2 + y_3 + y_4) - (x_1 + x_2)] \\ &+ [(y_1 + y_2 + y_3 + y_4) - (x_1 + x_2 + x_3)]^2 \\ &+ x_4 y_5\} \end{aligned} \tag{2}$$

Thus, the objective function is

$$F(\varphi, \psi) = (4\mu_s + n\gamma_s) + (5\mu_d + n\gamma_d) + f(X, Y). \tag{3}$$

Let the sets $A_1 = \{1, 2, \ldots, t_2^s\}$, $A_2 = \{t_2^s + 1, t_2^s + 2, \ldots, t_3^s\}$, $\ldots, A_u = \{t_u^s + 1, t_u^s + 2, \ldots, n\}$. The departure time of a delivery batch must belong to one of $A_1, A_2, \ldots, A_u$. In the above example, $A_1 = \{1, 2, \ldots, x_1\}$, $A_2 = \{x_1 + 1, \ldots, x_1 + x_2\}$, $A_3 = \{x_1 + x_2 + 1, \ldots, x_1 + x_2 + x_3\}$ and $A_4 = \{x_1 + x_2 + x_3 + 1, \ldots, n\}$. The numbers of delivery batches with departure times in A_1, A_2, A_3 and A_4 are 1, 1, 1 and 2. We define the numbers of delivery batches departing in u sets $A_1, A_2, \ldots, A_u$ as a *delivery batch distribution*. We denote a delivery batch distribution as $\Delta = (l_1, l_2, \ldots, l_u)$, where $l_1 + l_2 + \cdots + l_u = v$. If there is no delivery batch departing from the factory in A_k, then $l_k = 0$.

From (2) and (3), for given u and v, we see that if two solutions of the problem have the same delivery batch distribution, then their WIP inventory cost functions may be unified into a common expression. Different delivery batch distributions correspond to different expressions of the WIP inventory cost function. We may evaluate the number of different expressions from the following example. For example, $n = 2m$, $u = v = m$, and there are $C_{n-1}^m (= (n-1)!/m!(n-m-1)! > 2^{n/2-1})$ supply schemes. Obviously, there are at least C_{n-1}^m delivery batch distributions since each supply scheme corresponds to at least a delivery batch distribution by selecting suitable departure times for the $m - 1$ delivery batches. So, there may be at least C_{n-1}^m inventory cost expressions, which is an exponential function of n. For different expressions of the WIP inventory cost function, we have the following lemma.

Lemma 1. *For given u and v, the inventory cost functions of all the solutions have a unified expression.*

Proof. See Appendix A.

Utilizing Lemma 1, we may obtain a unified expression for the inventory cost function for any distribution. For a distribution Δ (see Fig. 2), $f(X, Y)$ is given by

$$\begin{aligned} f(X,Y) = \alpha\{&y_1^2 + (y_1 + y_2)y_2 + \cdots + (y_1 + \cdots + y_{v-1})y_{v-1} \\ &+ n[(n - (x_2 + \cdots + x_u)) - (y_1 + \cdots + y_{v-1})] \\ &+ x_2(x_2 + \cdots + x_u) + x_3(x_3 + \cdots + x_u) \\ &+ \cdots + x_{u-1}(x_{u-1} + x_u) + x_u^2\} \end{aligned} \tag{4}$$

Since $x_1 + \cdots + x_u = y_1 + \cdots + y_v = n$, there are $u - 1$ and $v - 1$ independent integer variables in vectors X and Y, respectively. For $f(X, Y)$ in (4), we may relax the integer variables in X and Y to be real numbers in order to search for the optimal solution. This means that the unit processing time may become fractional because a job may be split by a supply batch. We denote this situation as a *relaxed problem*. Thus, we have the following lemma.

Lemma 2. *For the relaxed problem, the supply batches in an optimal solution have the same batch size, and so do the delivery batches.*

Proof. For $f(X, Y)$ in (4), the partial derivative of $f(X, Y)$ with respect to X (except for the variable x_1) is

$$\frac{\partial f(X,Y)}{\partial x_k} = \alpha\{x_2 + \cdots + x_{k-1} + 2x_k + x_{k+1} + \cdots + x_u - n\}$$

for $k = 2, \ldots, u$.

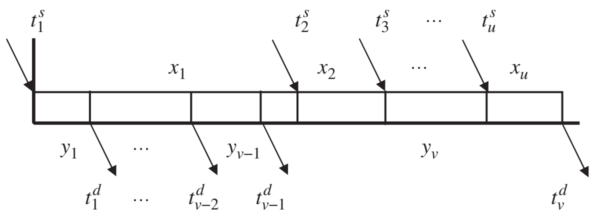

Fig. 2. A distribution Δ.

Setting

$$\frac{\partial f(X,Y)}{\partial x_k} = \alpha\{x_2 + \cdots + x_{k-1} + 2x_k + x_{k+1} + \cdots + x_u - n\} = 0$$
for $k = 2, \ldots, u$,

we obtain that $x_k = n/u$ for $k = 2, \ldots, u$. Since $x_1 = n - (x_2 + \cdots + x_u)$, $x_1 = n/u$.

The partial derivative of $f(X,Y)$ with respect to Y (except for the variable y_v) is

$$\frac{\partial f(X,Y)}{\partial y_h} = \alpha\{y_1 + \cdots + y_{h-1} + 2y_h + y_{h+1} + \cdots + y_{v-1} - n\}$$
for $h = 1, \ldots, v-1$.

Setting

$$\frac{\partial f(X,Y)}{\partial y_h} = \alpha\{y_1 + \cdots + y_{h-1} + 2y_h + y_{h+1} + \cdots + y_{v-1} - n\} = 0$$
for $h = 1, \ldots, v-1$,

we obtain that $y_h = n/v$ for $h = 1, \ldots, v-1$. Since $y_v = n - (y_1 + \cdots + y_{v-1})$, $y_v = n/v$.

Furthermore, taking the second partial derivatives of $f(X,Y)$ with respect to X and Y (except for x_1 and y_v), we obtain that the value of the Jacobian determinant of $f(X,Y)$ is $uv\alpha^{u+v-2}$, which is greater than zero. Therefore, when $x_k = n/u$ for $k = 1, 2, \ldots, u$ and $y_h = n/v$ for $h = 1, 2, \ldots, v$, the value of $f(X,Y)$ is the smallest. Thus, we reach the conclusion. $\square$

We now return to the original problem where the variables in X and Y are positive integers. For given u and v, we have $n = \lfloor n/u \rfloor u + l_1$ and $n = \lfloor n/v \rfloor v + l_2$, where $0 \le l_1 < u$, $0 \le l_2 < v$. Based on Lemma 2, we can easily obtain the following conclusion.

Lemma 3. *There exists an optimal solution for the problem in which the following conditions hold*:

(1) *The supply batch sizes are*:

$$(x_1, \ldots, x_{l_1}, x_{l_1+1}, \ldots, x_u) = \left(\left\lfloor\frac{n}{u}\right\rfloor + 1, \ldots, \left\lfloor\frac{n}{u}\right\rfloor + 1, \left\lfloor\frac{n}{u}\right\rfloor, \ldots, \left\lfloor\frac{n}{u}\right\rfloor\right).$$

(2) *The delivery batch sizes are*:

$$(y_1, \ldots, y_{l_2}, y_{l_2+1}, \ldots, y_v) = \left(\left\lfloor\frac{n}{v}\right\rfloor + 1, \ldots, \left\lfloor\frac{n}{v}\right\rfloor + 1, \left\lfloor\frac{n}{v}\right\rfloor, \ldots, \left\lfloor\frac{n}{v}\right\rfloor\right).$$

Lemma 3 shows that both supply batches and delivery batches have almost equal batch sizes in an optimal solution.

For given u and v, we determine the supply and delivery batches sizes according to Lemma 3. Then, the value of the inventory cost function (denoted as $\bar{f}(u,v)$) is calculated by (4) accordingly. We have:

$$\bar{f}(u,v) = \frac{1}{2}\left\{n\left(\left\lfloor\frac{n}{u}\right\rfloor + \left\lfloor\frac{n}{v}\right\rfloor\right) + \left(n - u\left\lfloor\frac{n}{u}\right\rfloor\right)\left(1 + \left\lfloor\frac{n}{u}\right\rfloor\right) + \left(n - v\left\lfloor\frac{n}{v}\right\rfloor\right)\left(1 + \left\lfloor\frac{n}{v}\right\rfloor\right)\right\}\alpha. \quad (5)$$

From (5), we notice that the contributions of supply batch and delivery batches to the optimal inventory cost are independent. Let

$$\bar{f}_s(u) = \frac{1}{2}\left\{n\left\lfloor\frac{n}{u}\right\rfloor + \left(n - u\left\lfloor\frac{n}{u}\right\rfloor\right)\left(1 + \left\lfloor\frac{n}{u}\right\rfloor\right)\right\}\alpha$$

and

$$\bar{f}_d(v) = \frac{1}{2}\left\{n\left\lfloor\frac{n}{v}\right\rfloor + \left(n - v\left\lfloor\frac{n}{v}\right\rfloor\right)\left(1 + \left\lfloor\frac{n}{v}\right\rfloor\right)\right\}\alpha.$$

Then, $\bar{f}(u,v) = \bar{f}_s(u) + \bar{f}_d(v)$.

In the following we develop an optimal algorithm for the case where the jobs have unit processing times.

Algorithm A1. *Step* 1. Calculate $\bar{f}_s(u)$ for $u = \lceil n/K_s \rceil, \ldots, n$ and $\bar{f}_d(v)$ for $v = \lceil n/K_d \rceil, \ldots, n$.

Step 2. Select u such that $\lambda_s^* = \min\{u\mu_s + n\gamma_s + \bar{f}_s(u) | u = \lceil n/K_s \rceil, \ldots, n\}$ and let $u^* = u$.

Step 3. Select v such that $\lambda_d^* = \min\{v\mu_d + n\gamma_d + \bar{f}_d(v) | v = \lceil n/K_d \rceil, \ldots, n\}$ and let $v^* = v$.

Step 4. For u^* and v^*, according to Lemma 3, determine a supply batch scheme φ^* and a delivery batch scheme ψ^*. Let $F(\varphi^*, \psi^*) = \lambda_s^* + \lambda_d^*$. Stop.

Obviously, the time complexity of Algorithm A1 is $O(n)$. Algorithm A1 searches all the possible values of u and v; and for the optimal values of u and v, we can obtain the optimal inventory cost $\bar{f}(u,v)$. Hence, we have the following conclusion.

Theorem 1. *For the problem with unit job processing times, Algorithm A1 produces an optimal solution in O(n) time.*

When all the jobs (orders) have identical processing times, our result shows that the supply scheme and the delivery scheme can be created separately. The supply batches have almost equal sizes, and so do the delivery batches, in an optimal solution. The supply scheme with almost equal batch sizes, to a certain extent, is similar the conventional economic ordering quality (EOQ) model for inventory management. The result is interesting because our model not only considers material supply and WIP inventory, but also takes product delivery into account. The full-truck-load strategy for supply and delivery is no more an optimal choice for the problem under study. It may make the transport cost smaller while it will result in higher inventory cost in the factory.

4. Special cases of the general problem

In this section we discuss some special cases of the general problem where the processing time of each job is a positive number. We consider the situation that the third party offers a supply (delivery) service with a high transport cost and a large vehicle capacity. We define an inventory cost constant β' as

$$\beta' = \max\left\{\alpha(n-\lambda)\sum_{i=1}^{\lambda} p_i | J_1, J_2, \ldots, J_\lambda \text{ are the } \lambda \text{ jobs with the largest processing times for } \lambda = 1, 2, \ldots, n\right\}$$

We say that the supply (delivery) cost is *high* if $\mu_s(\mu_d) \geq \beta'$ and the supply (delivery) vehicle capacity is *large* if $K_s(K_d) \geq n$. We also consider the situation that the third party offers a supply (delivery) service with a low transport cost. We say the supply (delivery) cost is *low* if $\mu_s(\mu_d)$ is no larger than an inventory cost constant $\beta'' = \min\{\alpha p_i | J_i \in N\}$. In the following we analyze four special cases:

Case I: High supply cost and large supply vehicle capacity.

Case II: High delivery cost and large delivery vehicle capacity.

Case III: Low supply cost.

Case IV: Low delivery cost.

4.1. Case I

For this case we can easily show that there is only one supply batch in an optimal solution because the manufacturer will incur a higher supply cost from increasing the number of supply batches, although the inventory cost may be reduced at the same time. This case models the actual situation where the manufacturer has prepared the materials of all the jobs for processing before a scheduling period. Hence, we have identified an optimal property for this case.

Lemma 4. *For Case I, there exists an optimal solution that has the following properties*:

(i) *The jobs are sequenced according to the shortest processing time (SPT) rule.*
(ii) *In the SPT schedule* $(J_1, J_2, \ldots, J_n)$, *if* $y_h p_{y_1+\cdots+y_h+1} \leq p_{y_1+\cdots+y_h+2} + \cdots + p_{y_1+\cdots+y_{h+1}}$ *and* $y_h < K_d$, *then job* $J_{y_1+\cdots+y_h+1}$ *should be delivered in the hth delivery batch for* $h = 1, 2, \ldots, v-1$.

We can easily prove (i) of Lemma 4 by swapping a pair of jobs in two adjacent delivery batches and (ii) by moving a job to the preceding delivery batch. Lemma 4 implies that the number of jobs in a delivery batch is no fewer than that of the next delivery batch. Next, we propose a solution algorithm for this case.

Algorithm A2. *Step* 1. Sequence the jobs according to the SPT rule. Denote the resulting schedule as $\pi = (J_1, J_2, \ldots, J_n)$.

Step 2. Initially, let $h = 1$ and $v = \lceil n/K_d \rceil$. If $v = 1$, then go to (4).

(1) Set $|B_1^d| = \lceil n/v \rceil, \ldots, |B_{l_2}^d| = \lceil n/v \rceil, |B_{l_2+1}^d| = \lfloor n/v \rfloor, \ldots, |B_v^d| = \lfloor n/v \rfloor$, where $l_2 = n - v\lfloor n/v \rfloor$.
(2) Set $h = 1$.
(3) If $y_h p_{y_1+\cdots+y_h+1} \leq p_{y_1+\cdots+y_h+2} + \cdots + p_{y_1+\cdots+y_{h+1}}$, $y_h < K_d$ and $y_{h+1} > 1$, then let $B_h^d = B_h^d \cup \{J_{y_1+\cdots+y_h+1}\}$, $y_h \leftarrow y_h + 1$ and $B_{h+1}^d = B_{h+1}^d \backslash \{J_{y_1+\cdots+y_h+1}\}$, $y_{h+1} \leftarrow y_{h+1} - 1$, go to (3); otherwise, $h \leftarrow h + 1$.
If $h < v$, then go to (3); otherwise, if $y_j p_{y_1+\cdots+y_j+1} > p_{y_1+\cdots+y_j+2} + \cdots + p_{y_1+\cdots+y_{j+1}}$ or $y_j = K_d$, for all $j = 1, 2, \ldots, v-1$, go to the next step; otherwise go to (2).
(4) According to (1), calculate $g(1, v) = F(\varphi, \psi)$. If $v < n$, then let $v \leftarrow v + 1$ and go to (1).

Step 3. Let

$$F^*(\varphi, \psi) = \min\left\{ g(1, v) | v = \left\lceil \frac{n}{K_d} \right\rceil, \quad \left\lceil \frac{n}{K_d} \right\rceil + 1, \ldots, n \right\}.$$

Stop.

Step 1 requires $O(n \log n)$ time to determine the SPT schedule. In Step 2, the first loop of $h = 1$ to $v - 1$ needs at most $n - v - \lfloor n/v \rfloor$ time to move jobs to the preceding delivery batch. The second loop of $h = 1$ to $v - 2$ needs at most $n - v - 2\lfloor n/v \rfloor + 1$ time to move jobs to the preceding delivery batch. Similarly, the $(v-1)$th loop of $h = 1$ needs at most $\lfloor n/v \rfloor$ or $\lfloor n/v \rfloor - 1$ time to move jobs to the preceding delivery batch. So, for a fixed v, the computational effort is $O((v-1)(n-v)/2)$. Since the algorithm can be applied to all the possible values of K_d and the complexity of the algorithm is dominated by Step 2, we see that the time complexity of Algorithm A2 is $O(n^3)$.

According to (i) of Lemma 4, we can determine the schedule of an optimal solution. Utilizing the necessary condition for an optimal solution provided by (ii) of Lemma 4, we search all the possible optimal delivery batch schemes in an optimal solution and select the best one in Algorithm A2, Therefore, we have the following result.

Theorem 2. *Algorithm A2 produces an optimal solution for Case I in* $O(n^3)$ *time.*

4.2. Case II

For this case we can easily show that there is only one delivery batch in an optimal solution because the manufacturer will incur a higher delivery cost from increasing the number of delivery batches, although the inventory cost may be reduced at the same time. This case models the actual situation where the manufacturer intends to deliver all the products to the customer after a scheduling period ends. Since this is a symmetry case of the case in Section 4.1, we only give the following results and omit the proofs.

Lemma 5. *For Case II, there exists an optimal solution that has the following properties*:

(i) *The jobs are sequenced according to the longest processing time (LPT) rule.*
(ii) *In the LPT schedule* $(J_1, J_2, \ldots, J_n)$, *if* $x_{k+1} p_{x_1+\cdots+x_k} \leq p_{x_1+\cdots+x_{k-1}+1} + \cdots + p_{x_1+\cdots+x_k-1}$ *and* $x_{k+1} < K_s$, *then job* $J_{x_1+\cdots+x_k}$ *should be delivered in the* $(k+1)$*th supply batch for* $k = 1, 2, \ldots, u-1$.

This case can be solved by the following algorithm.

Algorithm A3. *Step* 1. Sequence the jobs according to the LPT rule. Denote the resulting schedule as $\pi = (J_1, J_2, \ldots, J_n)$.

Step 2. Initially, let $k = 1$ and $u = \lceil n/K_s \rceil$. If $u = 1$, then go to (4).

(1) Set $|B_1^s| = \lfloor n/u \rfloor, \ldots, |B_{u-l_1}^s| = \lfloor n/u \rfloor, |B_{u-l_1+1}^s| = \lceil n/u \rceil, \ldots, |B_u^s| = \lceil n/u \rceil$, where $l_1 = n - u\lfloor n/u \rfloor$.
(2) Set $k = 1$.
(3) If $x_{k+1} p_{x_1+\cdots+x_k} \le p_{x_1+\cdots+x_{k-1}+1} + \cdots + p_{x_1+\cdots+x_k-1}$, $x_{k+1} < K_s$ and $x_k > 1$, then let $B_{k+1}^s = B_{k+1}^s \cup \{J_{x_1+\cdots+x_k}\}$, $x_{k+1} \leftarrow x_{k+1} + 1$ and $B_k^s = B_k^s \backslash \{J_{x_1+\cdots+x_k}\}$, $x_k \leftarrow x_k - 1$, go to (3); otherwise, $k \leftarrow k + 1$.
If $k < u$, then go to (3); otherwise, if $x_{j+1} p_{x_1+\cdots+x_j} > p_{x_1+\cdots+x_{j-1}+1} + \cdots + p_{x_1+\cdots+x_j-1}$ or $x_{j+1} = K_s$ for all $j = 1, 2, \ldots, u-1$, go to the next step; otherwise go to (2).
(4) According to (1), calculate $g(u, 1) = F(\varphi, \psi)$. If $u < n$, then let $u \leftarrow u + 1$ and go to (1).

Step 3. Let

$$F^*(\varphi, \psi) = \min\left\{ g(u, 1) | u = \left\lceil \frac{n}{K_s} \right\rceil, \left\lceil \frac{n}{K_s} \right\rceil + 1, \ldots, n \right\}.$$

Stop.

Theorem 3. *Algorithm A3 produces an optimal solution for Case II in $O(n^3)$ time.*

4.3. Case III

For this case each supply batch contains only one job since the inventory cost of any job is no less than the cost of a supply trip from the warehouse to the factory. This means that each job in an optimal solution should arrive at the factory just when it is about to begin processing.

For a given ν, we can convert the case into a ν-parallel identical machine scheduling problem to minimize the total earliness, where the due dates of all the jobs are unbounded and common. So, we have the following algorithm for this case.

Algorithm A4. *Step* 1. Sequence the jobs as an order set $\pi = (J_1, J_2, \ldots, J_n)$ such that $p_1 \le p_2 \le \cdots \le p_n$.

Step 2. For $\nu = \lceil n/K_d \rceil$ to n, repeat (1) and (2).

(1) Beginning with J_1, assign in turn the jobs in π to one of the ν machines in such a way that the job has the smallest earliness and the number of the assigned jobs on the machine is no larger than K_d.
(2) The jobs being processed on the same machine are taken as a delivery batch. The delivery batches $B_1^d, B_2^d, \ldots, B_\nu^d$ are sequenced randomly and there is no idle time between them. Supposing that $p_{j_1} \ge p_{j_2} \ge \cdots \ge p_{j_{y_h}}$ for jobs $J_{j_1}, J_{j_2}, \ldots, J_{j_{y_h}} \in B_h^d$ and $h = 1, 2, \ldots, \nu$, calculate the objective function:

$$g(n, \nu) = n(\mu_s + \gamma_s) + (\nu\mu_d + n\gamma_d) + \alpha \sum_{h=1}^{\nu} \sum_{J_{j_\lambda} \in B_h^d} \lambda p_{j_\ell}.$$

Step 3. Let

$$F^*(\varphi, \psi) = \min\left\{ g(n, \nu) | \nu = \left\lceil \frac{n}{K_d} \right\rceil, \left\lceil \frac{n}{K_d} \right\rceil + 1, \ldots, n \right\}.$$

Stop.

Step 1 requires $O(n \log n)$ time. In Step 2, for a given ν, the corresponding ν-parallel machine total earliness scheduling problem can be solved in $O(n)$ time. The algorithm is applied to all the possible values of K_d. Hence, the time complexity of the algorithm is $O(n^2)$. Obviously, we have the following result.

Theorem 4. *Algorithm A4 produces an optimal solution for Case III in $O(n^2)$ time.*

4.4. Case IV

For this case each delivery batch contains only one job since the inventory cost of any job is no less than the transport cost of a delivery trip from the factory to the customer. This means that each job in an optimal solution should depart from the factory once it finishes processing.

This case is similar to the problem studied by Qi (2005), which can be solved optimally by converting the case into $(n - \lceil n/K_s \rceil + 1)$ parallel-machine scheduling problems to minimize the total completion time. The computational complexity of this case is $O(n^2)$ time.

5. Conclusions

In this paper we studied a logistics scheduling problem with material supply and product delivery considerations. The objective is to minimize the sum of the WIP inventory cost and transport cost. When all the jobs have identical processing times, we showed that the expression of the WIP inventory cost function can be unified, and we proposed an $O(n)$ optimal algorithm to solve this case. We also examined several special cases of the general problem, identified their optimal properties, and developed polynomial-time optimal algorithms to solve them.

As for future logistics scheduling research, researchers need to build a scheduling model that integrates the three stages, i.e., supply, production and distribution, of the typical logistics activity. Such a model has very different characteristics from those of the two-stage models found in the existing literature that consider only production and distribution, or supply and production. Consideration of various machine processing environments and objectives for a three-stage logistics scheduling model is worthy of future study, too.

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Appendix A. Proof of Lemma 1

Proof. We notice that any two distributions can be transformed into each other by a procedure of changing in turn the departure times of some delivery batches from A_k to A_{k-1} or A_{k+1}. So, we will focus on discussing the effects on the WIP inventory cost function expressions for the following three types of transformation between two distributions.

(i) For two distributions $\Delta_1=(l_1,\ldots,l_{k-1},l_k,l_{k+1},\ l_{k+2},\ldots,l_u)$ and $\Delta_2=(l_1,\ldots,l_{k-1},l_k-1,l_{k+1}+1,l_{k+2},\ldots,l_u)$, where $l_k,\ l_{k+1}>1$.
(ii) For two distributions $\Delta_1=(l_1,\ldots,l_{k-1},1,l_{k+1},\ldots,l_u)$ and $\Delta_2=(l_1,\ldots,l_{k-1},0,l_{k+1}+1,l_{k+2},\ldots,l_u)$.
(iii) For two distributions $\Delta_1=(l_1,\ldots,l_{k-1},l_k,1,l_{k+2},\ldots,l_u)$ and $\Delta_2=(l_1,\ldots,l_{k-1},l_k+1,0,l_{k+2},\ldots,l_u)$.

For (i), we set the inventory cost functions of Δ_1 and Δ_2 as $f_1(X,Y)$ and $f_2(X,Y)$, respectively. Without loss of the generality, we assume that the arrival time to the factory of the ℓ_1th supply batch for Δ_1 is the same as that of Δ_2 for $\ell_1=1,2,\ldots,u$, and the departure time from the factory of the ℓ_2th delivery batch for Δ_1 is the same as that of Δ_2 for $\ell_2=1,2,\ldots,h-1,h+1,\ldots,v$. We also assume that the departure times of the hth delivery batch for Δ_1 and Δ_2 are just before and after time t^s_{k+1}, respectively. It is obvious that the inventory costs before time point t^d_{h-1} and after time point t^d_{h+1} are the same for Δ_1 and Δ_2. So, we set them as $\bar{f}(X,Y)$. Now, for Δ_1, we have (see Fig. 3(a)),

$$\begin{aligned}f_1(X,Y)&=\bar{f}(X,Y)+\alpha\{y_h[(y_1+\cdots+y_h)-(x_1+\cdots+x_{k-1})]\\&\quad+[(x_1+\cdots+x_k)-(y_1+\cdots+y_h)]\\&\quad\times[(y_1+\cdots+y_{h+1})-(x_1+\cdots+x_{k-1})]\\&\quad+[(y_1+\cdots+y_{h+1})-(x_1+\cdots+x_k)]^2\}\\&=\bar{f}(X,Y)+\alpha\{(y_h+y_{h+1}-x_k)\\&\quad\times[(y_1+\cdots+y_{h-1})-(x_1+\cdots+x_{k-1})]\\&\quad+(x_k^2+y_h^2+y_{h+1}^2-x_ky_h-x_ky_{h+1}+y_hy_{h+1})\}.\end{aligned}\tag{6}$$

For Δ_2, we have (see Fig. 3(b)),

$$\begin{aligned}f_2(X,Y)&=\bar{f}(X,Y)+\alpha\{[(x_1+\cdots+x_k)-(y_1+\cdots+y_{h-1})]\\&\quad\times[(y_1+\cdots+y_h)-(x_1+\cdots+x_{k-1})]\\&\quad+[(y_1+\cdots+y_h)-(x_1+\cdots+x_k)]^2\\&\quad+y_{h+1}[(y_1+\cdots+y_{h+1})-(x_1+\cdots+x_k)]\}\\&=\bar{f}(X,Y)+\alpha\{(y_h+y_{h+1}-x_k)\\&\quad\times[(y_1+\cdots+y_{h-1})-(x_1+\cdots+x_{k-1})]\\&\quad+(x_k^2+y_h^2+y_{h+1}^2-x_ky_h-x_ky_{h+1}+y_hy_{h+1})\}.\end{aligned}\tag{7}$$

Comparing (6) with (7), we can see that the expressions of $f_1(X,Y)$ and $f_2(X,Y)$ are identical.

Similar to (i), we can also prove that transformations between distributions Δ_1 and Δ_2 of types (ii) and (iii) do not change the expression of the inventory cost function. In fact, for given u and v, any one distribution can be transformed into another distribution by proceeding with a series of transformation of types (i)–(iii). Hence, we reach the conclusion. □

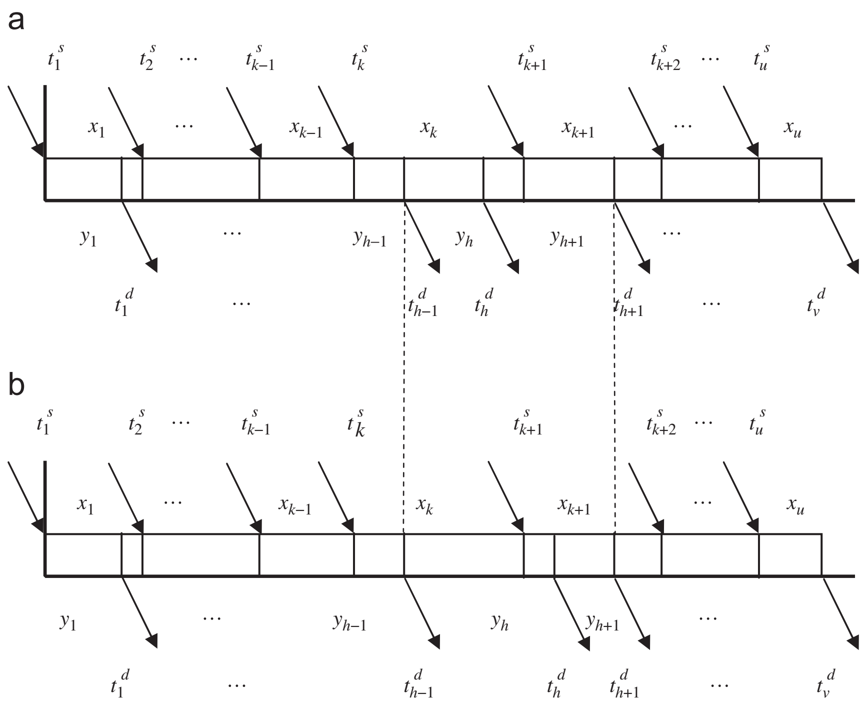

Fig. 3. (a) Distribution Δ_1 in (i) and (b) distribution Δ_2 in (i).

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