

**Problem Set 3 (Due September 29, 2023)**

1. This exercise contains three excerpts from a British quiz show called Golden Balls. After each clip, you are asked to use the game theoretical tools you learned in Lectures 4-5 to analyze the situations. you should listen carefully to what the contestants say in. the videos (apologies for the poor sound quality in some of them).
  - (a) Watch this **video clip 1** until 2:20 Draw the game between Lucy and Tony as a game matrix when Lucy and Tony care only about money. Continue next until the end of the clip. Based on their words and decisions, what do you think about their real preferences?
  - (b) In the second excerpt, Sarah and Steve have collected GBP 100 000 in the pot. This **link** takes you to the point where Sarah and Steve start talking. If you want, you can rewind to see the introduction to the game again. What do you conclude about Sarah's and Steve's preferences?
  - (c) In the last excerpt, Ibrahim and Nick play for GBP 13 600. This **video** begins at the point where Ibrahim and Nick start talking. One way to interpret what is going on is that Nick wants to convince Ibrahim that part of the game matrix is no longer relevant. How would this affect Ibrahim's optimal strategy?
2. One often hears talk about first mover advantage. Let's examine this with our two-player two-action games.
  - (a) A first mover chooses her action so that the other player observes the choice and then takes a best response to the observed action (assume that the second mover's best response to any action by the first mover is unique). Show that the first mover can get a payoff at least as large as in any Nash equilibrium of the simultaneous move game.

- (b) Give an example of a game where the second mover gets a higher payoff than the first mover.
  - (c) Suppose that the column player has a dominant strategy in the simultaneous game and that she is the second mover in a game where the row player is the first mover. Will the column player choose her dominant strategy? Suppose next that the row player has a dominant strategy in the simultaneous game and she is the first mover (and the column player does not have necessarily a dominant strategy). Will the row player always choose her dominant strategy when acting as a first mover? Explain your answer.
3. One of the purposes of having a police force is to prevent criminal behavior. In particular, the threat of getting caught and receiving a punishment steer citizens away from criminal activities. Let us consider this in a game theoretic setting where the government plays a game against potential criminals. The government decides whether to set up a police force at cost  $c > 0$ . The police force can solve any crimes that were committed resulting in the return of stolen goods to their lawful owners and in the punishment of the offenders. Without the police force, the crimes go unsolved. Let the damages from unsolved crime to the government be  $d > 0$  (so that the payoff from unsolved crimes is  $-d$ ). The benefit to the government from solved crimes is  $b > c$ . Fix the payoff to the government from no crime and no police force at 0. Finally the potential criminals decide whether to commit a crime or not. They get a gain of  $g > 0$  from their crimes if they are not caught and a punishment of  $p > 0$  if caught (so that getting caught yields a payoff of  $-p$ ). The criminals' payoff is fixed at 0 if they do not commit a crime.
- (a) Draw a game matrix for this situation. Assume first that the payoffs are simply the monetary payoffs to the players.
  - (b) Does this game have dominant strategies? What about Nash equilibria?
  - (c) Suppose that the government chooses first whether to have the police force or not and criminals decide on crime or not after see-

ing the government's decision. What is the backwards induction outcome for this game?

4. We talked about negative effects on other decision makers in our discussion of pollution and fossil fuel reduction. Let's be more positive for a change and discuss positive effects. These are slightly more open ended examples where insights from game theory are useful.
  - (a) Some sports clubs reserve a part of the stadium for a fan club. Discuss reasons for seating devoted fans next to each other.
  - (b) Discuss reasons for zoning and other building and maintenance restrictions in residential neighborhoods.
  - (c) Most Western countries mandate schooling until a fixed age. Discuss rationales for this practice.
5. Consider an economy country of 5 equal size groups  $i$  of individuals with annual income  $y_i$  for group  $i \in \{1, \dots, 5\}$ . Assume that the groups are ordered so that  $y_1 \leq y_2 \leq \dots \leq y_5$ . The Lorenz curve of this economy is given by the broken line connecting points:

$$(0, 0), \left(\frac{1}{5}, \frac{y_1}{y_1 + y_2 + y_3 + y_4 + y_5}\right), \left(\frac{2}{5}, \frac{y_1 + y_2}{y_1 + y_2 + y_3 + y_4 + y_5}\right), \\ \left(\frac{3}{5}, \frac{y_1 + y_2 + y_3}{y_1 + y_2 + y_3 + y_4 + y_5}\right), \left(\frac{4}{5}, \frac{y_1 + y_2 + y_3 + y_4}{y_1 + y_2 + y_3 + y_4 + y_5}\right), (1, 1)$$

- (a) Compute the Gini-coefficient for the case where  $y_1 = 30, y_2 = 40, y_3 = 50, y_4 = 60, y_5 = 70$ .
- (b) What happens to the Gini-coefficient following a 'Robin Hood Transfer' where an amount  $\Delta$  is transferred from the members of group  $i$  to the members of a poorer group  $j < i$  assuming that  $\Delta < y_i - y_{i-1}$ ?