ELEC-E8116 Model Based Control Systems
/exercise 1

Problem 1:

Let $f(x)$ be a scalar-valued function of the vector x and let A be a square matrix with an appropriate dimension. By using a simple example, study what kind of a function $f(x) = x^T A x$ is. Demonstrate that

$$\frac{d(Ax)}{dx} = A \qquad \text{and} \qquad \frac{df(x)}{dx} = \underline{x}^T (A + A^T)$$

when the gradient is considered to be a row vector (in the literature the gradient is sometimes regarded as a row vector and sometimes as a column vector).

Problem 2

Now consider the gradient of $f(x)$ as a column vector. Show by a simple example that

$$\frac{d(Ax)}{dx} = A^T \qquad \frac{d(x^T Ax)}{dx} = (A + A^T)x$$

Problem 3

Show that $x^T\left(A - A^T\right)x = 0$ holds, when x is a vector and A is a square matrix with an appropriate dimension.

Problem 4

Let the criterion to be minimized be given as

$$J = \int_0^T \left\{x(t)' Px(t) + u(t)' Qu(t)\right\} dt$$

where ’ denotes the transpose. Show that the square matrices P and Q can always be chosen as symmetric matrices.

Problem 5

Let A, B, C and D be nxn, nxm, mxn, mxm matrices. Prove the so-called *matrix-inverion lemma*

$$\left(A + BDC\right)^{-1} = A^{-1} - A^{-1}B(D^{-1} + CA^{-1}B)^{-1}CA^{-1}$$

where it is assumed that all inverse matrices exist.