ELEC-E8107 - Stochastic models, estimation and control

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September 22, 2023

Exercises Session 2

Exercise 1

A parameter x is measured with correlated (rather than independent) additive Gaussian noises, such that:

$$z_k = x + w_k,$$

where, $k = 1 \dots n$ and the mean value of the noise at instant k is $E[w_k] = 0$, while the variances are:

$$E[w_k w_j] = \begin{cases} 1, & \text{if } k = j \\ \rho, & \text{if } |k - j| = 1 \\ 0, & \text{if } |k - j| > 1 \end{cases}$$

1. Compute the likelihood function of the parameter x
2. Find the MLE of x.
3. Find the CRLB for the estimation of x.
4. Is the MLE efficient?

Exercise 2

Given $z = x + w$, where all the variables are n-vectors, with:

$$w \sim \mathcal{N}(0, P) \qquad x \sim \mathcal{N}(\bar{x}, P_0)$$

x and w are independent. Find the MAP estimator of x in terms of z and the covariance of this estimator.

Exercise 3

The model for a vehicle moving at a constant speed is $y_i = vt_i + e_i$. The position is measured as a function of time as shown in the Table below:

Time	0	1	2	3	4	10	12	18
Distance	4.71	9	15	19	20	45	55	78

The noise e_i are such that $E[e_i^2] = R_i = 0.9^{8-i}$. Use the batch least square method to estimate the velocity. (Write a Matlab script)

Exercise 4

Use the recursive least square method to estimate the velocity in the above problem. (Write a Matlab script)