NBE-E4310 D - Biomedical Ultrasonics 2023

This document contains the solutions for the Execise 1 of the BMUS course. The student solutions should have been posted no later than 13:00 on October 9.

Please, note that the code might be different than yours and might lead to slightly different solutions. A differently solved problem can sometimes be correct. However, this example should serve as a reference on how the exercise solutions should look like.

TASK 1 (8 points)

Please, consider the real pressure from a planar source that produces sound bursts that propagate in water. Then consider the vector t, where t represents the time points where the pressure was measured. Assume that the signal is measured at the spatial peak of the acoustic field. The files containing the pressure and time vectors can be found on MyCourses course website. The time is expressed in seconds and pressure in Pascals.

a. Plot the pressure as a function of time, including the labels with variable symbols and units. (1p)

In this part, we only need to plot the pressure over time as:

```
1  figure;
2  plot(t, pt);
3  xlabel('Time (s)');
4  ylabel('Pressure (Pa)');
5  title('Pressure as a Time function: ');
6  grid on;
```
Listing 1: Plot of the pressure over time.
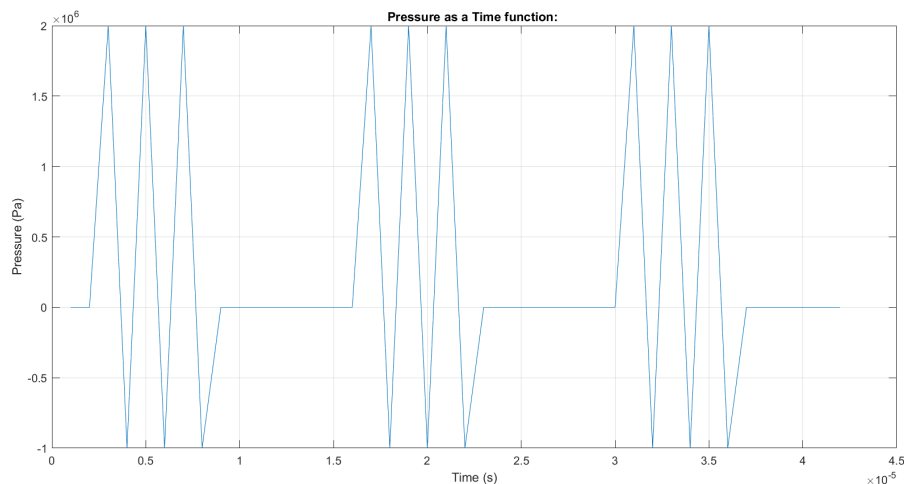
The result of the code is the following:

Figura 1: Pressure over time.

b. Plot the instantaneous intensity as a function of time, including the labels with variable symbols and units. (1p)

In this part, we need to calculate the intensity as a function of the pressure as:

$$I = p^2/\rho c \tag{1}$$

where p is the pressure, ρ is the density of the water and c is the speed of sound.

This is presented in the code as:

```matlab
% Assuming:
rho = 1000; % Density (water) in kg/m^3
c = 1500;   % Sound speed (water) in m/s

% And having into account Equation 1:
I = (pt.^2) / (rho * c);

% We plot the intensity as a function of time as:
figure;
plot(t, I);
xlabel('Time (s)');
ylabel('Instantaneous Intensity (W/m^2)');
title('Instantaneous Intensity as a function of Time: ');
grid on;
```

Listing 2: Plot of the pressure over time.
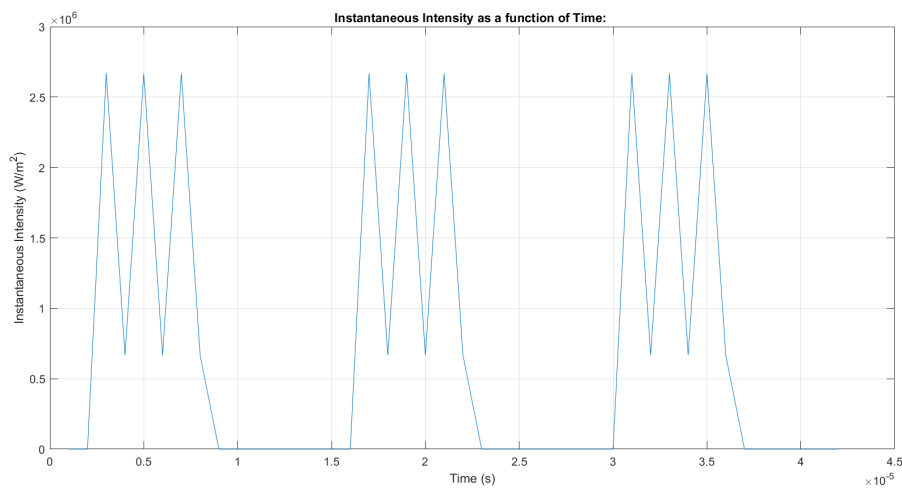
The result of the code is the following:

Figura 2: Instantaneous intensity over time.

Define then the following:

c. Period of the wave (1p)

The period of the wave is defined as the time it takes for two successive crests (one wavelength) to pass a specified point

From Figure 1 we see that the first burst starts at $t_0 = 2 \times 10^{-6}$ s and the finishes at $t_1 = 9 \times 10^{-6}$ s. We also see that the burst contains 3 wave cycles ($N_{cycle} = 3$). First, we define the pulse duration (PD) as:

$$PD = t_1 - t_0 \tag{2}$$

Then, the wave period is:

$$T = PD/N_{cycle} \qquad \rightarrow \qquad T = 2\mu s$$

d. PPP (1p)

The peak positive pressure (PPP) is defined as the highest (most positive) value of the pressure over time.

From Figure 1 can be found. Also, by coding we find that:

```
1  PNP = max(p);
2  fprintf('PPP (Peak Positive Pressure): %.6f MPa\n', PPP/1e6);
```
Listing 3: Finding PNP.

The result of the code is that:

$$PPP = 2MPa$$

e. PNP (1p)

The peak negative pressure (PNP) is defined as the lowest (most negative) value of the pressure over time.

From Figure 1 can be found. Also, by coding we find that:

```
1  PNP = min(p);
2  fprintf('PNP (Peak Negative Pressure): %.6f MPa\n', PNP/1e6);
```
Listing 4: Finding PNP.

The result of the code is that:

$$PNP = -1MPa$$

f. PRP (1p)

The pulse repetition period (PRP) is the time between the start of two consecutive bursts in a pressure wave over time.

From Figure 1 we see that the first pulse starts at $t_0 = 2 \times 10^{-6}$ s and the second pulse starts at $t_2 = 1.6 \times 10^{-5}$ s. Therefore, the PRP is:

$$PRP = t_2 - t_0 \qquad \rightarrow \qquad PRP = 14\mu s \qquad (3)$$

g. PRF (1p)

The pulse repetition frequency (PRF) is the number of times a pulsed activity occurs every second.

PRF is the inverse of PRP. Therefore, from Equation 3, the PRF is:

$$PRF = \frac{1}{PRP} = \frac{1}{t_2 - t_0} \qquad \rightarrow \qquad PRF = 71.4kHz \qquad (4)$$

h. PD (1p)

The Pulse Duration (PD) is defined as the interval of time between the start and the finish of a burst, as defined in Equation 2. As we found it before, the pulse duration is $7\mu s$.

TASK 2 (9 points)

Using the previous data from Task 1, define the following:

a. I_{SPTP} (3p)

The spatial-peak temporal-peak intensity (I_{SPTP}) is defined by the maximum value of the instantaneous intensity. We can calculate with a code as:

```
ISPTP = max(I);
fprintf('I_(SPTP) (Spatial Peak Temporal Peak Intensity): %.6f W/cm^2\n', ISPTP/1e4);
```
Listing 5: Finding ISPTP.

The result is:

$$I_{SPTP} = 266.7 W/cm^2$$

b. I_{SPPA} (3p)

The spatial peak pulse averaged acoustic intensity (I_{SPPA}) is the average acoustic intensity during the length of a burst, approximating the time interval of the integral to the 10% to 90% of the pulse duration. We do that because usually we don't know exactly where the pulse begins and finishes and we define the approximated pulse duration as:

$$PD_{ap} = 1.25 \times (t_{fin}^{90\%} - t_{ini}^{10\%}) \tag{5}$$

The integral of the I_{SPPA} follows:

$$I_{SPPA} = \int_0^{PD_{ap}} \frac{p^2(t)}{\rho c PD} \, dt \tag{6}$$

This is presented in the code as:

```
% The spatial-peak pulse-averaged intensity is defined by the instantaneous
% intensity between the 10% and the 90% time-interval integral. Then the
% pulse duration is defined as:

t0sppa = ini_burst + PD*0.1;
t1sppa = fin_burst - PD*0.1;
PDsppa = (t1sppa-t0sppa)*1.25;

% Define the time interval [t0, t1] for integration
t0 = t0sppa; % Starting time
t1 = t1sppa; % Ending time

% t0 = ini_burst; %If we wouldn't consider the approximation from Eq. 5
% t1 = fin_burst;

% Find the indices corresponding to t0 and t1 in the time vector t
index_t0 = find(t >= t0, 1); % Find the first index where t is g than or eq to t0
index_t1 = find(t <= t1, 1, 'last'); % Find the last index where t is l than or eq to t1

% Extract portion of time vector and the intensity values for integration
time_interval = t(index_t0:index_t1);
intensity_interval = I(index_t0:index_t1);

% Perform trapezoidal integration
ISPPA_int = trapz(time_interval, intensity_interval);
ISPPA = ISPPA_int/(PDsppa);

fprintf('I_(SPPA) (Spatial Peak Pulse Average Intensity): %.6f W/m^2\n', ISPPA);
```
Listing 6: Finding ISPPA.

The result is:

$$I_{SPPA} = 119.0 W/cm^2$$

c. I_{SPTA} (3p)

The spatial peak temporal averaged acoustic intensity (I_{SPTA}) is defined as the maximum spatial intensity occurring in an ultrasound beam averaged over the pulse repetition period. It can be calculated as the integral of the intensity considering the time interval as the PRP:

$$I_{SPTA} = \int_0^{PRP} \frac{p^2(t)}{\rho c PRP} \, dt \qquad (7)$$

This is presented in the code as:

```
1  % We initally define the burst duration:
2  t0spta = ini_burst;
3  t1spta= ini_2ndburst;
4  PDspta = (t1spta-t0spta);
5
6  % Define the time interval [t0, t1] for integration
7  t0 = t0spta; % Starting time
8  t1 = t1spta; % Ending time
9
10 % Find the indices corresponding to t0 and t1 in the time vector t
11 index_t0 = find(t >= t0, 1); % Find the first index where t is g than or eq to t0
12 index_t1 = find(t <= t1, 1, 'last'); % Find the last index where t is l than or eq to t1
13
14 % Extract portion of time vector and the intensity values for integration
15 time_interval = t(index_t0:index_t1);
16 intensity_interval = I(index_t0:index_t1);
17
18 % Perform trapezoidal integration
19 ISPTA_int = trapz(time_interval, intensity_interval); % Riemann integral is also ok, so
     trapezoidal integral is not absolutely necessary
20
21 ISPTA = ISPTA_int/PDspta;
22
23 fprintf('I_(SPTA) (Spatial Peak Temporal Average Intensity): %.6f W/m^2\n', ISPTA);
24
25
26 % Alternative method, approximated way of calculating ISPTA:
27 %DC = (fin_burst - ini_burst)/(ini_2ndburst - ini_burst); % duty cycle
28 %ISPTAaprox = ISPPA*(DC); % I^{aprox}_{SPTA} = 59.5 W/cm^2
29
30 fprintf('Approximate I_(SPTA) (Spatial Peak Temporal Average Intensity): %.6f W/m^2\n',
     ISPTAaprox);
```

Listing 7: Finding ISPPA.

The result is:

$$I_{SPTA} = 71.4 W/cm^2$$

TASK 3 (6 points)

a. Generate a 1D, continuous planar wave traveling along dimension x in longitudinal mode, operating at a frequency of 1 MHz and a pressure amplitude of 1 MPa. Calculate the pressure in complex form and define the real and absolute pressure for 5 cycles. (2p)

To solve this exercise, first we will define the parameters that define the behavior of the pressure wave (frequency $[f]$, pressure amplitude $[\hat{p}_r]$, number of cycles $[cycles]$, angular frequency $[w]$ and wave number $[k]$). Then, we will create the time and position vectors for plotting the real and absolute values of the pressure wave. This is presented in the code as:

```
% Parameters
f = 1e6;
p_hat_r = 1e6;
cycles = 5;
w = 2*pi*f;
k = 2*pi*f/c;

% Time and position vectors
t = linspace(0, cycles*(1/f), 1000);
x = linspace(0, 2*cycles*c/f, 1000);

% Pressure in complex form
P_complex = p_hat_r * exp(1i*(w*t - k*x));

% Real and absolute pressure
P_real = real(P_complex);
P_abs = abs(P_complex);

% Plot Real Pressure:
figure;
plot(x, P_real);
title('Real Pressure vs Displacement');
xlabel('x (m)');
xlim([0 x(end)])
ylabel('Pressure (Pa)');

% Plot Absolute Pressure:
figure;
plot(x, P_abs);
title('Absolute Pressure vs Displacement');
xlabel('x (m)');
xlim([0 x(end)])
ylabel('Pressure (Pa)');
```

Listing 8: Plotting the absolute and real pressures during 5 cycles.

The previous code results in Figure 3.

b. Generate the same wave as before, but from a depth from 0 to 5 cm, adding the attenuation of:

Now, we will add to the complex expression of the pressure wave the attenuation coefficient. It is defined as the a measure of the energy loss of sound propagation through an acoustic transmission medium, considering losses coming from absorption and scattering. It is dependent on the frequency (f) as:

$$\alpha = a \times f^b \tag{8}$$

As a and b are two coefficients that depend on the material. As we can see from Equation 8, the attenuation will increase with higher frequencies. This, as we will see in this exercise, has a greater effect on the attenuation of the wave with high values of α. When considering attenuation, the complex expression of the pressure changes as:

$$\hat{p}_z = \hat{p}_{z=0} 10^{-\frac{\alpha_{dB} z}{20}} \tag{9}$$

where z refers to the penetration depth in which we measure the wave and α_{dB} is the attenuation coefficient of the medium in decibels.
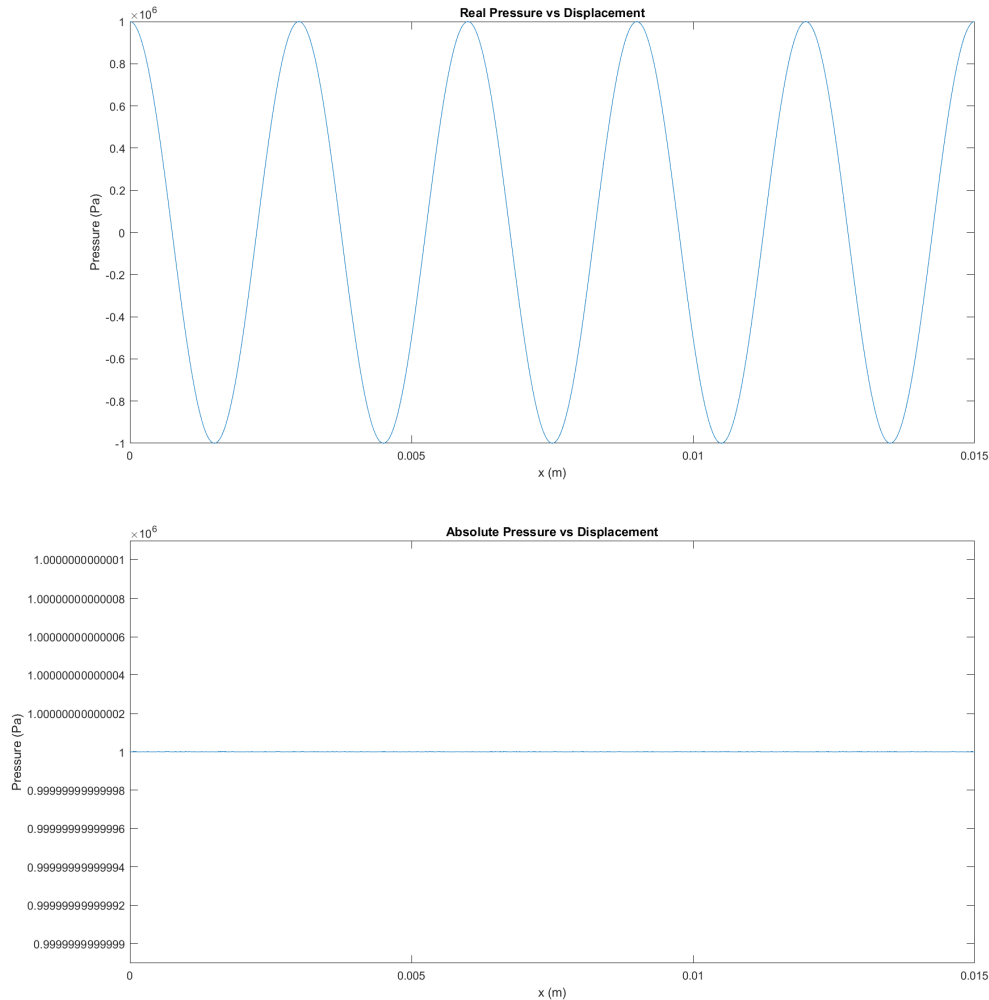
Figura 3: Real pressure (*Left*) and Absolute pressure (*Right*) for 5 cycles.

1. tissue (1p), 2. muscle (1p), 3. fat (1p), 4. bone (1p)

For simplicity, we will solve the 4 cases at the same time. Now, we just need to find in the slides, for each material, the values of α_{dB} and c and compute the complex expression of the pressure wave. Then, we will plot the real and absolute pressures in each case. This is presented in the code as:

```
1  % Define the depth and time vectors
2  depth = linspace(0, 0.05, 1000);
3  t = 0;
4
5  % Material speed of sound (m/s), wave number (1/m) and attenuation (dB/m):
6  % Brain tissue
7  c_brn = 1550;
8  alpha_brn = 0.85 * 100;
9  k_brn = w/c_brn;
10
11 % Muscle
12 c_msc = 1582.5;
13 alpha_msc = 2.2 * 100;
14 k_msc = w/c_msc;
15
16 % Fat
17 c_fat = 1450;
18 alpha_fat = 0.6 * 100;
19 k_fat = w/c_fat;
20
21 % Bone
22 c_bon = 4080;
23 alpha_bon = 20 * 100;
24 k_bon = w/c_bon;
25
```

```matlab
26 % Pressure in each material with complex expression
27 P_brn = p_hat_r * exp(1i*(0 - k_brn*depth)).*exp(-alpha_brn.*depth/20);
28 P_msc = p_hat_r * exp(1i*(0 - k_msc*depth)).*exp(-alpha_msc.*depth/20);
29 P_fat = p_hat_r * exp(1i*(0 - k_fat*depth)).*exp(-alpha_fat.*depth/20);
30 P_bon = p_hat_r * exp(1i*(0 - k_bon*depth)).*exp(-alpha_bon.*depth/20);
31
32 % Plot Real Pressure for all materials
33 figure;
34 sgtitle('Real pressure vs Depth for different materials')
35
36 subplot(2,2,1);
37 plot(depth, real(P_brn));
38 title('Brain Tissue');
39 xlabel('Depth (m)');
40 xlim([0 depth(end)])
41 ylabel('Pressure (Pa)');
42
43 subplot(2,2,2);
44 plot(depth, real(P_msc));
45 title('Muscle');
46 xlabel('Depth (m)');
47 xlim([0 depth(end)])
48 ylabel('Pressure (Pa)');
49
50 subplot(2,2,3);
51 plot(depth, real(P_fat));
52 title('Fat');
53 xlabel('Depth (m)');
54 xlim([0 depth(end)])
55 ylabel('Pressure (Pa)');
56
57 subplot(2,2,4);
58 plot(depth, real(P_bon));
59 title('Bone');
60 xlabel('Depth (m)');
61 xlim([0 depth(end)])
62 ylabel('Pressure (Pa)');
63
64 % Plot Absolute Pressure for all materials
65 figure;
66 sgtitle('Absolute pressure vs Depth for different materials')
67
68 subplot(2,2,1);
69 plot(depth, abs(P_brn));
70 title('Brain Tissue (alpha = 0.85)');
71 xlabel('Depth (m)');
72 xlim([0 depth(end)])
73 ylabel('Pressure (Pa)');
74
75 subplot(2,2,2);
76 plot(depth, abs(P_msc));
77 title('Muscle (alpha = 2.2)');
78 xlabel('Depth (m)');
79 xlim([0 depth(end)])
80 ylabel('Pressure (Pa)');
81
82 subplot(2,2,3);
83 plot(depth, abs(P_fat));
84 title('Fat (alpha = 0.6)');
85 xlabel('Depth (m)');
86 xlim([0 depth(end)])
87 ylabel('Pressure (Pa)');
88
89 subplot(2,2,4);
90 plot(depth, abs(P_bon));
91 title('Bone (alpha = 20)');
92 xlabel('Depth (m)');
93 xlim([0 depth(end)])
94 ylabel('Pressure (Pa)');
```

Listing 9: Plotting the absolute and real pressures to a depth of 5cm with the effect of attenuation.
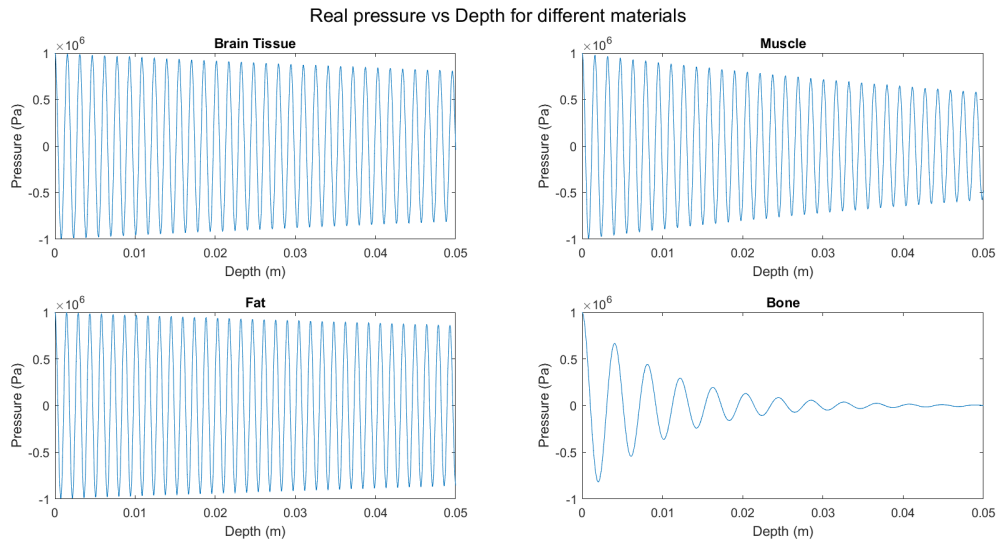
The previous code results in Figure 4 and 5.

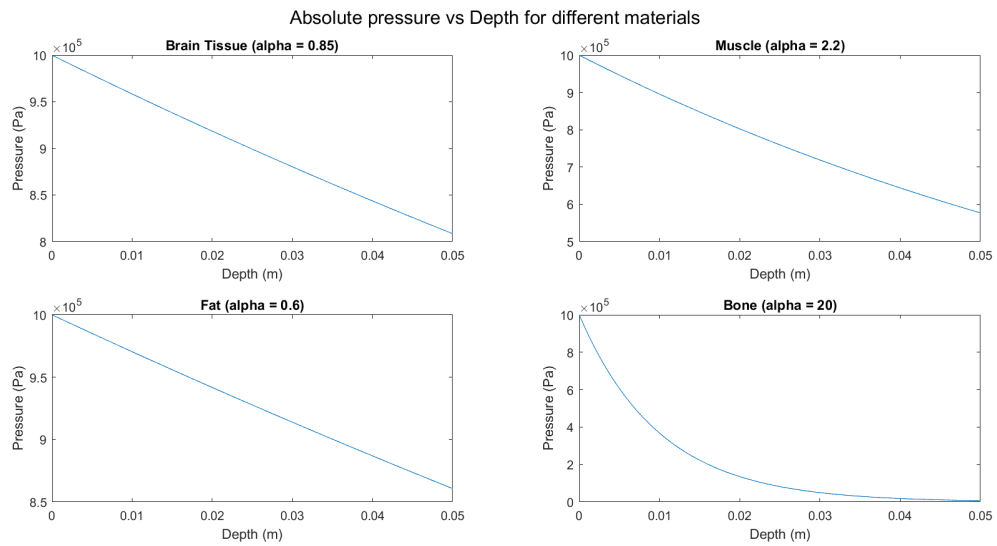Figura 4: Real pressure through different materials at a penetration depth of 5cm considering attenuation.

Figura 5: Absolute pressure through different materials at a penetration depth of 5cm considering attenuation.

TASK 4 (10 points)

Derive the geometric attenuation terms for intensity and pressure. Assume that there is only attenuation from the geometric factor (Hint! Assume conservation of energy in lossless media. Intensity is the energy flux):

To initially understand how the wave is propagated, we need to understand how the wave fronts change when moving away from the source in the two cases proposed.

a. For a line source (5p)

The spreading of the wave for a point source follows a polar geometry. Therefore, to simplify the derivation of the intensity and the geometric attenuation, we continue our calculations using this coordinates. Now, we are told that the energy is conserved, so the flux should be conserved too. The flux is defined by:

$$\phi = IA \tag{10}$$

where the I refers to the intensity and the A refers to the area of the 3D surface of the propagated wavefront. In this case, the area of a sphere, which is given by:

$$A_{sphere} = 4\pi r^2 \tag{11}$$

We know that $\phi_i = \phi_f$ (as per conservation of energy), so taking two different points from the source should lead to en equal flux going through the wavefront. In this case, we take $r = 1$m and an unknown r. Combining this with Equation 10 and 11, we get:

$$I_0 4\pi(1)^2 = I(r)4\pi(r)^2 \qquad \longrightarrow \qquad I(r) = \frac{I_0}{r^2} \tag{12}$$

where I_0 is the intensity at 1m from the source. Now, we just need to add the attenuation coefficient to Equation 12 as it is indicated in the slides seen in class and we get:

$$I_2 = I_1^{att} e^{-2\alpha r} = \frac{I_0}{r^2} e^{-2\alpha r} \tag{13}$$

where α is the attenuation coefficient defined in the slides.

From Equation 13 we can find the pressure by having into account Equation 1:

$$P_2 = P_1^{att} e^{-\alpha r} = \frac{P_0}{r} e^{-\alpha r} \tag{14}$$

b. For a point source (5p)

The spreading of the wave for a line source follows a cylindrical geometry for the near field, where we want to calculate the I. As we consider an infinite line source, the wave fronts in the far field will have the same cylindrical geometry. Therefore, to simplify the derivation of the intensity and the geometric attenuation, we continue our calculations using this coordinates. In this case, the wavefront will follow the area of a cylinder, which is given by:

$$A_{cylinder} = 2\pi rh \tag{15}$$

where h is the length of the linear source.

We preserve Equation 10 for this case and we know that $\phi_i = \phi_f$ (as per conservation of energy), so taking two different points from the source should lead to en equal flux going through the wavefront. In this case, we take again $r = 1$m and an unknown r. Combining this with Equation 10 and 15, we get:

$$I_0 2\pi(1)h = I(r)2\pi(r)h \qquad \longrightarrow \qquad I(r) = \frac{I_0}{r} \tag{16}$$

where I_0 is the intensity at 1m from the source. Now, we just need to add the attenuation coefficient to Equation 16 as it is indicated in the slides seen in class and we get:

$$I_2 = I_1^{att} e^{-2\alpha r} = \frac{I_0}{r} e^{-2\alpha r} \tag{17}$$

where α is the attenuation coefficient defined in the slides. From this same equation we can find the pressure by having into account Equation 1:

$$P_2 = P_1^{att} e^{-\alpha r} = \frac{P_0}{\sqrt{r}} e^{-\alpha r} \tag{18}$$