NBE-E4310 D - Biomedical Ultrasonics 2023

This document contains the solutions for the Execise 3 of the BMUS course. The student solutions should have been posted no later than 13:00 on December 4th.

Please, note that the code might be different than yours and might lead to slightly different solutions. A differently solved problem can sometimes be correct. However, this example should serve as a reference on how the exercise solutions should look like.

TASK 1 (14 points)

You have an acoustic fountain of water that has a maximum height of 3 mm.

a. What is the time-averaged intensity of the sound field?(3p)

When a focused wave creates an acoustic fountain on the air-water interface, water elevates and stays still for moderate power levels. In this configuration, as there is a balance of forces, the following equation should be fulfilled:

$$\sum \vec{F} = 0 \tag{1}$$

where $\vec{F}$ are the different forces contributing to the surface deformation. In this case, two force appear, the acoustic radiation force (ARF) and the hydrostatic force. Therefore, when balanced:

$$F_{Rad} + F_{Hyd} = 0 \tag{2}$$

Now, if we substitute Equation 2 for the values contributing to each force, we get:

$$P_{Lan}A = \Delta PA$$

Where P_{Lan} is the Langevin pressure, ΔP is the hydrostatic pressure and A are the area affected by the deformation. From here, we transform the equation into its original parameters and we get:

$$I_{SPTA} = \frac{\rho_0 g h c_0}{2} \tag{3}$$

where I_{SPTA} is the spatial-peak, time-averaged intensity of the ultrasonic wave, r_0 is the density of the medium deformed, g is the gravity acceleration, h is the height of the column deforming the air-water interface and c_0 is the speed of sound of the medium where the wave is traveling. Therefore, knowing that the medium where the wave travels is water and the height of the column of the acoustic fountain is 3 mm, we can solve Equation 3:

$$I_{SPTA}^{h=3mm} = 2.2W/cm^2$$

b. If the fountain is generated with a train of pulses that yield a duty cycle of 35%, what is the pulse-average intensity?(2p)

In this case, with the information we are given to solve this section, we can assume the following:

$$I_{SPTA} \simeq I_{SPPA} * DC \tag{4}$$

where I_{SPPA} is the spatial-peak, pulse-averaged intensity and DC is the percentage of duty cycle. Therefore, from Equation 4 and the data given from the exercise question we get:

$$I_{SPPA} = 6.3W/cm^2$$

c. Assuming linearity of the wave, what is the peak pressure of the wave?(3p)

Now, assuming linearity of the wave, we can relate the intensity and the pressure as:

$$p = \sqrt{I\rho_0 c_0} \tag{5}$$

where I refers to the spatial-peak, time-averaged and pulse-averaged intensities, respectively. Therefore, the peak pressure for the temporal averaged (continuous) and pulse averaged (burst) wave is:

$$P_{TA} = 0.18MPa P_{PA} = 0.31MPa$$

d. With the linearity assumption, what is the mechanical index? Assume f = 1 MHz.(3 p)

Now, with the information given by the exercise, we will assume that the peak pressure has the same absolute value in the negative and positive peak. Therefore, we now have the peak negative pressure (PNP), which relates to the mechanical index (MI) as:

$$MI = \frac{PNP}{\sqrt{f}} \tag{6}$$

where the PNP is expressed in MPa and f, the frequency, is expressed in MHz. Therefore, the MI for the temporal and averaged case is:

$$MI_{TA} = 0.18 MI_{PA} = 0.31$$

e. Is cavitation likely to be present?(2p)

From the results obtained in section d, we know that the mechanical index is an indicator to estimate if cavitation is likely to appear. For very sensitive applications, such as ophthalmology (*i.e.* the eye), the threshold of MI for cavitation is set at 0.23. In our case, the repeated pulse with a DC of 35% will be likely to produce cavitation and be a danger for the eye tissue.

f. How many times the intensity would be the intensity of Task *1a* if the fluid would be mercury?(3p)

In this case, we only need to change the density and speed of sound for equation 3. Therefore, we find that:

$$I_{SPTA}^{Hg} = 29W/cm^2$$

TASK 2 (11 points)

You have an object immersed in water. Consider that you have a planar source that travels in a direction normal to a flat surface and the wave meets that surface from water with impedance $Z_{water} = 1.5$ MRayl. Consider that the object has a varying impedance from Z_{object} = 1.5 MRayl to 15 MRayl.

a. Plot the reflection coefficient of pressure as a function of Z_{object}.(5 p)

The reflection coefficient can be calculated as:

$$R = \frac{p_{refl}}{p_{inc}} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \quad (7)$$

where p_{refl} and p_{inc} are the reflected and incident pressures and Z_1 and Z_2 are the impedances of the medium and the reflective object. In this case the medium is water and we will plot the R, the reflective coefficient of the object, as a function of its impedance. Therefore, we generate the following MATLAB code:

```
% Defining impedance
Z_wat = 1.5;
Z_obj = linspace(1.5, 15, 100);

% Equation
R = (Z_obj-Z_wat)./(Z_obj+Z_wat);

% Plotting
figure;
plot(Z_obj, R, LineWidth=2);
grid on
xlabel("Object impedance (MRayl)")
ylabel("Reflection coefficient")
```

Listing 1: Plotting the R vs Z.

And from the previous code, we get Figure 1 plot in blue.

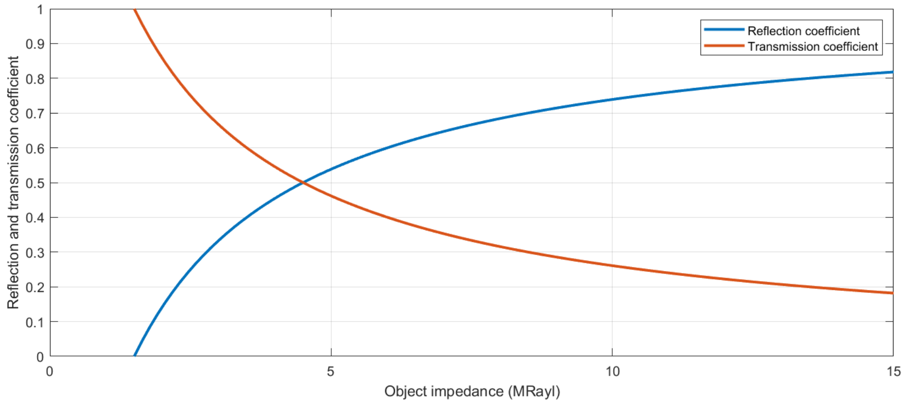

Figura 1: Reflection coefficient (blue) and transmission coefficient (red) vs object impedance.

b. Plot the transmission and reflection coefficients for pressure.(4 p)

The transmission coefficient is defined by:

$$T = 1 - R \quad (8)$$

where T and R are the transmission and reflection coefficients, respectively. Therefore, we can plot the two together using the following code:

```
% Equation
T = 1 - R;
```

```
% Plotting all together
figure;
plot(Z_obj, R);
hold on
plot(Z_obj, T)
grid on
hold off
legend(["Reflection coefficient", "Transmission coefficient"])
xlabel("Object impedance (MRayl)")
ylabel("Reflection and transmission coefficient")
```

Listing 2: Plotting the T vs Z.

And from the previous code, we get Figure 1 plot in red.

c. Plot the transmission and reflection coefficients for intensity.(2 p)

For the intensity coefficients, we have that:

$$R_{int} = R^2 \tag{9}$$

where R is the previous reflection coefficient and R_{int} is the reflection coefficient of intensity. Therefore, our code now will adapt the Equation 9:

```
% Equation
R_I = R.^2;
T_I = 1 - R_I;

% Plotting all together
figure;
plot(Z_obj, R_I);
hold on
plot(Z_obj, T_I)
grid on
hold off
legend(["Reflection coefficient", "Transmission coefficient"])
xlabel("Object impedance (MRayl)")
ylabel("Reflection and transmission coefficient of intensity")
```

Listing 3: Plotting the T vs Z.

And from the previous code, we get Figure 2.

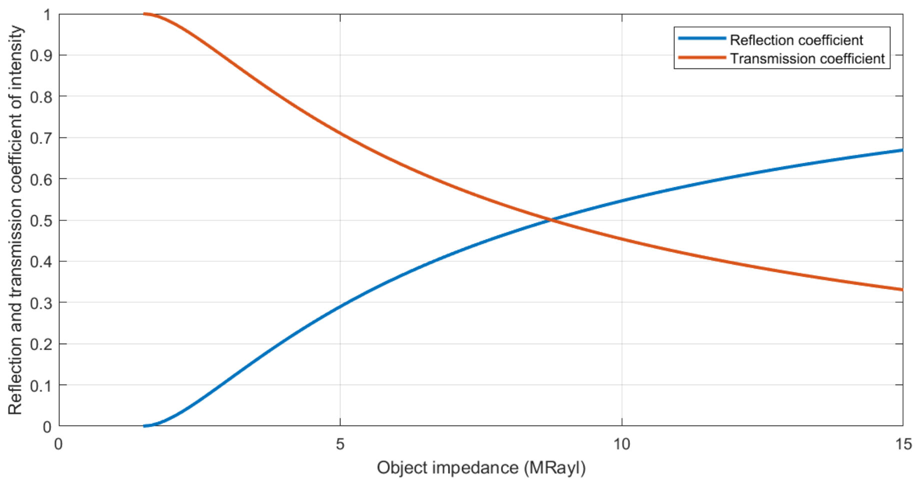

Figura 2: Reflection (blue) and transmission (red) coefficient of intensity vs object impedance.

TASK 3 (10 points)

Considering the case of Task 2, plot the magnitude of the drag coefficient as a function of Z_{object} if the transmitted wave is fully absorbed into the object.(10p)

Now, we have full absorption of the wave into the object. Because of this particular case, the total drag coefficient is expressed as a function of the reflection and transmission coeffcient as:

$$C_d = C_d^{pr} R_{int} + C_d^{pa} T_{int} \tag{10}$$

where C_d^{pr} is the drag coefficient in the case of a perfect reflection (*i.e.* drag coefficient equal to 2) and C_d^{pa} is the drag coefficient in the case of a perfect absortion (*i.e.* drag coefficient equal to 1). Therefore, we generate the following code:

```
% Equation
C_d = 2.*R_I + 1.*T_I

% Plotting all together
figure;
plot(Z_obj, C_d);
grid on
xlabel("Object impedance (MRayl)")
ylabel("Drag coefficient")
```

Listing 4: Plotting the T vs Z.

And from the previous code, we get Figure 3.

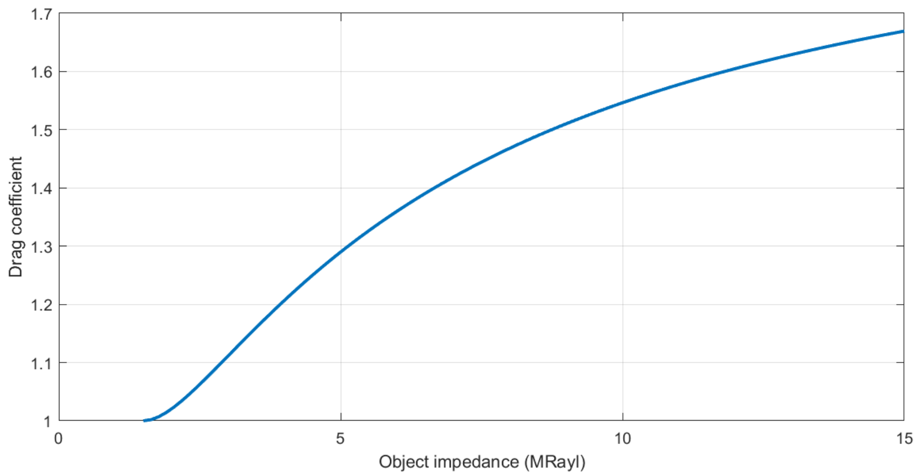

Figura 3: Drag coefficient of a fully absorbed wave vs object impedance.