

## NBE-E4310 D - Biomedical Ultrasonics 2023

This document contains the solutions for the Exercise 4 of the BMUS course. The student solutions should have been posted no later than 13:00 on December 11th.

Please, note that the code might be different than yours and might lead to slightly different solutions. A differently solved problem can sometimes be correct. However, this example should serve as a reference on how the exercise solutions should look like.

### TASK 1 (17 points)

Consider a circular ultrasound transducer of radius  $a = 8$  mm and operating frequency of  $f_0 = 1.5$  MHz in water.

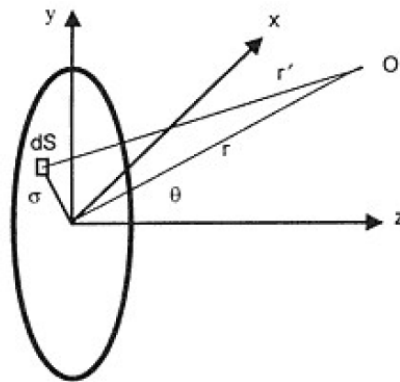


Figura 1: The geometry for the calculation of the pressure field  $p(r', t)$  at an observer point O, due to a plane circular piston source (Duck et al.)

#### a. What is the Rayleigh distance for this planar transducer (2p)

The Rayleigh distance is defined in the near field as:

$$Z_R = \frac{\pi a^2}{\lambda} \quad (1)$$

where  $a$  is the radius of the transducer and  $\lambda$  is the wavelength of the transmitted wave. In this case, the medium where the wave is propagating is water and the radius of the transducer is 8 mm. Therefore, having into account Equation 1, the Rayleigh distance is:

$$Z_R = 0.2m$$

#### b. Calculate the pressure $p_z$ along the center axis $Z$ from $z = 0$ to $1.5 \times$ Rayleigh distance as a complex presentation. Then plot the real part of the pressure. Use Equation 2. Consider that the pressure amplitude is $\hat{p} = 3$ kPa. (5p)

Following Equation 2, we can discretize the equation contributions into an area of a circle, leading to an addition of contributions to the pressure along the center axis of such circle.

$$p_Z(Z, r) = i \frac{\hat{p} k}{2\pi} \int_S \frac{e^{-ikr'}}{r'} dS \quad (2)$$

In this case, the code will follow:

```
1 % Defining parameters
2 a = 0.008;
```

```

3 f_0 = 1.5e06;
4 v = 1500;
5
6 % Define xyz field, transducer
7 x = linspace(-a,a,150);
8 y = linspace(-a,a,150);
9 z = linspace(0,1.5*Z_R,100);
10 [X,Y] = meshgrid(x,y);
11 scatter(X(:),Y(:), Marker=".")
12
13 N = 0;
14 x_t = [];
15 y_t = [];
16 for i = 1:150
17     for j = 1:150
18         dist = sqrt(x(i)^2 + y(j)^2);
19         if dist <= a
20             N = N + 1;
21             x_t = [x_t, i];
22             y_t = [y_t, j];
23         end
24     end
25 end
26 fprintf("Number of points inside the circle: %d pcs.", N)
27
28 % Circle to point approximation
29 A = pi*a^2;
30 A_app = A / N;
31
32 % Pressure:
33 p_hat = 3000;
34 k = (2*pi*f_0)/v;
35 p_z = zeros(100,1);
36 for j = 1:100
37     p_z(j) = 1i*(p_hat*k)./(2*pi).*sum(exp(-1i.*k.*sqrt(x(x_t()).^2 + y(y_t).^2 + z(j).^2)). ./ sqrt(x(x_t()).^2 + y(y_t).^2 + z(j)^2)*A_app);
38 end
39 figure()
40 plot(z, real(p_z))
41 xlabel("Z (m)")
42 xlim([0 0.3])
43 ylabel("Pressure (Pa)")

```

Listing 1: Plotting the P vs Z.

And from the previous code, we get Figure 2 plot in blue.

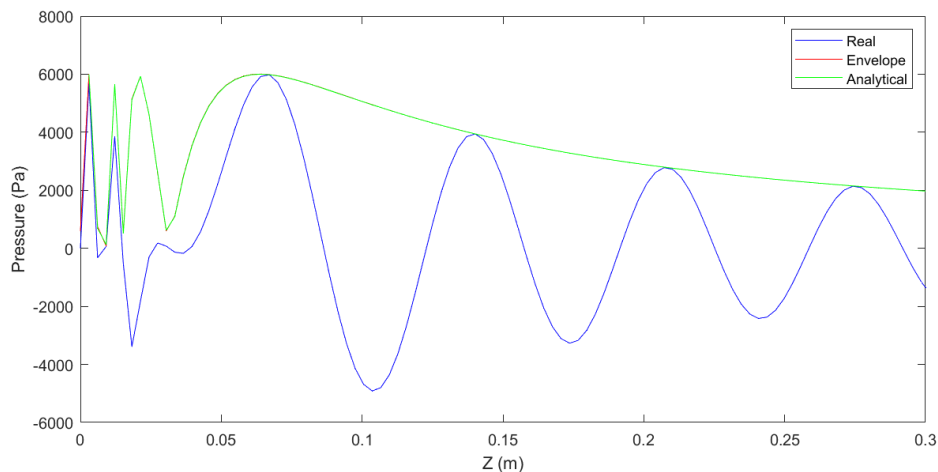


Figura 2: Real part (blue), Envelope (red) and analytical expression (green) of the pressure over the central axis.

c. Using Equation 2, calculate and plot the envelope of the pressure (hint: absolute of the complex pressure). You can neglect attenuation. (5p)

In this case, we only add the absolute command when plotting the pressure. The result is shown as the

red line in Figure 2.

**d. Plot the numerical expression from Task 1a and 1b and the pressure based on the analytical expression at the center axis in the same graph. The analytical expression is given in Equation 3. (5p)**

Following Equation 3, we develop a code that introduces the equation in the same manner as before.

$$p_z(z) = 2\hat{p} \left| \sin \left\{ \frac{kz}{2} \left[ \sqrt{1 + \left(\frac{a}{b}\right)^2} - 1 \right] \right\} \right| \quad (3)$$

We add the following to the previous code:

```

1 % Analytical expression
2 p_anal = zeros(100,1);
3 for j = 1:100
4     p_anal(j) = 2.*p_hat.*abs(sin((k.*z(j)/2 .* (sqrt(1+(a/z(j))^2)-1))));
5 end
6 figure()
7 plot(z,real(p_z), color='blue')
8 hold on
9 plot(z,abs(complex(p_z)), color='red')
10 plot(z,p_anal, color = 'green')
11 legend('Real', 'Envelope', 'Analytical')
12 xlabel("Z (m)");
13 xlim([0 0.3])
14 ylabel("Pressure (Pa)");

```

Listing 2: Plotting the analytical P vs Z.

And from the previous code, we get Figure 2 plot in green.

**TASK 2 (7 points)**

A focused continuous wave is travelling along the z-axis. The file (*I<sub>TA</sub>.txt*) contains a z-slice of intensity data in the x-y plane near the focus of the beam. Each element of the matrix corresponds to the time-averaged intensity (in W/cm<sup>2</sup>) in a 140 μm by 140 μm area. Apply the field to a perfect absorber. The entire power defined by *I<sub>TA</sub>.txt* is absorbed.

**a. What is the radiation force exerted on the absorber? (3p)**

In this exercise, the medium is not specified. For that reason, we will select water as our medium, but other mediums could have been picked.

Therefore, we can estimate the force exerted on the absorber as:

$$F_{rad} = \int \frac{\langle I_{SPTA} \rangle}{c_0} dS \quad (4)$$

where the  $\langle I_{SPTA} \rangle$  is the temporal averaged intensity given in the file of the exercise,  $c_0$  is the speed of sound in water and  $dS$  is the differential of area that will turn into the area affected by the focused wave. With that in mind, the generated code to solve the exercise follows:

```

1 % Parameters
2 I_ta = load("I_ta.txt");
3 A = length(I_ta(1,:)) * 140 * length(I_ta(1,:)) * 140;
4 A = A / 1e8;
5 c = 1484;
6
7 % Equation
8 I = sum(sum(I_ta));
9 F_rad = I*A / c; % N
10 disp(F_rad)

```

Listing 3: Calculation of ARF.

And from this code, we get that:

$$F_{rad} = 2.52N$$

**b. What is the maximum Langevin pressure applied to the interface, given that the interface is at the focus of the beam (2p)**

From the data of the file shared with the exercise, we can relate the intensity to the Langevin pressure as:

$$P_{Lan} = \frac{\langle I_{SPTA} \rangle}{c_0} \quad (5)$$

Therefore, the previous code will follow as:

```

1 max_I = max(I_ta(:));
2 max_P_Lan = max_I ./ c;
3 disp(max_P_Lan)

```

Listing 4: Calculation of maximum Langevin pressure.

And from this code, we get that:

$$P_{Lan} = 0.0013J/m^3$$

**c. Consider that in the previous case you are applying pulses with a duty cycle of 70%. What is the radiation force exerted on the absorber when the sound is on? (hint: think about the spatial peak pulse average) (2 p)**

Now, we just need to have into account that:

$$I_{SPTA} \simeq I_{SPPA} * DC \tag{6}$$

where  $I_{SPPA}$  is now the new intensity we will use for the exercise. The code will follow:

```
1 F_rad_pa = F_rad*0.7;  
2 disp(F_rad_pa)
```

Listing 5: Calculation of new ARF.

And from this code, we get that:

$$F_{rad} = 1.76N$$

**TASK 3 (10 points)**

Ultrasound can be used to heat the tissue to thermally damage it for therapeutic purposes, e.g. thermal ablation of cancer tissue. Pathologically, you can consider 30 and 240 equivalent minutes to represent edema and thermal necrosis, respectively. Please, study the review in the slides about the equations of the paper by Sapareto & Dewey and calculate the time to reach thermal dose of 30 and 240 equivalent minutes at temperatures:

a. 45 °C, b. 56 °C, c. 80 °C (6p)

For this exercise, we just need to have into account the equation from the literature:

$$t = \frac{TD}{R^{43-T}} \quad (7)$$

Where  $TD$  is the thermal dose,  $R$  is a constant equal to 0.5 (as  $T > 45$  °C) and  $T$  is the temperature. Therefore, the code will follow:

```

1 % Parameters
2 TD1 = 30;
3 TD2 = 240;
4 T1 = 45;
5 T2 = 56;
6 T3 = 80;
7 R = 0.5;
8
9 % A) T = 45 degC
10 t45_30 = TD1 / R^(43-T1);
11 t45_240 = TD2 / R^(43-T1);
12
13 % B) T = 56 degC
14 t56_30 = TD1 / R^(43-T2);
15 t56_240 = TD2 / R^(43-T2);
16
17 % C) T = 80 degC
18 t80_30 = TD1 / R^(43-T2);
19 t80_240 = TD2 / R^(43-T2);

```

Listing 6: Calculation of times.

And the results of this code are:

$$\begin{aligned}
 t_{30}^{40} &= 7.5min & ; & & t_{240}^{40} &= 60min \\
 t_{30}^{56} &= 3.7e - 3 & ; & & t_{240}^{56} &= 2.9e - 2 \\
 t_{30}^{80} &= 2.2e - 10 & ; & & t_{240}^{80} &= 1.8e - 9
 \end{aligned}$$