Exercise 1

Information from Exercise 1

Coffee	Blend of beans
Premium	Max 30% Haitian
Premium	Min 30% Colombian
Premium	Max 50% Peruvian

Coffee	Blend of beans
Deluxe	Min 40% Haitian
Deluxe	Min 40% Colombian
Deluxe	Max 20% Peruvian

Type of bean	Cost per kilo
Haitian	€ 1.00
Colombian	€ 2.50
Peruvian	€ 3.00

- Further information
  - Price of Premium blend (€ 6.00 per kg); deluxe blend (€ 4.50 per kg)
  - In total, 300 kg of Haitian, 500 kg of Colombian, 350 kg of Peruvian are available
  - Sell at least 400 kg of premium and 500 kg of deluxe blend

Step 1: Definition of variables

- **Step 1a: What are the variables?**
  - $X_{i,j}$: Total kg of beans of type i used in blend j, where I = 1,2 and 3 and j = 1,2

Premium	Deluxe
$X_{1,1}$ = kg of **Haitian** beans used for **premium** blend	$X_{1,2}$ = kg of **Haitian** beans used for **deluxe** blend
$X_{2,1}$ = kg of **Colombian** beans used for **premium** blend	$X_{2,2}$ = kg of **Colombian** beans used for **deluxe** blend
$X_{3,1}$ = kg of **Peruvian** beans used for **premium** blend	$X_{3,2}$ = kg of **Peruvian** beans used for **deluxe** blend

- **Step 1b: Indicate the valid range of all variables**
  - Real numbers (i.e., decimal values are possible)
  - Non-negative

Step 2: Define objective

- Step 2a: What do you want to achieve?
  - Maximize profit

- Step 2b: Express this mathematically by using variables and parameters
  - profit = revenue – costs

    revenue = price of kilo premium blend * amount premium blend + price of kilo deluxe blend * amount deluxe blend

    revenue = €6 ($X_{1,1}$ + $X_{2,1}$ + $X_{3,1}$) + €4.50 ($X_{1,2}$ + $X_{2,2}$ + $X_{3,2}$)

    cost = cost kilo Haitian beans * amount Haitian beans +

    cost kilo Colombian beans * amount Colombian beans +

    cost kilo Peruvian beans * amount Peruvian beans

    cost = €1 ($X_{1,1}$ + $X_{1,2}$) + €2.50 ($X_{1,2}$ + $X_{2,2,}$) + €3 ($X_{3,1}$ + $X_{3,2}$)
  - Maximize €5 $X_{1,1}$ + €3.50 $X_{2,1}$ + €3 $X_{3,1}$ + €3.50 $X_{1,2}$ + €2 $X_{2,2}$ + €1.50 $X_{3,2}$

Step 3: Formulate all constraints

- Restrictions on mix
  - Haitian beans in premium mix ≤ 30% total beans in premium mix
  - Colombian beans in premium mix ≥ 30% total beans in premium mix
  - Peruvian beans in premium mix ≤ 50% total beans in premium mix
  - Haitian beans in deluxe mix ≥ 40% total beans in deluxe mix
  - Colombian beans in deluxe mix ≥ 40% total beans in deluxe mix
  - Peruvian beans in deluxe mix ≤ 20% total beans in deluxe mix

Step 3: Formulate all constraints

- Restrictions on mix
  - $X_{1,1}$ ≤ 30% $(X_{1,1} + X_{2,1} + X_{3,1})$
  - Colombian beans in premium mix ≥ 30% total beans in premium mix
  - Peruvian beans in premium mix ≤ 50% total beans in premium mix
  - Haitian beans in deluxe mix ≥ 40% total beans in deluxe mix
  - Colombian beans in deluxe mix ≥ 40% total beans in deluxe mix
  - Peruvian beans in deluxe mix ≤ 20% total beans in deluxe mix

Step 3: Formulate all constraints

- Restrictions on mix
  - $X_{1,1} \leq 30\% (X_{1,1} + X_{2,1} + X_{3,1})$
  - $X_{2,1} \geq 30\% (X_{1,1} + X_{2,1} + X_{3,1})$
  - Peruvian beans in premium mix ≤ 50% total beans in premium mix
  - Haitian beans in deluxe mix ≥ 40% total beans in deluxe mix
  - Colombian beans in deluxe mix ≥ 40% total beans in deluxe mix
  - Peruvian beans in deluxe mix ≤ 20% total beans in deluxe mix

Step 3: Formulate all constraints

- Restrictions on mix
  - $X_{1,1} \leq 30\% (X_{1,1} + X_{2,1} + X_{3,1})$
  - $X_{2,1} \geq 30\% (X_{1,1} + X_{2,1} + X_{3,1})$
  - $X_{3,1} \leq 50\% (X_{1,1} + X_{2,1} + X_{3,1})$
  - $X_{1,2} \geq 40\% (X_{1,2} + X_{2,2} + X_{3,2})$
  - $X_{2,2} \geq 40\% (X_{1,2} + X_{2,2} + X_{3,2})$
  - $X_{3,2} \leq 20\% (X_{1,2} + X_{2,2} + X_{3,2})$

Step 3: Formulate all constraints

- Restrictions on mix (reformulation)
  - $0.7\ X_{1,1} - 0.3\ X_{2,1} - 0.3\ X_{3,1} \leq 0$
  - $-0.3\ X_{1,1} + 0.7\ X_{2,1} - 0.3\ X_{3,1} \geq 0$
  - $-0.5\ X_{1,1} - 0.5\ X_{2,1} + 0.5\ X_{3,1} \leq 0$
  - $0.6\ X_{1,2} - 0.4\ X_{2,2} - 0.4\ X_{3,2} \geq 0$
  - $-0.4\ X_{1,2} + 0.6\ X_{2,2} - 0.4\ X_{3,2} \geq 0$
  - $-0.2\ X_{1,2} - 0.2\ X_{2,2} + 0.8\ X_{3,2} \leq 0$

Step 3: Formulate all constraints

- Restrictions availability of beans
  - Haitian beans in premium + Haitian beans in deluxe ≤ 300
  - Colombian beans in premium + Colombian beans in deluxe ≤ 500
  - Peruvian beans in premium + Peruvian beans in deluxe ≤ 350

Step 3: Formulate all constraints

- Restrictions availability of beans
  - $X_{1,1} + X_{1,2} \leq 300$
  - Colombian beans in premium + Colombian beans in deluxe ≤ 500
  - Peruvian beans in premium + Peruvian beans in deluxe ≤ 350

Step 3: Formulate all constraints

- Restrictions availability of beans
  - $X_{1,1} + X_{1,2} \leq 300$
  - $X_{2,1} + X_{2,2} \leq 500$
  - $X_{3,1} + X_{3,2} \leq 350$

Step 3: Formulate all constraints

- Minimal production of each blend
  - total beans premium ≥ 400
  - total beans deluxe ≥ 500

Step 3: Formulate all constraints

- Minimal production of each blend
  - $X_{1,1} + X_{2,1} + X_{3,1} \geq 400$
  - total beans deluxe ≥ 500

Step 3: Formulate all constraints

- Minimal production of each blend
  - $X_{1,1} + X_{2,1} + X_{3,1} \geq 400$
  - $X_{1,2} + X_{2,2} + X_{3,2} \geq 500$

Answer Exercise 1 (the variables)

<u>Variables</u>

$X_{1,1}$ = kg of Haitian beans used for premium blend
$X_{2,1}$ = kg of Colombian beans used for premium blend
$X_{3,1}$ = kg of Peruvian beans used for premium blend
$X_{1,2}$ = kg of Haitian beans used for deluxe blend
$X_{2,2}$ = kg of Colombian beans used for deluxe blend
$X_{3,2}$ = kg of Peruvian beans used for deluxe blend

<u>Valid range</u>

$X_{1,1}, X_{2,1}, X_{3,1}, X_{1,2}, X_{2,2}, X_{3,2} \geq 0$

Answer Exercise 1 (the model)

Maximize $Z = 5X_{1,1} + 3.5X_{2,1} + 3X_{3,1} + 3.5X_{1,2} + 2X_{2,2} + 1.5X_{3,2}$

Subject to

$$0.7\,X_{1,1} - 0.3\,X_{2,1} - 0.3\,X_{3,1} \leq 0$$
$$-0.3\,X_{1,1} + 0.7\,X_{2,1} - 0.3\,X_{3,1} \geq 0$$
$$-0.5\,X_{1,1} - 0.5\,X_{2,1} + 0.5\,X_{3,1} \leq 0$$
$$0.6\,X_{1,2} - 0.4\,X_{2,2} - 0.4\,X_{3,2} \geq 0$$
$$-0.4\,X_{1,2} + 0.6\,X_{2,2} - 0.4\,X_{3,2} \geq 0$$
$$-0.2\,X_{1,2} - 0.2\,X_{2,2} + 0.8\,X_{3,2} \leq 0$$
$$X_{1,1} + X_{1,2} \leq 300$$
$$X_{2,1} + X_{2,2} \leq 500$$
$$X_{3,1} + X_{3,2} \leq 350$$
$$X_{1,1} + X_{2,1} + X_{3,1} \geq 400$$
$$X_{1,2} + X_{2,2} + X_{3,2} \geq 500$$

$X_{1,1}, X_{2,1}, X_{3,1}, X_{1,2}, X_{2,2}, X_{3,2} \geq 0$

Exercise 2

Information from Exercise 2

Production times	Work station 1	Work station 2	Work station 3	Work station 4
Product 1	5	7	4	10
Product 2	6	12	8	15
Product 3	13	14	9	17

Profit contribution	Work station 1	Work station 2	Work station 3	Work station 4
Product 1	10	8	6	9
Product 2	18	20	15	17
Product 3	15	16	13	17

- Further information
  - Per week, 35 working hours at each workstation (=2100 mins)
  - Minimum production:
    - 100 units product 1
    - 150 units product 2
    - 100 units product 3

Step 1: Definition of variables

- Step 1a: What are the variables?
  - 12 variables (x_{ij}), one variable for each combination of product type (i = 1, 2, 3) and workstation (j = 1, 2, 3, 4)

  - $X_{1,1}$ = quantity of units **product type 1** on **workstation 1**
  - $X_{1,2}$ = quantity of units **product type 1** on **workstation 2**
  - ⋮
  - ⋮
  - $X_{3,4}$ = quantity of units **product type 3** on **workstation 4**

- Step 1b: Indicate the valid range of all variables
  - All variables are integer and non-negative

Information from Exercise 2

Production times				
	Work station 1	Work station 2	Work station 3	Work station 4
Product 1	5	7	4	10
Product 2	6	12	8	15
Product 3	13	14	9	17

Profit contribution				
	Work station 1	Work station 2	Work station 3	Work station 4
Product 1	10	8	6	9
Product 2	18	20	15	17
Product 3	15	16	13	17

- Further information
  - Per week, 35 working hours at each workstation (=2100 mins)
  - Minimum production:
    - 100 units product 1
    - 150 units product 2
    - 100 units product 3

Step 2: Define objective

- 2a: What do you want to achieve?
  - Maximise profit

- 2b: Express this mathematically by using variables and parameters
  - Profit Z = sum {profit per "workstation–product type combination" * quantity of product type i produced on workstation j}
  - Maximize $Z = 10\,x_{1,1} + 8\,x_{1,2} + 6\,x_{1,3} + 9\,x_{1,4} + 18\,x_{2,1} +$
    $20\,x_{2,2} + 15\,x_{2,3} + 17\,x_{2,4} + 15\,x_{3,1} + 16\,x_{3,2} + 13\,x_{3,3} + 17\,x_{3,4}$

Information from Exercise 2

	Production times			
	Work station 1	Work station 2	Work station 3	Work station 4
Product 1	5	7	4	10
Product 2	6	12	8	15
Product 3	13	14	9	17

	Profit contribution			
	Work station 1	Work station 2	Work station 3	Work station 4
Product 1	10	8	6	9
Product 2	18	20	15	17
Product 3	15	16	13	17

- Further information
  - Per week, 35 working hours at each workstation (=2100 mins)
  - Minimum production:
    - 100 units product 1
    - 150 units product 2
    - 100 units product 3

Step 3: Formulate all constraints

- Availability of production time
  - Production time on workstation 1 ≤ 2100 min

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  - Sum of all products produced on workstation 1 ≤ 2100 min

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  - $5\ x_{1,1} + 6\ x_{2,1} + 13\ x_{3,1} \leq 2100$ min
- Similar for other workstations
  - $7\ x_{1,2} + 12\ x_{2,2} + 14\ x_{3,2} \leq 2100$ min
  - $4\ x_{1,3} + 8\ x_{2,3} + 9\ x_{3,3} \leq 2100$ min
  - $10\ x_{1,4} + 15\ x_{2,4} + 17\ x_{3,4} \leq 2100$ min

Step 3: Formulate all constraints

- Minimal production of each product type
  - number of product 1 produced ≥ 100 units

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  - sum of all units of product type 1 (produced on any of the 4 workstations) ≥ 100 units

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  - $X_{1,1} + x_{1,2} + x_{1,3} + x_{1,4} \geq 100$ units

- Similar for other product types
  - $X_{2,1} + x_{2,2} + x_{2,3} + x_{2,4} \geq 150$ units
  - $X_{3,1} + x_{3,2} + x_{3,3} + x_{3,4} \geq 100$ units

Answer Exercise 2 (the variables)

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Variables

12 variables ($x_{i,j}$), one variable for each combination of product type (i = 1, 2, 3) and workstation (j = 1, 2, 3, 4)

$X_{1,1}$ = quantity of units product type 1 on workstation 1

$X_{1,2}$ = quantity of units product type 1 on workstation 2

⋮

$X_{3,4}$ = quantity of units product type 3 on workstation 4

Valid range

$x_{i,j}$ are integer and non-negative

Answer Exercise 2 (the model)

Maximise $Z = 10x_{11} + 8x_{12} + 6x_{13} + 9x_{14} + 18x_{21} + 20x_{22} + 15x_{23} + 17x_{24} + 15x_{31} + 16x_{32} + 13x_{33} + 17x_{34}$

Subject to

$5x_{1,1} + 6x_{2,1} + 13x_{3,1} \leq 2100$ minutes

$7x_{1,2} + 12x_{2,2} + 14x_{3,2} \leq 2100$ minutes

$4x_{1,3} + 8x_{2,3} + 9x_{3,3} \leq 2100$ minutes

$10x_{1,4} + 15x_{2,4} + 17x_{3,4} \leq 2100$ minutes

$x_{1,1} + x_{1,2} + x_{1,3} + x_{1,4} \geq 100$

$x_{2,1} + x_{2,2} + x_{2,3} + x_{2,4} \geq 150$

$x_{3,1} + x_{3,2} + x_{3,3} + x_{3,4} \geq 100$

$x_{i,j} \geq 0$ for all i and j

$x_{i,j}$ integer

Exercises 3 & 6

Information from exercise 3

Month	3	4	5	6	7	8
Demand	5000	6000	6500	7000	8000	9500

Additional information:

- Deterioration at the end of each month:
  - 11% if produced in current month (T)
  - 47% if produced in previous month (T - 1)
  - 100% if produced two months before (T - 2)
- 1000 kg in stock from month 2 and 2000 from month 1
- Deterioration per kg cost $25
- Production cost per kg is $15, and inventory cost per kg/month is $0,75
- Max production capacity per month is 7000 kg

Step 1: Definition of variables

- Step 1a: What are the variables?
  - 18 variables ($P_{i,j}$), each variable is the amount of kg produced in month i which will be sold in month j:

  - $P_{3,3}$ = kg produced in month 3 meant to be sold in month 3
  - $P_{3,4}$ = kg produced in month 3 meant to be sold in month 4

  ⋮
  - $P_{8,8}$ = kg produced in month 8 meant to be sold in month 8

  Additionally, we need to create variables to model the leftovers from months 1 and 2:

  - $P_{1,3}$ = kg produced in month 1 meant to be sold in month 3
  - $P_{2,3}$ = kg produced in month 2 meant to be sold in month 3
  - $P_{2,4}$ = kg produced in month 2 meant to be sold in month 4

- Step 1b: Indicate the valid range of all variables
  - All variables are non-negative

Step 3: Formulate all constraints

- Capacity of production
  - Production on month 3 ≤ 7000

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  - $P_{3,3} + P_{3,4} + P_{3,5} \leq 7000$
- Similar for other months:
  - $P_{4,4} + P_{4,5} + P_{4,6} \leq 7000$
  - $P_{5,5} + P_{5,6} + P_{5,7} \leq 7000$
  - $P_{6,6} + P_{6,7} + P_{6,8} \leq 7000$
  - $P_{7,7} + P_{7,8} \leq 7000$
  - $P_{8,8} \leq 7000$

Step 3: Formulate all constraints

- Satisfying Demand :
  - kg to be sold on month **3** = 5000 kg

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  - $0,47P_{1,3} + 0,89P_{2,3} + P_{3,3} = 5000$

- Same for other months:
  - $0,47P_{2,4} + 0,89P_{3,4} + P_{4,4} = 6000$
  - $0,47P_{3,5} + 0,89P_{4,5} + P_{5,5} = 6500$
  - $0,47P_{4,6} + 0,89P_{5,6} + P_{6,6} = 7000$
  - $0,47P_{5,7} + 0,89P_{6,7} + P_{7,7} = 8000$
  - $0,47P_{6,8} + 0,89P_{7,8} + P_{8,8} = 9500$

Step 3: Formulate all constraints

- Leftover inventory:
  - Leftovers from month 1 = 2000 kg (at this point only 47% of what was initially produced is left)

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  - $P_{1,3}$ = 2000/0,47 = 4256

- Similar for month 2, but these inventories have longer shelf life so they can be used on months 3 or 4:
  - $P_{2,3} + P_{2,4}$ = 1000/0,89= 1123

Step 2: Define objective

- 2a: What do you want to achieve?
  - Minimize total cost

- 2b: Express this mathematically by using variables and parameters
  - Total cost = Production cost + inventory cost + damaged product cost
  - Production cost = 15 * ($P_{3,3}$ + $P_{3,4}$ + $P_{3,5}$ + $P_{4,4}$ … + $P_{8,8}$)

Inventory at the end of month 2 **Inventory at the end of month 3**

  - Inventory cost = 0,75 * (0,47$P_{1,3}$ + 0,89$P_{2,3}$ + 0,89$P_{2,4}$) + 0,75 * (0,47$P_{2,4}$ + 0,89$P_{3,4}$ + 0,89$P_{3,5}$) +…
  - Waste cost = 25 * (0,53$P_{1,3}$ + 0,11$P_{2,3}$ + 0,11$P_{2,4}$) + 15 * (0,53$P_{2,4}$ + 0,11$P_{3,4}$ + 0,11$P_{3,5}$) +…

Answer Exercise 3 (the model)

Minimize $Z = \mathbf{15} * (P_{3,3} + P_{3,4} + P_{3,5} + P_{4,4} \ldots + P_{8,8}) + \ldots + \mathbf{0,75} * (0,47P_{2,4} + 0,89P_{3,4} + 0,89P_{3,5}) + \ldots + \mathbf{25} * (0,53P_{2,4} + 0,11P_{3,4} + 0,11P_{3,5}) + \ldots$

s.t.

$P_{3,3} + P_{3,4} + P_{3,5} \leq 6000$

$P_{4,4} + P_{4,5} + P_{4,6} \leq 6000$

$P_{5,5} + P_{5,6} + P_{5,7} \leq 6000$

$P_{6,6} + P_{6,7} + P_{6,8} \leq 6000$

$P_{7,7} + P_{7,8} \leq 6000$

$P_{8,8} \leq 6000$

$0,47P_{2,4} + 0,89P_{3,4} + P_{4,4} = 6000$

$0,47P_{3,5} + 0,89P_{4,5} + P_{5,5} = 6500$

$0,47P_{4,6} + 0,89P_{5,6} + P_{6,6} = 7000$

$0,47P_{5,7} + 0,89P_{6,7} + P_{7,7} = 8000$

$0,47P_{6,8} + 0,89P_{7,8} + P_{8,8} = 9500$

$P_{1,3} = 4256$

$P_{2,3} + P_{2,4} = 1123$

$P_{i,j} \geq 0$ *for all i and j (integer)*

Are there any questions?