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# Session 6 (cont.): <br> Vehicle routing problems: heuristics $35 E 00750$ Logistics Systems and Analytics 

## Solving a TSP to optimality?

- An optimal algorithm would be:
- Try all combinations and select cheapest
- For TSP, assume n cities:
- Start in city 1
- Second city: $(n-1)$ options
- Third city: $(n-2)$ options
- ...

Hence, we have to consider $(n-1)(n-2) \ldots 1=(n-1)$ !
 options to find an optimal route connecting all cities

## TSP heuristics

- Constructive heuristics
- Starts with an empty solution and extends solution step by step until a complete solution is obtained
- Example within this course: nearest neighbor
- Improvement or exchange procedures
- Start with a (random) solution and repeatedly consider various changes
- Examples: Clarke \& Wright savings heuristic
- Examples: simulated annealing, genetic algorithms


## Heuristics for the VRP

A VRP may consist of multiple routes. VRP heuristics are thus typically a stepwise process, such as:

- Cluster-first; route-second
- Step 1: Construct clusters of locations
- Step 2: Solve one TSP per cluster
- Route-first; cluster-second
- Step 1: Relax constraints on vehicle capacity to build one enormous TSP tour
- Step 2: Split the enormous tour into feasible routes

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## Nearest neighbor (insertion heuristic)

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- Nearest neighbor (n cities):
- Starting location/city is known
- Select a city from those not visited yet with the min. distance from the last city in the tour
- Continue as long as cities remain unvisited
- You could consider some stopping criterion
- Total route length
- Maximum driving time
- Vehicle (weight/volume) capacity

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## Nearest neighbor (example)

- The objective is to find the route visiting all customers exactly once in the shortest time
- The route starts and ends at the depot

|  | Depot | C1 | C2 | C3 | C4 | C5 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Depot | - | 35 | 43 | 23 | 33 | 19 |
| C1 | 35 | - | 47 | 42 | 21 | 26 |
| C2 | 43 | 47 | - | 36 | 31 | 30 |
| C3 | 23 | 42 | 36 | - | 50 | 17 |
| C4 | 33 | 21 | 31 | 50 | - | 45 |
| C5 | 19 | 26 | 30 | 17 | 45 | - |

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## Nearest neighbor (example)

| To |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| From | Depot | 1 | C2 | 3 | C5 |  |
| 1) Depot |  | 35 | 43 | 23 | 33 | 19 |
| C1 |  | - | 47 | 42 | 21 | 26 |
| C2 |  | 47 | - | 36 | 31 | 30 |
| C3 |  | 42 | 36 | - | 50 | 17 |
| C4 |  | 21 | 31 | 50 | - | 45 |
| C5 |  | 26 | 30 | 17 | 45 | - |

## Nearest neighbor (example)

|  | $\begin{aligned} & \text { To } \\ & \mid \text { Depot } \end{aligned}$ |  | C2 | C3 | C4 | C5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| From $\qquad$ |  |  |  |  |  |  |
| 1) ${ }_{\text {C1 }}$ |  | 35 | 43 47 | 23 42 | 33 21 |  |
| C2 |  | 47 | - | 36 | 31 |  |
| C3 |  | 42 | 36 | - | 50 |  |
| C4 |  | 21 | 31 | 50 | - |  |
| 12) C 5 |  | 26 | 30 | 17 | 45 |  |

## Nearest neighbor (example)

|  | To Depot | C1 | C2 | C3 | C4 | C5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1) Depot |  | 35 | 43 |  | 33 |  |
| C1 |  | - | 47 |  | 21 |  |
| C2 |  | 47 | - |  | 31 |  |
| C3 |  | 42 | 36 |  | 50 |  |
| C4 |  | 21 | 31 |  | - |  |
| 2) C 5 |  | 26 | 30 |  | 45 |  |

## Nearest neighbor (example)

|  | To Depot | C1 | C2 | C3 | C4 | C5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1) Depot |  | 35 |  |  | 33 |  |
| C1 |  | - |  |  | 21 |  |
| $4{ }^{4} \mathrm{C} 2$ |  | 47 |  |  | 31 |  |
| 3 C3 |  | 42 |  |  | 50 |  |
| C4 |  | 21 |  |  | - |  |
| 2) C 5 |  | 26 |  |  | 45 |  |

## Nearest neighbor (example)



## Nearest neighbor (example)

|  |  | To | Depot C1 | C2 | C3 | C4 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| From | C5 |  |  |  |  |  |
| 1) |  |  |  |  |  |  |
| Depot | - |  |  |  |  |  |
| C1 | 35 |  |  |  |  |  |
| C2 |  |  |  |  |  |  |
| C3 |  |  |  |  |  |  |
| C5 |  |  |  |  |  |  |
| C5 |  |  |  |  |  |  |

## Nearest neighbor (example)

|  | Depot C1 |  | C2 | C3 | C4 | C5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -1) Depot | - | 35 | 43 | 23 | 33 | 19 |
| -6) C 1 | 35 | - | 47 | 42 | 21 | 26 |
| 4) C 2 | 43 | 47 | - | 36 | 31 | 30 |
| 3) C 3 | 23 | 42 | 36 | - | 50 | 17 |
| 5) C 4 | 33 | 21 | 31 | 50 | - | 45 |
| $\stackrel{\text { ¢ }}{ }$ C5 | 19 | 26 | 30 | 17 | 45 | - |

- Route: \{Depot, 5, 3, 2, 4, 1, Depot\}
- Total distance: $19+17+36+31+21+35=159$

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## Thank you!

## Questions?

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