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Tutorial 2: Facility Location and Network Design

Supply Chain Network Design

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Contents of Tutorial 2

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Exercise 1

Information on Exercise 1

Location Factor	Weight (w_j)	Scores Amsterdam	Scores The Hague	Scores Utrecht	Scores Rotterdam
1. Warehouse utilization	20	8	6	4	7
2. Average time per trip from warehouse to retailers	15	7	5	7	5
3. Employee preferences	15	1	7	8	3
4. Accessibility to major highways	10	7	4	9	5
5. Land costs	10	2	3	1	4
6. Quality of life	15	5	6	9	5
7. Taxes	15	3	5	5	4

a) Which is the best site?

- ◊ Assume that a higher score is more desirable than a lower one
- ◊ Support your answer with calculations

Answer to Exercise 1a

Location Factor	Weight (w_i)	Scores Amsterdam	Scores The Hague	Scores Utrecht	Scores Rotterdam
1. Warehouse utilization	20	8	6	4	7
2. Average time per trip from warehouse to retailers	15	7	5	7	5
3. Employee preferences	15	1	7	8	3
4. Accessibility to major highways	10	7	4	9	5
5. Land costs	10	2	3	1	4
6. Quality of life	15	5	6	9	5
7. Taxes	15	3	5	5	4
		490	535	615	485

Answer to Exercise 1a

Location Factor	Weight (w_i)	Scores Amsterdam	Scores The Hague	Scores Utrecht	Scores Rotterdam
1. Warehouse utilization	20	8	6	4	7
2. Average time per trip from warehouse to retailers	15	7	5	7	5
3. Employee preferences	15	1	7	8	3
4. Accessibility to major highways	10	7	4	9	5
5. Land costs	10	2	3	1	4
6. Quality of life	15	5	6	9	5
7. Taxes	15	3	5	5	4
		490	535	615	485

Information on Exercise 1b

Location Factor	Weight (w_j)	Scores Amsterdam	Scores The Hague	Scores Utrecht	Scores Rotterdam
1. Warehouse utilization	20	8	6	4	7
2. Average time per trip from warehouse to retailers	15	7	5	7	5
3. Employee preferences	15	1	7	8	3
4. Accessibility to major highways	10	7	4	9	5
5. Land costs	10	2	3	1	4
6. Quality of life	15	5	6	9	5
7. Taxes	15	3	5	5	4

b) Range of values for w_1 such that the site from Exercise 1a remains the best

- ◊ Assume that all other weights and all scores keep their current values
- ◊ Support your answer with calculations

Answer to Exercise 1b

- Weight of warehouse utilization = w_1
  - Amsterdam: $w_1 \times 8 + 15 \times 7 + 15 \times 1 + 10 \times 7 + 10 \times 2 + 15 \times 5 + 15 \times 3 = 330 + 8w_1$
  - The Hague: $6w_1 + 415$
  - Rotterdam: $7w_1 + 345$
  - Utrecht: $4w_1 + 535$ (best site in Exercise 1a)
- $4w_1 + 535$ needs to be larger than all values for other cities, so:
  - $4w_1 + 535 > 330 + 8w_1; w_1 < 51.25$
  - $4w_1 + 535 > 6w_1 + 415; w_1 < 60$
  - $4w_1 + 535 > 7w_1 + 345; w_1 < 63.3$
- If range of values for this weight: $0 \leq w_1 < 51.25$ then Utrecht remains the best site

Answer to Exercise 1b

- If $0 \leq w_1 < 51.25$ then Utrecht remains the best site.
  - Note that even if the weight w_1 changes, the other weights do not. So it may happen that the sum of all weights may not be 100 anymore.
  - But this is not an issue. As long as all options are considered with the same weight composition, the comparison of total scores is fair.

Exercise 2

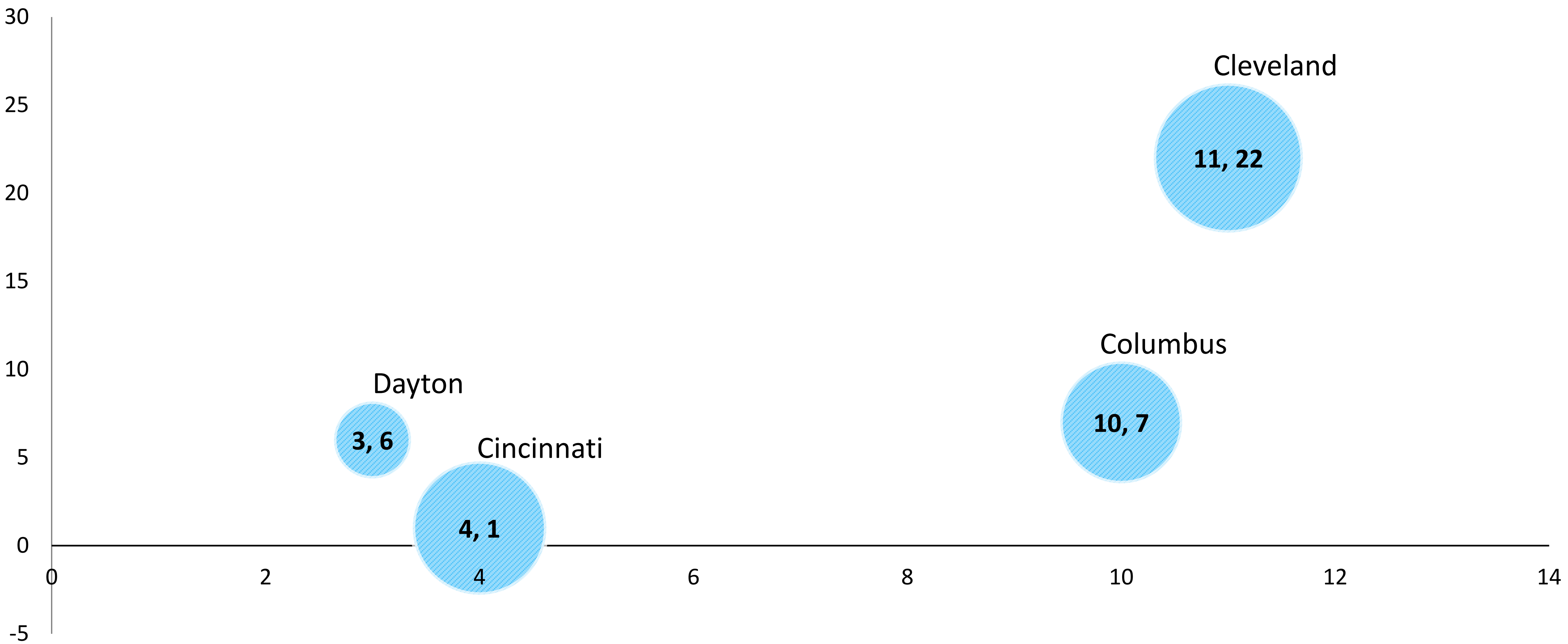
Information on Exercise 2

- Matrix Manufacturing Inc. services four stores located in four Ohio cities

Store Location	Coordinates (x, y)	Load volumes
Cleveland	(11, 22)	15
Columbus	(10, 7)	10
Cincinnati	(4, 1)	12
Dayton	(3, 6)	4
Total		41

- It considers two warehouse locations:
  - Mansfield, Ohio (coordinates: x=11, y=14)
  - Springfield, Ohio (coordinates: x=6, y=6.5)
- Which of the two locations is the most suitable location?

Answer to Exercise 2



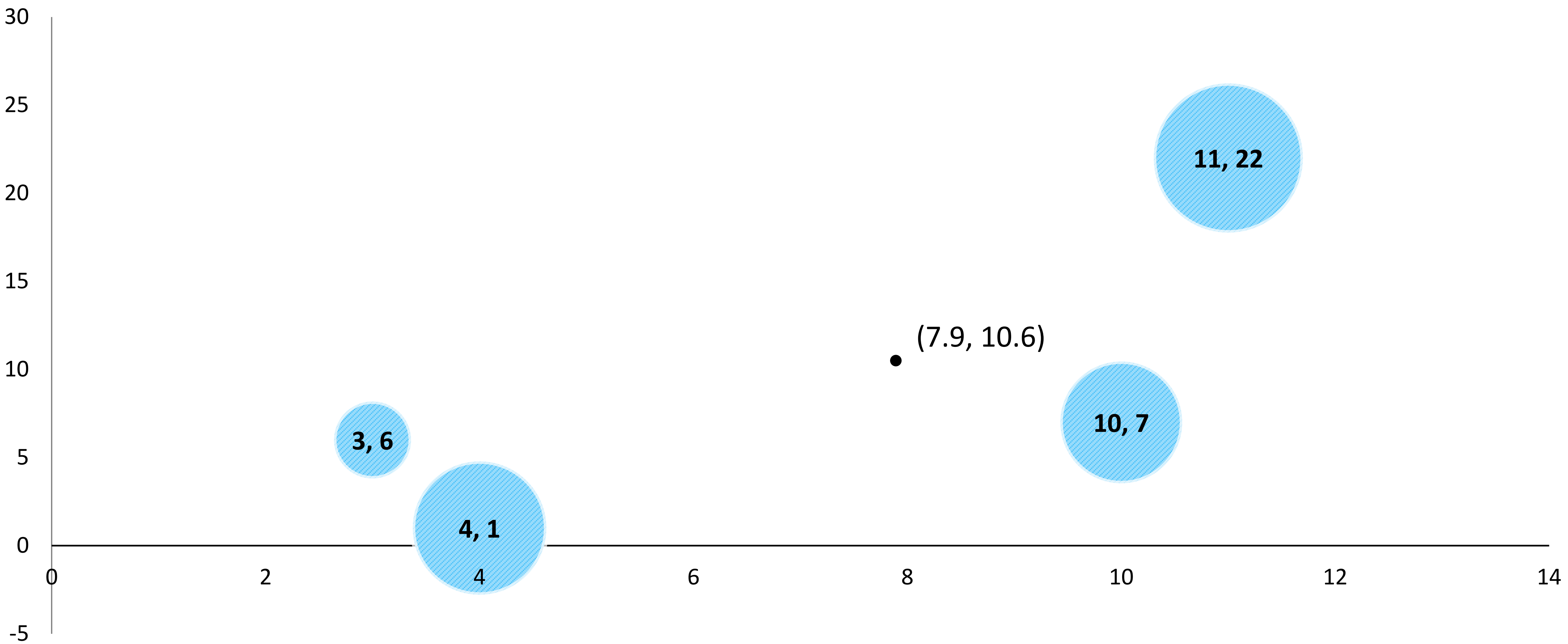
Answer to Exercise 2

- Use center-of-gravity method

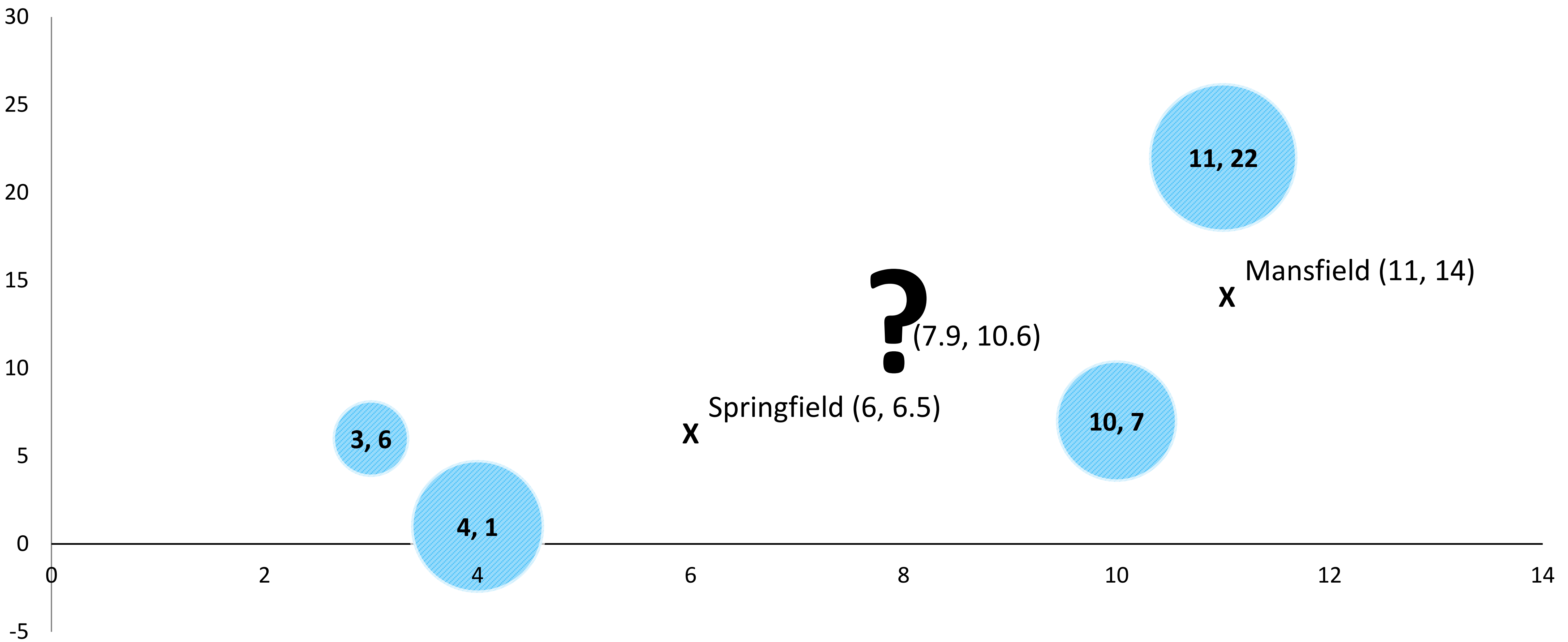
Store Location	Coordinates (x, y)	Load volumes	$d_{ix}W_i$	$d_{iy}W_i$
Cleveland	(11, 22)	15	165	330
Columbus	(10, 7)	10	100	70
Cincinnati	(4, 1)	12	48	12
Dayton	(3, 6)	4	12	24
Total		41	325	436

- $C_x = \frac{\sum_i d_{ix}W_i}{\sum_i W_i} = \frac{325}{41} = 7.9$
- $C_y = \frac{\sum_i d_{iy}W_i}{\sum_i W_i} = \frac{436}{41} = 10.6$

Answer Exercise 2



Further information on Exercise 2



Answer to Exercise 2

- Optimal location seems to be in the middle of Mansfield and Springfield (candidate locations). How to continue?
- For example, compute the (Euclidean) distances between optimal location and candidate locations
  - $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
  - Distance between optimal (7.9, 10.6) and Springfield (6, 6.5) = 4.52
  - Distance between optimal (7.9, 10.6) and Mansfield (11, 14) = 4.60
- So, Springfield is the most suitable option
  - With very close margin. If any other aspects play a role, those should probably be more decisive.

Exercise 3

Information from Exercise 3a

- 4 potential locations for a warehouse to serve demand in 5 cities
  - Fixed costs for opening a new distribution centre at each of the candidate locations known
  - Variable distribution costs associated with supplying one unit of demand of a city from that location is known

Potential warehouse location	Fixed cost	City 1	City 2	City 3	City 4	City 5
1	175	5	4.5	8	2.5	5
2	410	3	4	6	6	7
3	200	5.5	9	3	5	4
4	160	2	10	5	7	6
Demand		15	22	11	25	22

- Formulate this as an <u>uncapacitated</u> facility location problem

Steps to formulate an LP model

- Read the problem; then read it again!
- Step 1: Definition of decision variables
  - 1a: Decision needs to be made on?
    - Express this by using, for example, x_1, x_2 (clearly explaining each variable)
  - 1b: Indicate valid range of all variables
    - Binary, integer, real; positive, (non-)negative
- Step 2: Define objective function
  - 2a: What do you want to achieve? Choose between minimize and maximize
  - 2b: Express this mathematically using variables and parameters
- Step 3: Formulate all constraints
  - Develop mathematical relationships to describe constraints (using either $<$, $>$, $=$, $\leq$ or $\geq$)

Step 1: Definition of variables

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Sets:

I set of 5 cities (customers) $I = \{1,2,3,4,5\}$

J set of 4 potential locations $J = \{1,2,3,4\}$

Parameters:

F_j Fixed costs for opening facility at location j

c_{ij} The (variable) distribution cost for supplying 1 unit of customer i 's demand from the facility at j

D_i Demand of customer i

Step 1: Definition of variables

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- Step 1a: What are the variables?
  - y_j : Whether or not to open a warehouse at location *j*
  - x_{ij} : Volume of demand of customer i served from location *j*

- Step 1b: Indicate the valid range of all variables
  - $y_j \in \{0,1\} \; \forall j$ (binary: y_j values are 0 or 1 for all j)
  - $x_{ij} \geq 0, \forall \; i, j$ (x_{ij} non-negative for all i and j)

Step 2: Define objective

- Step 2a: What do you want to achieve?
  - Minimize the total distribution costs of this network

- Step 2b: Express mathematically
  - The total costs include a fixed cost and the variable distribution costs associated with a certain location
    - Fixed costs: $175y_1 + 410y_2 + 200y_3 + 160y_4$
    - Variable distribution costs: $5x_{11} + 3x_{12} + 5.5x_{13} + 2x_{14} + 4.5x_{21} + 4x_{22} + 9x_{23} + 10x_{24} + 8x_{31} + 6x_{32} + 3x_{33} + 5x_{34} + 2.5x_{41} + 6x_{42} + 5x_{43} + 7x_{44} + 5x_{51} + 7x_{52} + 4x_{53} + 6x_{54}$
  - $Min\ 175y_1 + 410y_2 + 200y_3 + 160y_4 + 5x_{11} + 3x_{12} + 5.5x_{13} + 2x_{14} + 4.5x_{21} + 4x_{22} + 9x_{23} + 10x_{24} + 8x_{31} + 6x_{32} + 3x_{33} + 5x_{34} + 2.5x_{41} + 6x_{42} + 5x_{43} + 7x_{44} + 5x_{51} + 7x_{52} + 4x_{53} + 6x_{54}$

Step 2: Define objective

- Step 2a: What do you want to achieve?
  - Minimize the total distribution costs of this network

- Step 2b: Express mathematically
  - The total costs include a fixed cost and the variable distribution costs associated with a certain location
    - Fixed costs: $\sum_{j \in J} F_j y_j$
    - Variable distribution costs: $\sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij}$
  - Overall: $\sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} + \sum_{j \in J} F_j y_j$

Step 3: Formulate all constraints

- Satisfy all demand from all customers
  - (For customer 1, $i = 1$) $x_{11} + x_{12} + x_{13} + x_{14} = 15$
  - (For customer 2, $i = 2$) $x_{21} + x_{22} + x_{23} + x_{24} = 22$
  - (For customer 3, $i = 3$) $x_{31} + x_{32} + x_{33} + x_{34} = 11$
  - (For customer 4, $i = 4$) $x_{41} + x_{42} + x_{43} + x_{44} = 25$
  - (For customer 5, $i = 5$) $x_{51} + x_{52} + x_{53} + x_{54} = 22$

$$\sum_{j \in J} x_{ij} = D_i \quad \forall i \in I$$

- Customer can only be served from facility that is opened ($M = \sum_{i \in I} D_i = 95$)
  - (For facility 1, $j = 1$) $x_{11} + x_{21} + x_{31} + x_{41} + x_{51} \leq 95y_1$
  - (For facility 2, $j = 2$) $x_{12} + x_{22} + x_{32} + x_{42} + x_{52} \leq 95y_2$
  - (For facility 3, $j = 3$) $x_{13} + x_{23} + x_{33} + x_{43} + x_{53} \leq 95y_3$
  - (For facility 4, $j = 4$) $x_{14} + x_{24} + x_{34} + x_{44} + x_{54} \leq 95y_4$

$$\sum_{i \in I} x_{ij} \leq M y_j \quad \forall j \in J$$

Complete Model (Answer to Exercise 3a)

$Min\ 175y_1 + 410y_2 + 200y_3 + 160y_4 + 5x_{11} + 3x_{12} + 5.5x_{13} + 2x_{14} + 4.5x_{21} + 4x_{22} + 9x_{23} + 10x_{24} + 8x_{31} + 6x_{32} + 3x_{33} + 5x_{34} + 2.5x_{41} + 6x_{42} + 5x_{43} + 7x_{44} + 5x_{51} + 7x_{52} + 4x_{53} + 6x_{54}$

$$\sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij} + \sum_{j\in J} F_j y_j$$

s.t.

$x_{11} + x_{12} + x_{13} + x_{14} = 15$

$x_{21} + x_{22} + x_{23} + x_{24} = 22$

$x_{31} + x_{32} + x_{33} + x_{34} = 11$

$x_{41} + x_{42} + x_{43} + x_{44} = 25$

$x_{51} + x_{52} + x_{53} + x_{54} = 22$

$$\sum_{j\in J} x_{ij} = D_i \quad \forall i \in I$$

$x_{11} + x_{21} + x_{31} + x_{41} + x_{51} \le 95y_1$

$x_{12} + x_{22} + x_{32} + x_{42} + x_{52} \le 95y_2$

$x_{13} + x_{23} + x_{33} + x_{43} + x_{53} \le 95y_3$

$x_{14} + x_{24} + x_{34} + x_{44} + x_{54} \le 95y_4$

$$\sum_{i\in I} x_{ij} \le My_j \quad \forall j \in J$$

$y_j = 0\ or\ 1\ \forall j \in J$ and $x_{ij} \ge 0, \forall\ i \in I, \forall\ j \in J$

$y_j = 0\ or\ 1\ \ \forall j \in J$

$x_{ij} \ge 0\ \ \forall\ i \in I, \forall\ j \in J$

Information on Exercises 3b & 3c

- Required data for <u>capacitated</u> versus uncapacitated?

Location	Fixed cost	Capacity	City 1	City 2	City 3	City 4	City 5
1	175	30	5	4.5	8	2.5	5
2	410	50	3	4	6	6	7
3	200	40	5.5	9	3	5	4
4	160	30	2	10	5	7	6
Demand			15	22	11	25	22

- The table now shows:
  - <u>K_j: Capacity of facility at location *j*</u>
  - F_j: Fixed costs for opening facility at location *j*
  - c_{ij}: The distribution cost per unit demand for supplying customer *i* from the facility at *j*
  - D_i: Demand of customer i

Step 1: Definition of variables

- Step 1a: What are the variables?
  - y_j : Whether or not to open a warehouse at location *j*
  - x_{ij} : Volume of demand of customer i served from location *j*

- Step 1b: Indicate the valid range of all variables
  - $y_j \in \{0,1\}\ \forall j$ (binary: y_j values are 0 or 1 for all j)
  - $x_{ij} \geq 0, \forall\ i, j$ (x_{ij} non-negative for all i and j)

Answer Exercise 3c

- Satisfy all demand from all customers
  - (For customer 1, $i = 1$) $x_{11} + x_{12} + x_{13} + x_{14} = 15$
  - (For customer 2, $i = 2$) $x_{21} + x_{22} + x_{23} + x_{24} = 22$
  - (For customer 3, $i = 3$) $x_{31} + x_{32} + x_{33} + x_{34} = 11$
  - (For customer 4, $i = 4$) $x_{41} + x_{42} + x_{43} + x_{44} = 25$
  - (For customer 5, $i = 5$) $x_{51} + x_{52} + x_{53} + x_{54} = 22$

$$\sum_{j \in J} x_{ij} = D_i \quad \forall i \in I$$

- Two-purpose constraints:
  - The quantity of demand of customers served from location j must meet capacity limits
  - If a facility is not opened, it cannot serve any customers.
  - (For facility 1, $j = 1$) $x_{11} + x_{21} + x_{31} + x_{41} + x_{51} \leq 30y_1$
  - (For facility 2, $j = 2$) $x_{12} + x_{22} + x_{32} + x_{42} + x_{52} \leq 50y_2$
  - (For facility 3, $j = 3$) $x_{13} + x_{23} + x_{33} + x_{43} + x_{53} \leq 40y_3$
  - (For facility 4, $j = 4$) $x_{14} + x_{24} + x_{34} + x_{44} + x_{54} \leq 30y_4$

$$\sum_{i \in I} x_{ij} \leq K_j y_j \quad \forall j \in J$$

Test Your Skills

Information

- ErikNok seeks a good location for its distribution center:
  - Supply mobile phones once a week to stores in Amsterdam, Utrecht, Maastricht and Assen
  - Expected return flow from a recycling station in Apeldoorn.

City	Number of trucks	X-coordinate	Y-coordinate
Amsterdam	10	-10	10
Utrecht	7	0	0
Maastricht	2	15	-75
Assen	2	30	20
Apeldoorn	**?**	10	10

- Determine the number of trucks with recycled phones that are to be returned from Apeldoorn to make Amersfoort the best distribution center location?
  - Assume the x- and y-coordinates of Amersfoort are (5, 5).

Answer

- Consider the coordinates of Amersfoort (C_x =5, C_y = 5)
- No. of trucks to the cities is fixed, except the number of trucks from Apeldoorn
  - z = the number of trucks from Apeldoorn

City	Number of trucks	X-coordinate	Y-coordinate
Amsterdam	10	-10	10
Utrecht	7	0	0
Maastricht	2	15	-75
Assen	2	30	20
Apeldoorn	z	10	10

- x-coordinate
  - $C_x = \frac{\sum_i d_{ix} W_i}{\sum_i W_i} = 5 = \frac{(10*(-10))+(7*0)+(2*15)+(2*30)+(z*10)}{21+z}$

Answer

- x-coordinate
  - $5 = \frac{(10\times(-10))+(7\times0)+(2\times15)+(2\times30)+(z\times10)}{21+z}$
  - $-100 + 30 + 60 + 10z = 105 + 5z$
  - $-10 + 10z = 105 + 5z$
  - $5z = 115$
  - $z = 23$
- 23 trucks from Apeldoorn would render Amersfoort (5, 5) the best location.
- Would work with the y-coordinate as well:
  - $5 = \frac{(10\times10)+(7\times0)+(2\times(-75))+(2\times20)+(z\times10)}{21+z}$
  - $100 - 150 + 40 + 10z = 105 + 5z$
  - $5z = 115$
  - $z = 23$

Exercise 4

Information on Exercise 4

- Consider the following network:
  - Single commodity, uncapacitated arcs
  - Nodes 1, 2, 3, 4, 5
    - Demand nodes: 4 and 5
    - Supply nodes: 1, 2, and 3
  - Fixed cost of opening an arc = 20

Travel Cost	1	2	3	4	5
1	0	3	11	5	8
2	3	0	8	3	6
3	11	8	0	6	6
4	5	3	6	0	7
5	8	6	6	7	0

Node (i)	Demand Quantity (D_i)	Supply Quantity (S_i)
1	-	2
2	-	5
3	-	3
4	4	-
5	6	-

Answer to Exercise 4a: Variables and Parameters

- Decision Variables
  - $y_{ij} = \begin{cases} 1, \text{if arc } (i,j) \text{ is opened} \\ 0, \text{otherwise} \end{cases}$
  - x_{ij}: Quantity transported on arc (i,j)
    - No commodity index k, since there is only one commodity
- Parameters
  - $D = \{4, 5\}$ (Set of demand nodes)
  - $S = \{1, 2, 3\}$ (Set of supply nodes)
  - $N = \{1, 2, 3, 4, 5\}$ (Set of all nodes)

Uncapacitated Network Design Problem

$$min \sum_{i \in N} \sum_{j \in N} f_{ij} y_{ij} + \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ij}$$

s.t.

$$\sum_{j \in N} x_{ij} - \sum_{j \in N} x_{ji} = \begin{cases} S_i & if\ i \in S \\ -D_i & if\ i \in D \end{cases} \qquad \forall i \in N$$

$$x_{ij} \leq M y_{ij}, \qquad \forall i \in N, \forall j \in N$$

$$y_{ij} \in \{0,1\}, \qquad \forall i \in N, \forall j \in N$$

$$x_{ij} \geq 0, \qquad \forall i \in N, \forall j \in N$$

Answer to Exercise 4a: Objective

- Objective

$$\min \sum_{i \in N} \sum_{j \in N} f_{ij} y_{ij} + \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ij}$$

- Write term-by-term:

$$\begin{aligned}\min \quad & 20y_{12} + 20y_{13} + 20y_{14} + 20y_{15} + 20y_{21} + 20y_{23} + 20y_{24} + 20y_{25} + 20y_{31} + 20y_{32} \\ & + 20y_{34} + 20y_{35} + 20y_{41} + 20y_{42} + 20y_{43} + 20y_{45} + 20y_{51} + 20y_{52} + 20y_{53} + 20y_{54} \\ & + 3x_{12} + 11x_{13} + 5x_{14} + 8x_{15} + 3x_{21} + 8x_{23} + 3x_{24} + 6x_{25} + 11x_{31} + 8x_{32} + 6x_{34} + 6x_{35} \\ & + 5x_{41} + 3x_{42} + 6x_{43} + 7x_{45} + 8x_{51} + 6x_{52} + 6x_{53} + 7x_{54}\end{aligned}$$

Answer to Exercise 4a: Constraints

- Balance Constraints

$$\sum_{j \in N} x_{ij} - \sum_{j \in N} x_{ji} = \begin{cases} S_i & if\ i \in S \\ -D_i & if\ i \in D \end{cases} \qquad \forall i \in N$$

- Write for all $i \in N$ (where $D = \{4, 5\}$ and $S = \{1, 2, 3\}$):
  - (For node 1, $i = 1$) $x_{12} + x_{13} + x_{14} + x_{15} - (x_{21} + x_{31} + x_{41} + x_{51}) = 2$
  - (For node 2, $i = 2$) $x_{21} + x_{23} + x_{24} + x_{25} - (x_{12} + x_{32} + x_{42} + x_{52}) = 5$
  - (For node 3, $i = 3$) $x_{31} + x_{32} + x_{34} + x_{35} - (x_{13} + x_{23} + x_{43} + x_{53}) = 3$
  - (For node 4, $i = 4$) $x_{41} + x_{42} + x_{43} + x_{45} - (x_{14} + x_{24} + x_{34} + x_{54}) = -4$
  - (For node 5, $i = 5$) $x_{51} + x_{52} + x_{53} + x_{54} - (x_{15} + x_{25} + x_{35} + x_{45}) = -6$

Answer to Exercise 4a: Constraints

- What should the big-M value be for the arc capacity constraints? (The arcs are uncapacitated in this problem!)
  - $x_{ij} \leq My_{ij} \qquad \forall (i,j) \in A$

Node (i)	Demand Quantity (D_i)	Supply Quantity (S_i)
1	-	2
2	-	5
3	-	3
4	4	-
5	6	-

  - M must be at least 10, since the total demand quantity (and the total supply quantity) is 10.

Answer to Exercise 4a: Constraints

- If an arc is not opened, it cannot transport anything.

$$x_{ij} \leq M y_{ij}, \qquad \forall i \in N, \forall j \in N$$

  - Write for all arcs (i,j)
    - Arc (1,2), $(i,j) = (1,2)$: $x_{12} \leq 10y_{12}$
    - Arc (1,3), $(i,j) = (1,3)$: $x_{13} \leq 10y_{13}$
    - Arc (1,4), $(i,j) = (1,4)$: $x_{14} \leq 10y_{14}$
    - Arc (1,5), $(i,j) = (1,5)$: $x_{15} \leq 10y_{15}$
    - Arc (2,1), $(i,j) = (2,1)$: $x_{21} \leq 10y_{21}$
    - Arc (2,3), $(i,j) = (2,3)$: $x_{23} \leq 10y_{23}$
    - Arc (2,4), $(i,j) = (2,4)$: $x_{24} \leq 10y_{24}$
    - Arc (2,5), $(i,j) = (2,5)$: $x_{25} \leq 10y_{25}$
    - Arc (3,1), $(i,j) = (3,1)$: $x_{31} \leq 10y_{31}$
    - Arc (3,2), $(i,j) = (3,2)$: $x_{32} \leq 10y_{32}$
    - Arc (3,4), $(i,j) = (3,4)$: $x_{34} \leq 10y_{34}$
    - Arc (3,5), $(i,j) = (3,5)$: $x_{35} \leq 10y_{35}$
    - Arc (4,1), $(i,j) = (4,1)$: $x_{41} \leq 10y_{41}$
    - Arc (4,2), $(i,j) = (4,2)$: $x_{42} \leq 10y_{42}$
    - Arc (4,3), $(i,j) = (4,3)$: $x_{43} \leq 10y_{43}$
    - Arc (4,5), $(i,j) = (4,5)$: $x_{45} \leq 10y_{45}$
    - Arc (5,1), $(i,j) = (5,1)$: $x_{51} \leq 10y_{51}$
    - Arc (5,2), $(i,j) = (5,2)$: $x_{52} \leq 10y_{52}$
    - Arc (5,3), $(i,j) = (5,3)$: $x_{53} \leq 10y_{53}$
    - Arc (5,4), $(i,j) = (5,4)$: $x_{54} \leq 10y_{54}$

Answer to Exercise 4a

- Formulation

$$
\begin{aligned}
\min \quad & 20y_{12} + 20y_{13} + 20y_{14} + 20y_{15} + 20y_{21} + 20y_{23} + 20y_{24} + 20y_{25} + 20y_{31} + 20y_{32} \\
& + 20y_{34} + 20y_{35} + 20y_{41} + 20y_{42} + 20y_{43} + 20y_{45} + 20y_{51} + 20y_{52} + 20y_{53} + 20y_{54} \\
& + 3x_{12} + 11x_{13} + 5x_{14} + 8x_{15} + 3x_{21} + 8x_{23} + 3x_{24} + 6x_{25} + 11x_{31} + 8x_{32} + 6x_{34} + 6x_{35} \\
& + 5x_{41} + 3x_{42} + 6x_{43} + 7x_{45} + 8x_{51} + 6x_{52} + 6x_{53} + 7x_{54} \\
s.t. \quad & x_{12} + x_{13} + x_{14} + x_{15} - x_{21} - x_{31} - x_{41} - x_{51} = 2 \\
& x_{21} + x_{23} + x_{24} + x_{25} - x_{12} - x_{32} - x_{42} - x_{52} = 3 \\
& x_{31} + x_{32} + x_{34} + x_{35} - x_{13} - x_{23} - x_{43} - x_{53} = 5 \\
& x_{41} + x_{42} + x_{43} + x_{45} - x_{14} - x_{24} - x_{34} - x_{54} = -4 \\
& x_{51} + x_{52} + x_{53} + x_{54} - x_{15} - x_{25} - x_{35} - x_{45} = -6 \\
& x_{12} \leq 10y_{12}, \quad x_{13} \leq 10y_{13}, \quad x_{14} \leq 10y_{14}, \quad x_{15} \leq 10y_{15} \\
& x_{21} \leq 10y_{21}, \quad x_{23} \leq 10y_{23}, \quad x_{24} \leq 10y_{24}, \quad x_{25} \leq 10y_{25} \\
& x_{31} \leq 10y_{31}, \quad x_{32} \leq 10y_{32}, \quad x_{34} \leq 10y_{34}, \quad x_{35} \leq 10y_{35} \\
& x_{41} \leq 10y_{41}, \quad x_{42} \leq 10y_{42}, \quad x_{43} \leq 10y_{43}, \quad x_{45} \leq 10y_{45} \\
& x_{51} \leq 10y_{51}, \quad x_{52} \leq 10y_{52}, \quad x_{53} \leq 10y_{53}, \quad x_{54} \leq 10y_{54} \\
& x_{12}, x_{13}, x_{14}, x_{15}, x_{21}, x_{23}, x_{24}, x_{25}, x_{31}, x_{32}, x_{34}, x_{35}, x_{41}, x_{42}, x_{43}, x_{45}, x_{51}, x_{52}, x_{53}, x_{54} \geq 0 \\
& y_{12}, y_{13}, y_{14}, y_{15}, y_{21}, y_{23}, y_{24}, y_{25}, y_{31}, y_{32}, y_{34}, y_{35}, y_{41}, y_{42}, y_{43}, y_{45}, y_{51}, y_{52}, y_{53}, y_{54} \in \{0, 1\}
\end{aligned}
$$

Answer to Exercise 4b

- First, enter the data:
  - $M = \sum_{i \in D} D_i = 10$

(Fixed) Arc Opening Costs					
	1	2	3	4	5
1	-	20	20	20	20
2	20	-	20	20	20
3	20	20	-	20	20
4	20	20	20	-	20
5	20	20	20	20	-

(Variable) Transportation Costs					
	1	2	3	4	5
1	-	3	11	5	8
2	3	-	8	3	6
3	11	8	-	6	6
4	5	3	6	-	7
5	8	6	6	7	-

Arc Capacities					
	1	2	3	4	5
1	-	10	10	10	10
2	10	-	10	10	10
3	10	10	-	10	10
4	10	10	10	-	10
5	10	10	10	10	-

	Demand	Supply
1	-	2
2	-	5
3	-	3
4	4	-
5	6	-

- Set up the decision variables:

y_{ij}					
	1	2	3	4	5
1	0	0	0	0	1
2	0	0	0	1	1
3	0	0	0	0	1
4	0	0	0	0	0
5	0	0	0	0	0

x_{ij}					
	1	2	3	4	5
1	0	0	0	0	2
2	0	0	0	4	1
3	0	0	0	0	3
4	0	0	0	0	0
5	0	0	0	0	0

Answer to Exercise 4b

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- Set up the objective:

Objective	=SUMPRODUCT(B3:F7, B11:F15) + SUMPRODUCT(I3:M7, I11:M15)

- Set up the balance constraints:

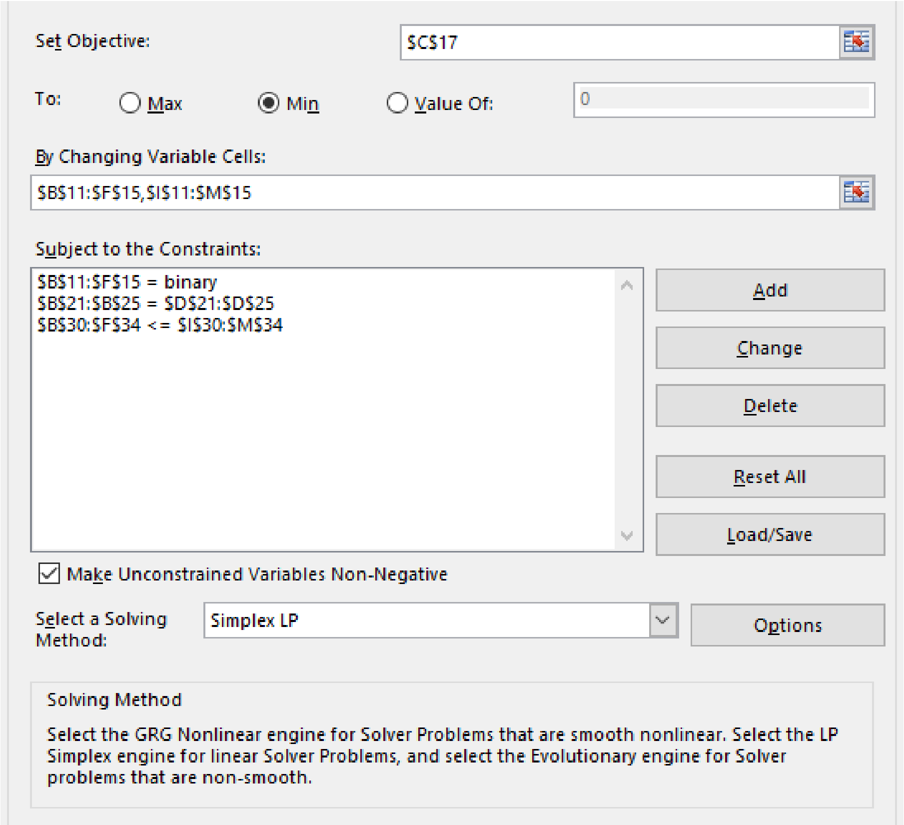
Balance Constraints			
Node	LHS	Sign	RHS
1	=SUM(I11:M11) - SUM(I11:I15)	=	=X3
2	=SUM(I12:M12) - SUM(J11:J15)	=	=X4
3	=SUM(I13:M13) - SUM(K11:K15)	=	=X5
4	=SUM(I14:M14) - SUM(L11:L15)	=	=-W6
5	=SUM(I15:M15) - SUM(M11:M15)	=	=-W7

- Set up the capacity constraints:

Capacity Constraints												
	LHS					Sign	RHS					
	1	2	3	4	5			1	2	3	4	5
1	0	=J11	=K11	=L11	=M11	<=	1	0	=Q3 * C11	=R3 * D11	=S3 * E11	=T3 * F11
2	=I12	0	=K12	=L12	=M12	<=	2	=P4 * B12	0	=R4 * D12	=S4 * E12	=T4 * F12
3	=I13	=J13	0	=L13	=M13	<=	3	=P5 * B13	=Q5 * C13	0	=S5 * E13	=T5 * F13
4	=I14	=J14	=K14	0	=M14	<=	4	=P6 * B14	=Q6 * C14	=R6 * D14	0	=T6 * F14
5	=I15	=J15	=K15	=L15	0	<=	5	=P7 * B15	=Q7 * C15	=R7 * D15	=S7 * E15	0

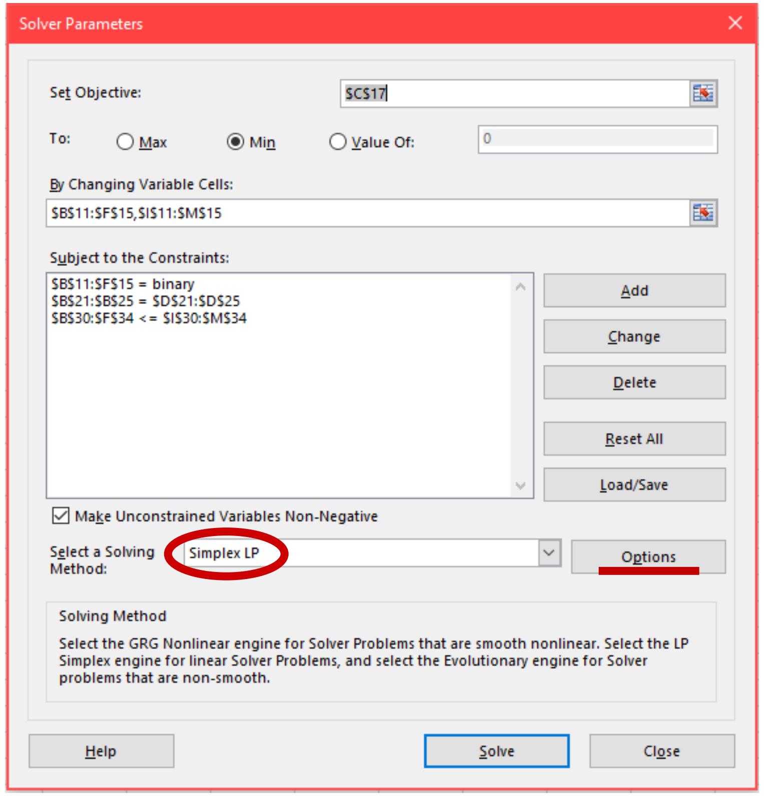
Answer to Exercise 4b

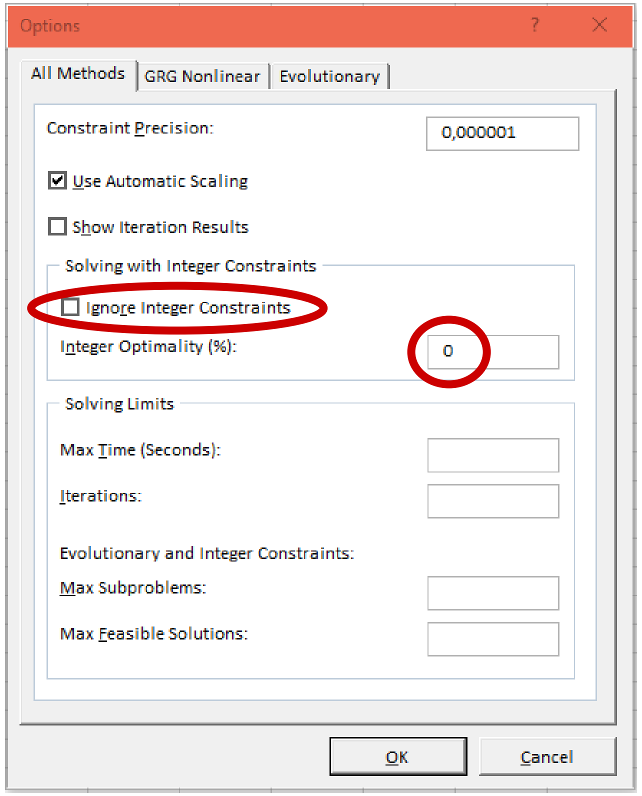
- Build the LP in Excel Solver:



- See Excel file "Tutorial 2 Calculations.xlsx" for more details.

Excel Solver Reminder

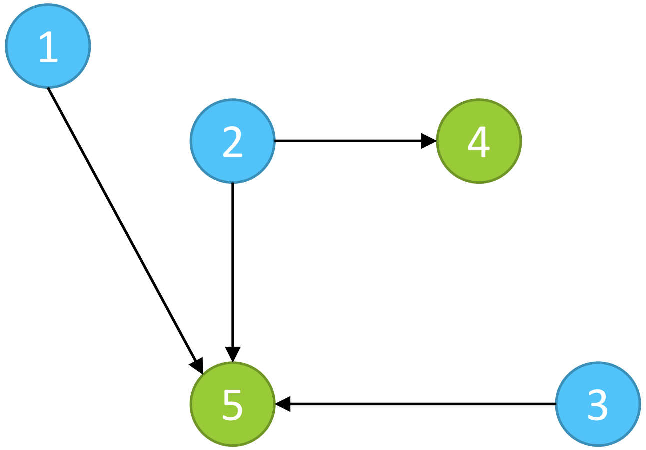




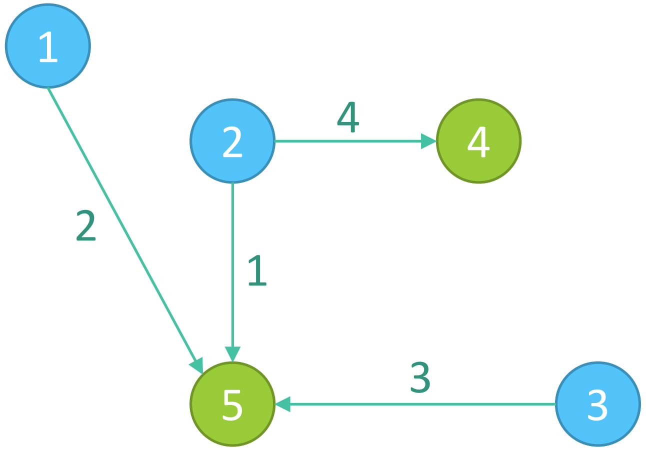
- Use “Simplex LP” solving method first. You can try the other methods to check/compare.
- In “Options” dialog box:
  - If you’re using integer variables,
    - Make sure the “Ignore Integer Constraint” box is unchecked.
    - Make sure “Integer Optimality (%)” is set to 0.
      - See the Excel Solver video for the potential errors you can encounter here.

Answer to Exercise 4c

- The open arcs in the optimal solution



- The transported quantities in the optimal solution

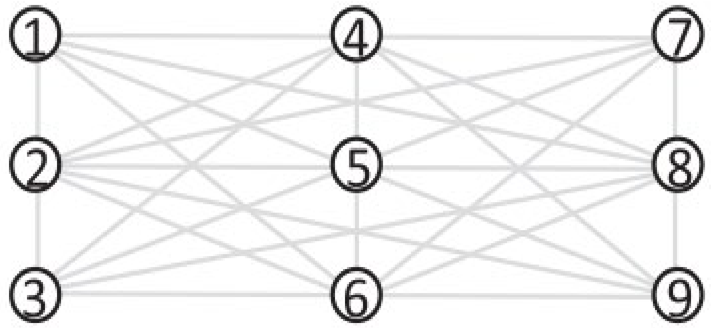


- Total cost in the optimal solution = 132
  - Fixed cost = 4 x 20 = 80
  - Transportation cost = 2 x 8 + 4 x 3 + 1 x 6 + 3 x 6 = 52

Exercise 5

Information on Exercise 5

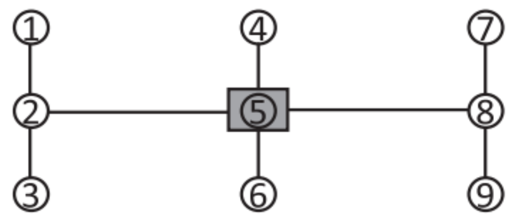
- Consider the given network



  - Each demand node has a demand of 1 unit, to be met from any of the facilities.
  - Facility opening cost, F_j, is 5 for each candidate location.
  - Link opening cost, f_{ij}, is equal to d_{ij}.
    - d_{ij} is the Euclidean distance.
  - Routing (transportation) cost per 1 unit transported, c_{ij}, is 0.
- Calculate the total cost of 5 given solutions, and decide which one is best.

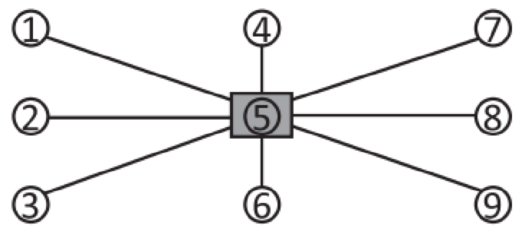
Answer to Exercise 5

- Solution S1
  - Facility opening cost = 5
  - Link opening cost = $1 \times 6 + 2 \times 2 = 10$
  - Routing (transportation) cost = 0
  - TOTAL = 15



Solution S1

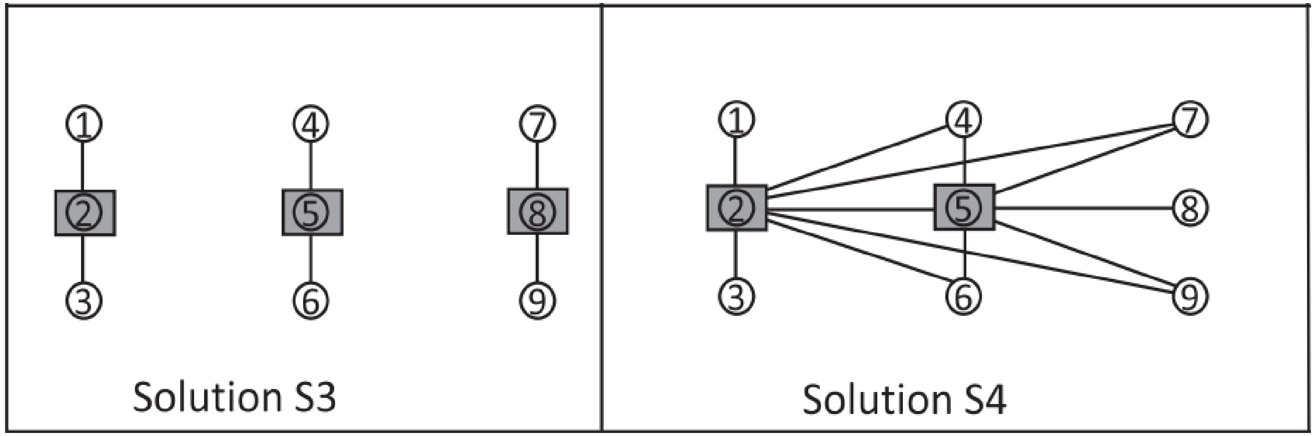
- Solution S2
  - Facility opening cost = 5
  - Link opening cost = $4 \times \sqrt{2^2 + 1^2} + 2 \times 2 + 2 \times 1 = 14.94$
  - Routing cost = 0
  - TOTAL = 19.94



Solution S2

Answer to Exercise 5

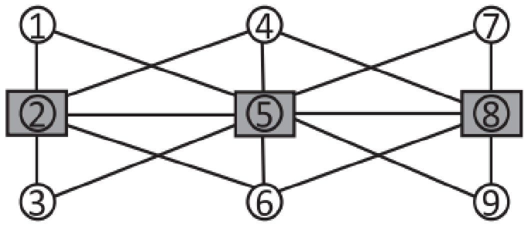
- Solution S3
  - Facility opening cost = $3 \times 5 = 15$
  - Link opening cost = $6 \times 1 = 6$
  - Routing cost = 0
  - TOTAL = 21



- Solution S4
  - Facility opening cost = $2 \times 5 = 10$
  - Link opening cost = $4 \times \sqrt{2^2 + 1^2} + 2 \times \sqrt{4^2 + 1^2} + 2 \times 2 + 4 \times 1 = 25.19$
  - Routing cost = 0
  - TOTAL = 35.19

Answer to Exercise 5

- Solution S5
  - Facility opening cost = $3 \times 5 = 15$
  - Link opening cost = $8 \times \sqrt{2^2 + 1^2} + 2 \times 2 + 6 \times 1 = 27.89$
  - Routing cost = 0
  - TOTAL = 42.89



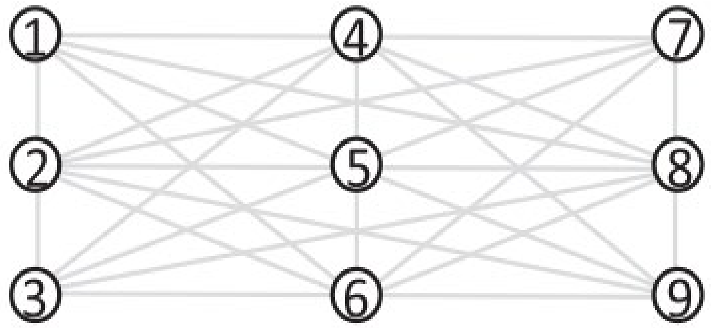
Solution S5

- Solution S1 has the lowest cost, and therefore is the most preferable.

Exercise 6

Information on Exercise 6

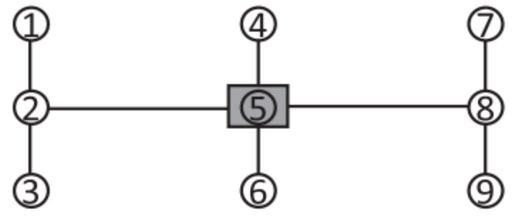
- Consider the given network



  - Each demand node has a demand of 1 unit, to be met from any of the facilities.
  - Facility opening cost, F_j, is 5 for each candidate location.
  - Link opening cost, f_{ij}, is 0.
  - Routing (transportation) cost per 1 unit transported, c_{ij}, is equal to d_{ij}.
    - d_{ij} is the Euclidean distance.
- Calculate the total cost of 5 given solutions, and decide which one is best.

Answer to Exercise 6

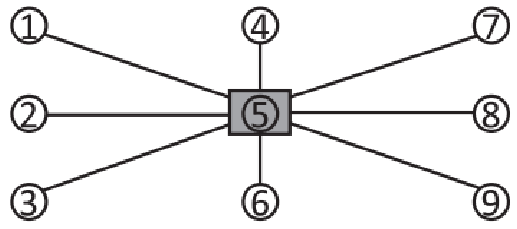
- Solution S1
  - Facility opening cost = 5
  - Link opening cost = 0
  - Routing (transportation) cost = 18
    - Node 1 (Route 5→2→1) = 2 + 1 = 3
    - Node 2 (Route 5→2) = 2
    - Node 3 (Route 5→2→3) = 2 + 1 = 3
    - Node 4 (Route 5→4) = 1
    - Node 5 = 0
    - Node 6 (Route 5→6) = 1
    - Node 7 (Route 5→8→7) = 2 + 1 = 3
    - Node 8 (Route 5→8) = 2
    - Node 9 (Route 5→8→9) = 2 + 1 = 3
  - TOTAL = 23



Solution S1

Answer to Exercise 6

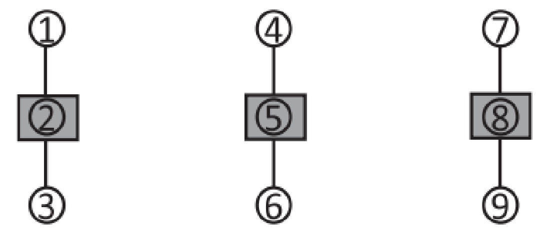
- Solution S2
  - Facility opening cost = 5
  - Link opening cost = 0
  - Routing (transportation) cost = 14.96
    - Node 1 (Route 5→1) = 2.24
    - Node 2 (Route 5→2) = 2
    - Node 3 (Route 5→3) = 2.24
    - Node 4 (Route 5→4) = 1
    - Node 5 = 0
    - Node 6 (Route 5→6) = 1
    - Node 7 (Route 5→7) = 2.24
    - Node 8 (Route 5→8) = 2
    - Node 9 (Route 5→9) = 2.24
  - TOTAL = 19.96



Solution S2

Answer to Exercise 6

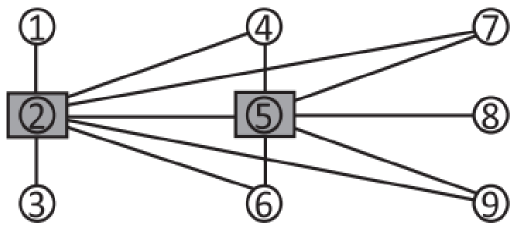
- Solution S3
  - Facility opening cost = $3 \times 5 = 15$
  - Link opening cost = 0
  - Routing (transportation) cost = 6
    - Node 1 (Route 2→1) = 1
    - Node 2 = 0
    - Node 3 (Route 2→3) = 1
    - Node 4 (Route 5→4) = 1
    - Node 5 = 0
    - Node 6 (Route 5→6) = 1
    - Node 7 (Route 8→7) = 1
    - Node 8 = 0
    - Node 9 (Route 8→9) = 1
  - TOTAL = 21



Solution S3

Answer to Exercise 6

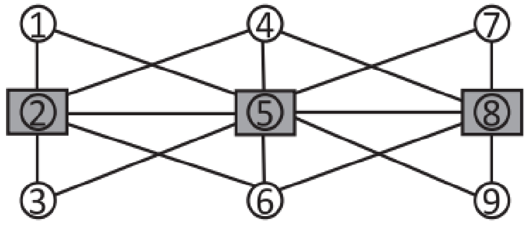
- Solution S4
  - Facility opening cost = $2 \times 5 = 10$
  - Link opening cost = 0
  - Routing (transportation) cost = 10.48
    - Node 1 (Route 2→1) = 1
    - Node 2 = 0
    - Node 3 (Route 2→3) = 1
    - Node 4 (Route 5→4) = 1
    - Node 5 = 0
    - Node 6 (Route 5→6) = 1
    - Node 7 (Route 5→7) = 2.24
    - Node 8 (Route 5→8) = 2
    - Node 9 (Route 5→9) = 2.24
  - TOTAL = 20.48



Solution S4

Answer to Exercise 6

- Solution S5
  - Facility opening cost = $3 \times 5 = 15$
  - Link opening cost = 0
  - Routing (transportation) cost = 6
    - Node 1 (Route 2→1) = 1
    - Node 2 = 0
    - Node 3 (Route 2→3) = 1
    - Node 4 (Route 5→4) = 1
    - Node 5 = 0
    - Node 6 (Route 5→6) = 1
    - Node 7 (Route 8→7) = 1
    - Node 8 = 0
    - Node 9 (Route 8→9) = 1
  - TOTAL = 21



Solution S5

Answer to Exercise 6

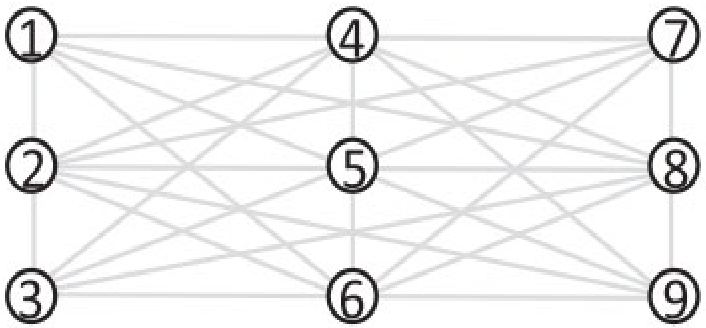
- S1 = 23
- S2 = 19.96
- S3 = 21
- S4 = 20.48
- S5 = 21

- Solution S2 has the lowest cost, and therefore is the most preferable.

Exercise 7

Information on Exercise 7

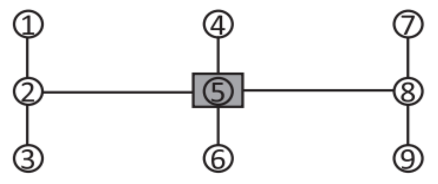
- Consider the given network



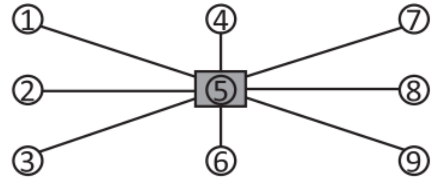
  - Each demand node has a demand of 1 unit, to be met from any of the facilities.
  - Facility opening cost, F_j, is 5 for each candidate location.
  - Link opening cost, f_{ij}, is 3 for each link.
  - Routing (transportation) cost per 1 unit transported, c_{ij}, is equal to d_{ij}.
- Consider solutions S1, S2, S3, S4, and S5
  - Are there redundant open links? If so, how would you improve the solution (and how much would the proposed improvement reduce total cost)?

Answer to Exercise 7

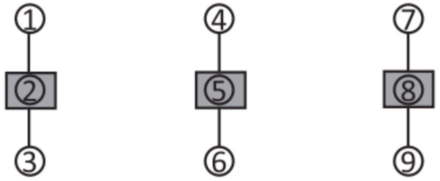
- Solution S1, S2, and S3
  - There are no redundant links, all links are used for transportation!



Solution S1



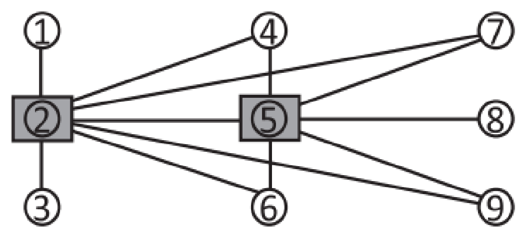
Solution S2



Solution S3

Answer to Exercise 7

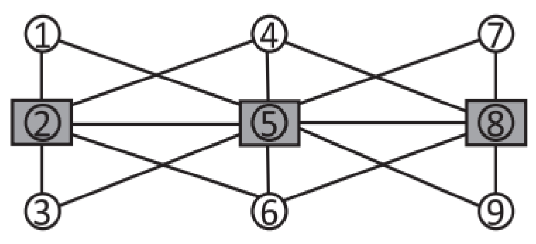
- Solution S4
  - Routes:
    - Node 1 (Route 2→1)
    - Node 2
    - Node 3 (Route 2→3)
    - Node 4 (Route 5→4)
    - Node 5
    - Node 6 (Route 5→6)
    - Node 7 (Route 5→7)
    - Node 8 (Route 5→8)
    - Node 9 (Route 5→9)
  - Links (2,4), (2,5), (2,6), (2,7), (2,9) are redundant!
  - Closing them would save $5 \times 3 = 15$, in link opening costs.



Solution S4

Answer to Exercise 7

- Solution S5
  - Routes:
    - Node 1 (Route 2→1)
    - Node 2
    - Node 3 (Route 2→3)
    - Node 4 (Route 5→4)
    - Node 5
    - Node 6 (Route 5→6)
    - Node 7 (Route 8→7)
    - Node 8
    - Node 9 (Route 8→9)
  - Links (2,4), (2,5), (2,6), (1,5), (3,5), (4,8), (6,8), (5,7), (5,8), (5,9) are redundant!
  - Closing them would save $10 \times 3 = 30$, in link opening costs.

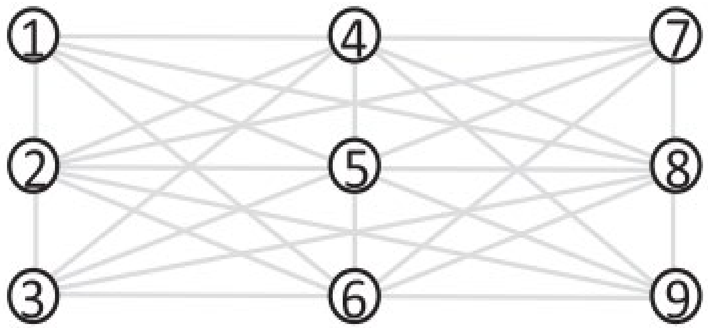


Solution S5

Exercise 8

Information on Exercise 8

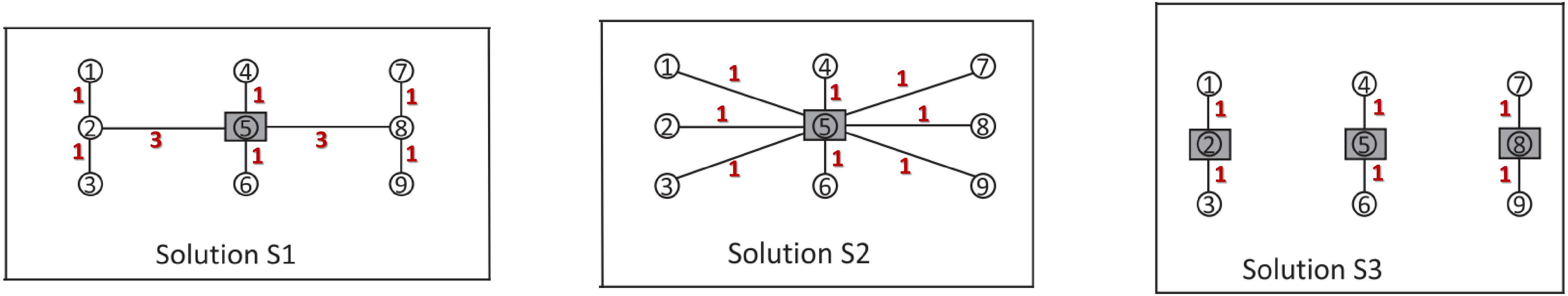
- Consider the given network



  - Each demand node has a demand of 1 unit, to be met from any of the facilities.
- Consider solutions S1, S2, S3, S4, S5, and S6.
  - Each link has a capacity of 1 unit.
  - Are all solutions S1, … , S6 feasible with respect to link capacities?

Answer to Exercise 8

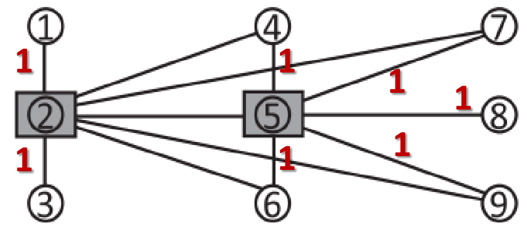
- The number of units transported on each link are given in red.



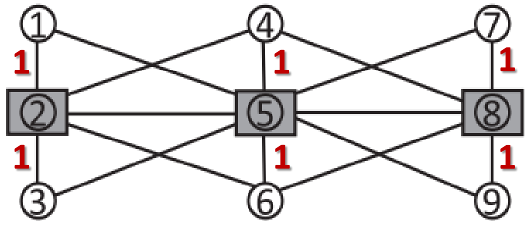
  - Solutions S2 and S3 are feasible, since no link exceeds its capacity.
  - Solution S1 is infeasible, since links (5,2) and (5,8) exceed their capacity.

Answer to Exercise 8

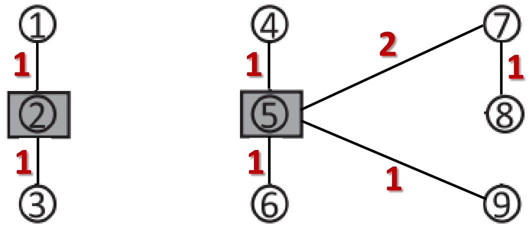
- The number of units transported on each link are given in red.
  - Some links in solutions S4 and S5 has no red number on them, because these links are not being used for transportation (they are redundant).



Solution S4



Solution S5



Solution S6

  - Solutions S4 and S5 are feasible, since no link exceeds its capacity.
  - Solution S6 is infeasible, since link capacity of link (5,7) is exceeded.