**35E00750 Logistics Systems and Analytics**
**2023-2024**

# Session 9. Facility Location Problem

## Exercise 1

You are asked to provide your supply chain director with input to determine the best possible warehouse location. After extensive research, you came up with four candidate cities and you collected data on several location factors for each of them. From discussions with the supply chain director, you have deduced the importance of each factor and given weights accordingly.

| Location factor | | Weight ($w_i$) | Score Amsterdam | Score The Hague | Score Utrecht | Score Rotterdam |
|---|---|---|---|---|---|---|
| 1 | Warehouse utilization | 20 | 8 | 6 | 4 | 7 |
| 2 | Average time per trip from warehouse to retailers | 15 | 7 | 5 | 7 | 5 |
| 3 | Employee preferences | 15 | 1 | 7 | 8 | 3 |
| 4 | Accessibility to major highways | 10 | 7 | 4 | 9 | 5 |
| 5 | Land costs | 10 | 2 | 3 | 1 | 4 |
| 6 | Quality of life | 15 | 5 | 6 | 9 | 5 |
| 7 | Taxes | 15 | 3 | 5 | 5 | 4 |

a. Which is the best site? Assume that a higher score is more desirable than a lower one. Support your answer with calculations.

b. For what range of values for the weight "warehouse utilization" (currently $w_1 = 20$) does the site given as the answer to question (a) remain the best site? Assume that all other weights and all scores keep their current values. Support your answer with calculations.

## Exercise 2

Matrix Manufacturing Inc. is considering where to locate its warehouse in order to service its four stores located in four Ohio cities: Cleveland, Columbus, Cincinnati, and Dayton. Two possible sites for the warehouse are being considered. One is in Mansfield, Ohio (coordinates: $x = 11, y = 14$); and the other is in Springfield (coordinates: $x = 6, y = 6.5$), Ohio. The following information is given:

| **Store Location** | **Coordinates (x, y)** | **Load volumes** |
|---|---|---|
| Cleveland | (11, 22) | 15 |
| Columbus | (10, 7) | 19 |
| Cincinnati | (4, 1) | 12 |
| Dayton | (3, 6) | 4 |
| **Total** | | 41 |

Using the center of gravity method, which of the two locations considered (Mansfield or Springfield) is the most suitable location?

## Exercise 3

A grocery company considers 4 candidate locations for opening one or more distribution centers to serve its customers in five nearby cities. The table below shows the fixed costs for opening a new distribution center at each of the locations under consideration. It also shows, for each candidate location, the (variable) distribution costs associated with supplying a unit volume of customer demand of a city from that location. In the bottom row of the table, the demand volumes of the cities are given.

| **Location** | **Fixed cost** | **City 1** | **City 2** | **City 3** | **City 4** | **City 5** |
|---|---|---|---|---|---|---|
| 1 | 175 | 5 | 4.5 | 8 | 2.5 | 5 |
| 2 | 410 | 3 | 4 | 6 | 6 | 7 |
| 3 | 200 | 5.5 | 9 | 3 | 5 | 4 |
| 4 | 160 | 2 | 10 | 5 | 7 | 6 |
| Demand | | 15 | 22 | 11 | 25 | 22 |

a. Formulate this uncapacitated facility location problem as a linear programming model, assuming the retailer's objective is to minimize total cost. Explain your answers briefly.

With the following data, the problem can also be translated into a capacitated facility location problem:

| Location | Fixed cost | Capacity | City 1 | City 2 | City 3 | City 4 | City 5 |
|---|---|---|---|---|---|---|---|
| 1 | 175 | 30 | 5 | 4.5 | 8 | 2.5 | 5 |
| 2 | 410 | 50 | 3 | 4 | 6 | 6 | 7 |
| 3 | 200 | 40 | 5.5 | 9 | 3 | 5 | 4 |
| 4 | 160 | 30 | 2 | 10 | 5 | 7 | 6 |
| Demand | | | 15 | 22 | 11 | 25 | 22 |

b. Explain the implications for the input data and meaning of variables when formulating the problem as a capacitated facility location problem, compared to the uncapacitated variant addressed in question (a).

c. Reformulate the constraints of the linear programming model, to reflect the capacitated facility location problem. Briefly explain the changes.

## Exercise 4

Consider a network with 5 terminals, where terminals 1, 2, and 3 are supply terminals (with supply quantities of 2, 5, and 3, respectively), and terminals 4 and 5 are demand terminals (with demand quantities of 4 and 6). You are asked to solve a network design problem to decide on which arcs to use (open), and what quantities to transport over these arcs. Suppose there is only one commodity.

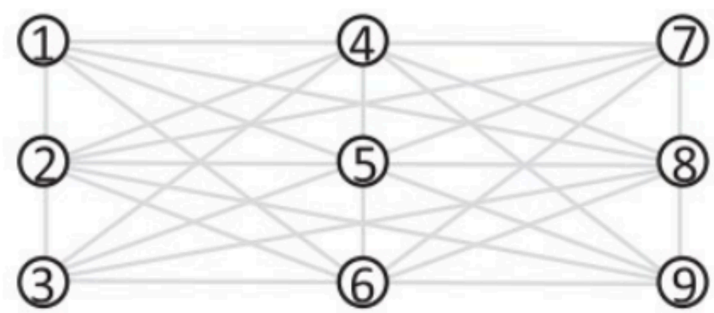
| From \ To | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 0 | 3 | 11 | 5 | 8 |
| 2 | 3 | 0 | 8 | 3 | 6 |
| 3 | 11 | 8 | 0 | 6 | 6 |
| 4 | 5 | 3 | 6 | 0 | 7 |
| 5 | 8 | 6 | 6 | 7 | 0 |

The fixed cost of opening an arc is 20 for all arcs in the network, and the transportation costs are given in the table below. Furthermore, the arcs are uncapacitated, so if open, the arcs can carry as much demand as possible.

a) Formulate this network design problem.
b) Solve this network design problem using Excel.
c) Visualize (draw) the optimal solution you found in Excel, and manually verify the total cost.

## Exercise 5

The figure below depicts a network where each node represents a demand node, as well as a candidate location for a facility.



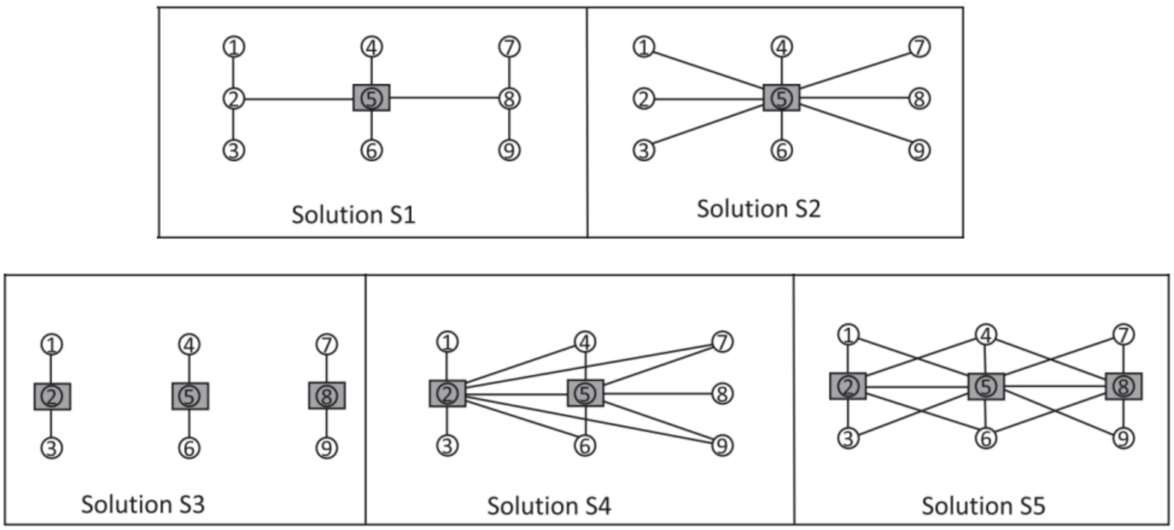

Consider an integrated facility location and network design problem for this network. Let $d_{ij}$ denote the distance between node $i$ and node $j$. The distance of any vertical link (between nodes 1 and 2, for example) is 1, and the length of any horizontal link (between nodes 1 and 4, for example) is 2. All other links have Euclidean distances; for example, $d_{16} = \sqrt{2^2 + 2^2} = 2\sqrt{2} \cong 2.83$. The goal is to minimize the total cost; including facility opening cost, link opening (or setup) cost, and routing cost.

Consider the following details:

- Each demand node has a demand of 1 unit, to be met from any of the facilities.
- Facility opening cost, $F_j$, is 5 for each candidate location.
- Link opening cost, $f_{ij}$, is equal to $d_{ij}$.

- Routing (transportation) cost per 1 unit transported, $c_{ij}$, is 0.

Calculate the total costs of the following 5 solutions and decide which one is best. (Note that nodes drawn as a rectangle represent facilities, and the drawn edges represent links that are opened.




## Exercise 6

Consider the same setting as the previous exercise, with the following differences:

- Link opening cost, $f_{ij}$, is 0.
- Routing (transportation) cost per one unit transported, $c_{ij}$, is equal to $d_{ij}$.

Calculate the total costs of solutions S1, S2, S3, S3, and S5; and decide which one is best.

## Exercise 7

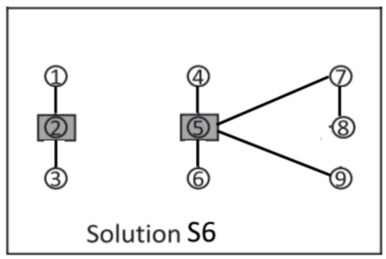
Consider the same setting as the previous exercise, with the following differences:

- Link opening cost, $f_{ij}$, is 3 for each link.
- Routing (transportation) cost per 1 unit transported, $c_{ij}$, is equal to $d_{ij}$.

Consider solutions S1, S2, S3, S3, and S5: Are there redundant open links? If so, how would you improve the solution (and how much would the proposed improvement reduce total cost)?

## Exercise 8

Consider the network and solutions S1, ..., S5 given in Exercise 5. Additionally, consider solution S6 given on the right. Now, suppose that each link has a capacity of 1, which means at most 1 unit can be transported on each link. Which solutions among S1, ..., S6 are feasible?




*Courtesy of dr. P. Buijs, dr. I. Bakir, and dr. J.A. Lopez Alvarez from the Department of Operations, Faculty of Economics and Business, University of Groningen*

**Contact person:**
**Tri Tran**
Assistant Professor of Operations Management
Department of Operations
Faculty of Economics and Business
University of Groningen

For Aalto students: tri.tran@aalto.fi