ELEC-E8412 **Power Electronics**
Exercise No 01 28.09.2023

Problem 1:

A sinusoidal voltage source of $v(t) = 40\cos(2\pi 50t)$ V is applied to a nonlinear load, resulting in a non-sinusoidal current which is expressed in Fourier series as;

$i(t) = 3 + 7\cos(2\pi 50t + 20°) + 4\cos(6\pi 50t + 15°) + 3\cos(8\pi 50t + 25°)$ A.

Determine

a) power absorbed by the load,
b) power factor of the load
c) total harmonic distortion of the load current.

Solution :

a)

$$p(t) = v(t)i(t)$$

$$P = \frac{40}{\sqrt{2}} \cdot \frac{7}{\sqrt{2}} \cos(20) = 131.56\ W$$

b)

$$V_{rms} = \frac{40}{\sqrt{2}} = 28.28\ V$$

$$I_{rms} = \sqrt{(3)^2 + \left(\frac{7}{\sqrt{2}}\right)^2 + \left(\frac{4}{\sqrt{2}}\right)^2 + \left(\frac{3}{\sqrt{2}}\right)^2} = \sqrt{46} = 6.78\ A$$

$$pf = \frac{P}{V_{rms} I_{rms}} = \frac{131.56}{28.28 * 6.78} = 0.686$$

c)

$$THD = \frac{\sqrt{(3)^2 + \left(\frac{4}{\sqrt{2}}\right)^2 + \left(\frac{3}{\sqrt{2}}\right)^2}}{\frac{7}{\sqrt{2}}} = \frac{4.637}{4.95} = 0.936$$

Problem 2:

The voltage and current for a circuit element are;

$v(t) = 3 + 5\cos(2\pi 60t + 15^\circ) + 2\cos(4\pi 60t)$ V

$i(t) = 2 + 7\cos(2\pi 60t + 45^\circ) + 3\cos(6\pi 60t + 25^\circ)$ A.

Determine

a) rms values of voltage and current.
b) power absorbed by the element.
c) total harmonic distortion of the load current.

Solution :

a)

$$V_{rms} = \sqrt{(3)^2 + \left(\frac{5}{\sqrt{2}}\right)^2 + \left(\frac{2}{\sqrt{2}}\right)^2} = 4.85\ V$$

$$I_{rms} = \sqrt{(2)^2 + \left(\frac{7}{\sqrt{2}}\right)^2 + \left(\frac{3}{\sqrt{2}}\right)^2} = 5.74\ A$$

b)

$$P = 6 + \frac{5}{\sqrt{2}}.\frac{7}{\sqrt{2}}\cos\left(\frac{30}{180}\pi\right) = 21.15\ W$$

c)

$$\mathrm{THD} = \sqrt{\frac{I_{\mathrm{rms}}^2 - I_{1,\mathrm{rms}}^2}{I_{1,\mathrm{rms}}^2}}$$

Problem 3:

The voltage across a **10Ω** resistor is $v(t) = 170 \text{ Sin } (377t)$

Determine

a) instantaneous power.
b) average power.
c) peak power.

Solution :

a)

$$i(t) = \frac{v(t)}{R}$$

$$i(t) = \frac{170\, Sin(377t)}{10} = 17\, Sin(377t)$$

$$p(t) = 170\, Sin(377t)\ x\ 17\, Sin(377t)$$

$$p(t) = 2890\, Sin^2(377t)\ W$$

b)

$$P = \frac{1}{T}\int_0^T p(t)\,dt$$

$$\sin^2 t = \frac{1 - Cos\,2t}{2}$$

$$P = \frac{1}{2\pi}\int_0^{2\pi} 2890\, Sin^2(377t)\, dt$$

$$P = \frac{2890}{2\pi}\int_0^{2\pi} \frac{1 - Cos\ 2(377)t}{2}\, dt$$

$$P = \frac{2890}{4\pi}\int_0^{2\pi} 1 - Cos\ (754)t\ dt$$

$$P = \frac{2890}{4\pi}\left[t - \frac{Sin\ 754t}{754}\right]$$

$$P = \frac{2890}{4\pi}\left[(2\pi - 0) - \left(\frac{Sin\ 754 * 2\pi}{754} - \frac{Sin\ 754 * 0}{754}\right)\right]$$

$$P = \frac{2890}{4\pi}[6.27] = 1444.74\ W$$

c) $PeakPower = 2890W$

Problem 4:
The voltage and current of a circuit is given by;

$v(t)$= 3 + 5 Cos (2π60t+15°) + 2 Cos (4π60t)

$i(t)$= 2 + 7 Cos (2π60t+45°) + 3 Cos (6π60t+25°)
Determine

a) rms values of voltage and current.
b) power absorbed by the element.
c) power factor of the load.

Solution :

a)

$$V_{rms} = \sqrt{(3)^2 + \left(\frac{5}{\sqrt{2}}\right)^2 + \left(\frac{2}{\sqrt{2}}\right)^2} = 4.85\ V$$

$$I_{rms} = \sqrt{(2)^2 + \left(\frac{7}{\sqrt{2}}\right)^2 + \left(\frac{3}{\sqrt{2}}\right)^2} = 5.74\ A$$

b)

$$P = V_0 I_0 + \sum V_{rms} I_{rms}\ Cos(\theta - \varphi)$$

$$\therefore V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$P = (3)(2) + \left(\frac{5}{\sqrt{2}}\right)\left(\frac{7}{\sqrt{2}}\right)\cos(15 - 45) + \left(\frac{2}{\sqrt{2}}\right)(0) + (0)\left(\frac{3}{\sqrt{2}}\right)$$

$$P = 21.15\ W$$

c)

$$PF = \frac{P}{S} = \frac{P}{V_{in\ (rms)} I_{in\ (rms)}}$$

$$PF = \frac{21.15}{4.85\ x\ 5.74}$$

$$PF = 0.7597$$

Instantaneous Power

The instantaneous power for any device is computed from the voltage across it and the current in it. *Instantaneous power* is

$$p(t) = v(t)i(t)$$

Average Power

Periodic voltage and current functions produce a periodic instantaneous power function.

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)\,dt = \frac{1}{T}\int_{t_0}^{t_0+T} v(t)i(t)\,dt \qquad t_o = 0 \qquad P = \frac{1}{T}\int_{0}^{T} p(t)\,dt$$

Total average power is the sum of the powers at the frequencies in the Fourier series.

$$P = V_0 I_0 + \sum_{n=1}^{\infty}\left(\frac{V_{n,\max} I_{n,\max}}{2}\right)\cos(\theta_n - \phi_n)$$

EFFECTIVE VALUES: RMS

The effective value of a voltage or current is also known as the root-mean-square (rms) value. (Sinusoids)

$$V_{\text{rms}} = V_m/\sqrt{2}, I_{\text{rms}} = I_m/\sqrt{2},$$

If a voltage is the sum of more than two periodic voltages, all orthogonal, the rms value is

$$V_{\text{rms}} = \sqrt{V_{1,\text{rms}}^2 + V_{2,\text{rms}}^2 + V_{3,\text{rms}}^2 + \ \ldots}$$

$$I_{\text{rms}} = \sqrt{I_{1,\text{rms}}^2 + I_{2,\text{rms}}^2 + I_{3,\text{rms}}^2 + \ \ldots}$$

Apparent Power *S*

$$S = V_{\text{rms}} I_{\text{rms}}$$

Power Factor

The *power factor* of a load is defined as the ratio of average power P to apparent power S:

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{\text{rms}} I_{\text{rms}}}$$

Total harmonic distortion (**THD**) quantify the non-sinusoidal property of a waveform. THD is the ratio of the rms value of all the nonfundamental frequency terms to the rms value of the fundamental frequency term.

$$\text{THD} = \sqrt{\frac{I_{\text{rms}}^2 - I_{1,\text{rms}}^2}{I_{1,\text{rms}}^2}}$$