The dc component V_o of the output voltage is the average value of a half-wave rectified sinusoid

$$V_o = V_{avg} = \frac{1}{2\pi}\int_0^{\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{V_m}{\pi}$$

The dc component of the current for the purely resistive load is $I_o = \frac{V_o}{R} = \frac{V_m}{\pi R}$

Average power absorbed by the resistor in half wave rectifier P = I^2_{rms} * R = V^2_{rms} / R. (voltage and current are half-wave rectified sine waves)

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} [V_m \sin(\omega t)]^2 d(\omega t)} = \frac{V_m}{2}$$

$$I_{rms} = \frac{V_m}{2R}$$

steady-state current can be found from phasor analysis

$$Z = \sqrt{R^2 + (\omega L)^2} \qquad \text{and} \qquad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

Adding the forced and natural responses gets the complete solution

$$i(t) = i_f(t) + i_n(t) = \frac{V_m}{Z}\sin(\omega t - \theta) + Ae^{-t/\tau}$$

It is often convenient to write the function in terms of the angle wt rather than time.

$$i(\omega t) = \frac{V_m}{Z}\left[\sin(\omega t - \theta) + \sin(\theta) e^{-\omega t/\omega\tau}\right]$$

Substituting wt = β, $i(\beta) = \frac{V_m}{Z}\left[\sin(\beta - \theta) + \sin(\theta) e^{-\beta/\omega\tau}\right] = 0$

$$\boxed{\sin(\beta - \theta) + \sin(\theta) e^{-\beta/\omega\tau} = 0}$$

$$i(\omega t) = \begin{cases} \frac{V_m}{Z}\left[\sin(\omega t - \theta) + \sin(\theta) e^{-\omega t/\omega\tau}\right] & \text{for } 0 \le \omega t \le \beta \\ 0 & \text{for } \beta \le \omega t \le 2\pi \end{cases}$$

$$\text{where} \quad Z = \sqrt{R^2 + (\omega L)^2} \quad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right) \quad \text{and} \quad \tau = \frac{L}{R}$$

The average power absorbed by the load is $I_{2rms} R$, since the average power absorbed by the inductor is zero. The rms value of the current is:

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2(\omega t)\, d(\omega t)} = \sqrt{\frac{1}{2\pi}\int_0^{\beta} i^2(\omega t)\, d(\omega t)}$$

Average current is

$$I_o = \frac{1}{2\pi}\int_0^{\beta} i(\omega t)\, d(\omega t)$$