

**Problem 1:**

For the half-wave rectifier, the source is a sinusoid of 300 V rms at a frequency of 50 Hz. The load resistor is 25.

Determine

**a) the average load current**

The voltage across the resistor in a half-wave rectified sine wave with peak value  $V_m = V_{rms} * \sqrt{2} = 300\sqrt{2} = 424.4 V$ . The average voltage in a half-wave rectifier is  $V_m/\pi$ , and average current is:

$$\langle i_o \rangle = \frac{V_m}{\pi R} = \frac{300(\sqrt{2})}{25\pi} = 5.4 A$$

**b) the power absorbed by the load**

The rms voltage across the resistor for a half-wave rectified sinusoid is:

$$V_{rms} = \frac{V_m}{2} = \frac{300(\sqrt{2})}{2} = 212.132 V$$

$$I_{rms} = \frac{V_m}{2R} = \frac{300\sqrt{2}}{2*25} = 8.488 A$$

The power absorbed by the resistor is:

$$P = R \times I_{rms}^2 = \frac{V_{rms}^2}{R} = \frac{(212.132)^2}{25} = 1800 W$$

The rms current in the resistor is 8.488 A, and the power could also be calculated from  $RI_{rms}^2 = 25 \times (8.488)^2 = 1800 W$

**c) the apparent power supplied by the source**

$$\text{Apparent Power} = V_{in-rms} * I_{in-rms} = 300 * 8.488 = 2545$$

**d) the power factor of the circuit**

$$pf = \frac{\text{average power}}{\text{apparent power}} = \frac{P}{V_{in-rms} \times I_{in-rms}} = \frac{1800}{300 \times 8.488} = 0.707$$

**Problem 2:**

For the half-wave rectifier with R-L load,  $R=100\Omega$ ,  $L=0.1$  H,  $\omega=377$  rad/s, and  $V_m=100$ V.  $\beta=3.5$  rad

Determine

a) expression for the current in this circuit,

$$i(\omega t) = \frac{V_m}{Z} [\sin(\omega t - \theta) + \sin(\theta)e^{-\omega t/\omega\tau}] \quad Z = \sqrt{R^2 + (\omega L)^2} \quad \text{and} \quad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$Z = \sqrt{(100)^2 + (0.1 * 377)^2} = 106.87$$

$$\theta = \tan^{-1}\left(\frac{L\omega}{R}\right) = \tan^{-1}\left(\frac{37.7}{100}\right) = \tan^{-1}(0.377) = 0.36 \text{ rad} = 20.64^\circ$$

$$i(\omega t) = \frac{100}{106.87} \sin(\omega t - 0.36) + \frac{100 * \sin(0.36)}{106.87} e^{-\omega t/0.377}$$

$$i(\beta) = \frac{V_m}{Z} [\sin(\beta - \theta) + \sin(\theta)e^{-\beta/\omega\tau}] = 0$$

Substituting  $\omega t = \beta$

$\beta = 3.5$  rad is given by the problem, so

$$i(\omega t) = 0.936 \sin(\omega t - 0.36) + 0.33e^{-\omega t/0.377} \quad \text{for } 0 \leq \omega t \leq 3.5$$

$$i(\omega t) = 0 \quad \text{for } 3.5 \leq \omega t \leq 2\pi$$

b) the average current,

$$I_o = \frac{1}{2\pi} \int_0^{\beta} i(\omega t) d(\omega t) = \frac{1}{2\pi} \int_0^{3.5} [0.936 \sin(\omega t - 0.36) + 0.33e^{-\omega t/0.377}] d(\omega t)$$

$$= 0.308 \text{ A}$$

c) the rms current

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} i^2(\omega t) d(\omega t)} = \sqrt{\frac{1}{2\pi} \int_0^{\beta} i^2(\omega t) d(\omega t)}$$

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi} \int_0^{3.5} [0.936 \sin(\omega t - 0.36) + 0.33e^{-\omega t/0.377}]^2 d(\omega t)} = 0.474 \text{ A}$$

d) the power absorbed by the resistor,

$$P = R * I_{\text{rms}}^2 = 100 * (0.474)^2 = 22.47 \text{ W}$$

e) the power factor

$$Pf = \frac{P}{V_{in-rms} * I_{in-rms}} = \frac{22.47}{\frac{100}{\sqrt{2}} * 0.474} = 0.67$$

**Problem 3:**

For a half wave rectifier, the source voltage is 120 V-RMS at 60 Hz. The load resistance is 5  $\Omega$ .

Determine

**a) Average load current.**

$$I_o (avg) = \frac{V_o (avg)}{R}$$
$$\therefore V_o (avg) = \frac{V_m}{\pi}$$
$$I_o (avg) = \frac{V_m}{\pi R}$$
$$I_o (avg) = \frac{V_{rms} \times \sqrt{2}}{\pi R} = \frac{120 \times \sqrt{2}}{\pi \times 5} = 10.8 \text{ A}$$

**b) Average power absorbed by load**

$$P_{out} = \frac{V_o^2 (rms)}{R}$$
$$V_o (rms) = \frac{V_m}{2} = \frac{120 \times \sqrt{2}}{2} = 84.9 \text{ V}$$
$$P_{out} = \frac{(84.9)^2}{5} = 1441 \text{ W}$$

**c) Power factor.**

$$PF = \frac{P}{S} = \frac{P}{V_{in} (rms) I_{in} (rms)}$$
$$P_{out} = I_{(rms)}^2 \times R$$
$$I_{in(rms)} = \sqrt{\frac{P_{out}}{R}} = \sqrt{\frac{1441}{5}} = 16.9 \text{ A}$$
$$PF = \frac{1441}{120 \times 16.97} = 0.7$$