The extinction angle β is defined as the angle at which the current reaches zero,

The first positive value of ωt that results in zero current is called the extinction angle β

α be the value of ωt that causes the source voltage to be equal to Vdc,

The diode starts to conduct at $\omega t = \alpha$

For RC circuits

$$\theta = \tan^{-1}(-\omega RC) = -\tan^{-1}(\omega RC) + \pi$$

For circuits where time constant is large

$$\theta \approx \frac{\pi}{2} \quad \text{and} \quad V_m \sin\theta \approx V_m$$

When $\omega RC \gg 1$ The ripple voltage can be approximated as

$$\Delta V_o \approx V_m\left(\frac{2\pi}{\omega RC}\right) = \frac{V_m}{fRC}$$

When $\omega RC > 1$ (exact relation)

$$\Delta V_o = V_m - V_m \sin\alpha = V_m(1 - \sin\alpha)$$

$$\sin\alpha - (\sin\theta)e^{-(2\pi + \alpha - \theta)/\omega RC} = 0$$

The output voltage ripple is reduced by increasing the filter capacitor C. As C increases, the conduction interval for the diode decreases. Therefore, increasing the capacitance to reduce the output voltage ripple results in a larger peak diode current.

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi}[V_m \sin(\omega t)]^2 d(\omega t)} = \frac{V_m}{2}$$

$$I_{rms} = \frac{V_m}{2R}$$

$$I_{C,\text{peak}} = \omega C V_m \cos(2\pi + \alpha) = \omega C V_m \cos\alpha$$

Peak diode current is

$$I_{D,\text{peak}} = \omega C V_m \cos\alpha + \frac{V_m \sin\alpha}{R} = V_m\left(\omega C\cos\alpha + \frac{\sin\alpha}{R}\right)$$

Average dc voltage across resistor is

$$V_o = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{V_m}{2\pi}(1 + \cos\alpha)$$

The power absorbed by the resistor is V_{rms}^2/R, where the rms voltage across the resistor is computed from

Power factor of controlled rectifier

$$\text{pf} = \sqrt{\frac{1}{2} - \frac{\alpha}{2\pi} + \frac{\sin(2\alpha)}{4\pi}}$$

$$V_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v_o^2(\omega t)\,d(\omega t)}$$

$$= \sqrt{\frac{1}{2\pi}\int_\alpha^{\pi} [V_m \sin(\omega t)]^2\,d(\omega t)}$$

$$= \frac{V_m}{2}\sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}}$$