

Problem 1:

A half-wave rectifier with a 1-k Ω load has a parallel capacitor. The source is 120 V_{rms}, 60 Hz. Determine the peak-to-peak ripple of the output voltage when the capacitor is

a) 4000 μ F

When $\omega RC \gg 1$ The ripple voltage can be approximated as

$$\Delta V_o \approx V_m \left(\frac{2\pi}{\omega RC} \right) = \frac{V_m}{fRC}$$

When $\omega RC > 1$ (exact relation)

$$\Delta V_o = V_m - V_m \sin \alpha = V_m(1 - \sin \alpha)$$

$$C=4000 \mu\text{F}$$

$$\omega RC = 377 * 1000 * 4 * 10^{-3} = 1508 \gg 1$$

$$\Delta V_o \approx \frac{V_m}{fRC} = \frac{120\sqrt{2}}{60 * 1000 * 4 * 10^{-3}} = 0.707 \text{ V}$$

b) 20 μ F.

$$C=20 \mu\text{F} \quad \omega RC = 377 * 1000 * 20 * 10^{-6} = 7.54 > 1 \text{ (approximation is not valid here)}$$

$$\theta = \pi - \tan^{-1}(RC\omega) = \pi - \tan^{-1}(7.54) = 1.7027 \text{ rad}$$

$$\sin \alpha - \sin \theta \cdot e^{-(2\pi + \alpha - \theta)/\omega RC} = 0$$

$$\alpha = 30.2^\circ = 0.5275 \text{ rad}$$

$$\Delta V_o = V_m(1 - \sin \alpha) = 120\sqrt{2}(1 - 0.5034) = 83.26 \text{ V} \text{ (correct answer)}$$

α here is calculated using matlab code. In exam, its value will be given.

Problem 2:

A half-wave rectifier with a capacitor filter has $V_m=200$ V, $R=10$ k Ω , $C=1000$ μ F, and $\omega=377$.

Determine

- a) Peak-to peak ripple voltage using the exact equations.

$$\theta = \tan^{-1}(-\omega RC) = -\tan^{-1}(\omega RC) + \pi = 1.5711 \text{ rad}$$

$$\sin\alpha - \sin\theta \cdot e^{-(2\pi+\alpha-\theta)/\omega RC} = 0$$

$$\alpha = 86.7^\circ = 1.513 \text{ rad}$$

$$\Delta V_0 = V_m(1 - \sin\alpha) = 200(1 - \sin\alpha) = 200(1 - 0.9983) = 0.34 \text{ V}$$

- b) Peak-to peak ripple voltage using the approximate formula.

$$\Delta V_0 \approx \frac{V_m}{fRC} = \frac{200}{60 * 10000 * 1000 * 10^{-6}} = 0.333 \text{ V}$$

Problem 3:

For the controlled half-wave rectifier with resistive load, the source is $120 V_{rms}$ at 60 Hz. The resistance is 100Ω , and the delay angle α is 45.

Determine

a) average voltage across the resistor

$$V_o = \frac{V_m}{2\pi} (1 + \cos\alpha) = \frac{120\sqrt{2}}{2\pi} (1 + \cos 45) = 46.11 V$$

b) power absorbed by the resistor

$$P = \frac{V_{rms}^2}{R}$$

$$V_{rms} = \frac{V_m}{2} \sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}} = \frac{120\sqrt{2}}{2} \sqrt{1 - \frac{0.785}{\pi} + \frac{\sin 2\alpha}{2\pi}} = \frac{120\sqrt{2}}{2} * 0.953 = 80.91V$$

$$\alpha = 45^\circ = \frac{\pi}{4} rad = 0.785 rad$$

$$P = \frac{V_{rms}^2}{R} = \frac{(80.91)^2}{100} = 65.46W$$

c) power factor

$$pf = \sqrt{\frac{1}{2} - \frac{\alpha}{2\pi} + \frac{\sin 2\alpha}{4\pi}} = 0.67$$

Problem 4:

A half-wave rectifier has a 120 V rms, 60 Hz ac source. The load is 750 Ω . Determine

- a) value of a filter capacitor to keep the peak-to-peak ripple across the load to less than 2 V.

$$\Delta V_0 \approx \frac{V_m}{fRC} \quad C > \frac{120\sqrt{2}}{60 * 750 * 2} = 1885 \mu F$$

- b) peak values of diode current

Peak diode current is

$$I_{D, \text{peak}} = \omega C V_m \cos \alpha + \frac{V_m \sin \alpha}{R} = V_m \left(\omega C \cos \alpha + \frac{\sin \alpha}{R} \right)$$

$$I_{D, \text{peak}} = 120 * \sqrt{2} (2 * \pi * 60 * 1885 * 10^{-6} * \cos(81.2) + \sin(81.2) / 750)$$

$$I_{D, \text{peak}} = 18.7 \text{ A}$$