Aalto University
School of Engineering

Power Electronics

ELEC-E8412 Power Electronics, 5 ECTS

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Course Objectives

At the end of this course, you will be able to:

generally analyse the uncontrolled and controlled half-wave rectifiers with different loads and apply the power computation concepts from the previous chapter to these circuits.

Half-Wave Rectifiers

A **rectifier** converts **AC** to **DC**. The half-wave rectifier is used most often in **low-power** applications.

1. The Uncontrolled Half-Wave Rectifiers:

Assuming you have an AC source, and you are going to supply a DC load. For this process, you need an AC/DC rectifier.

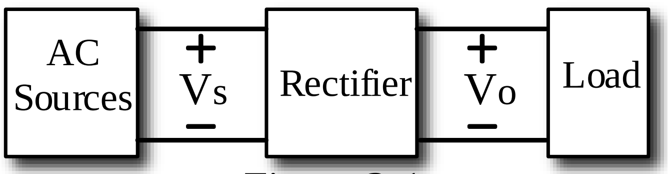
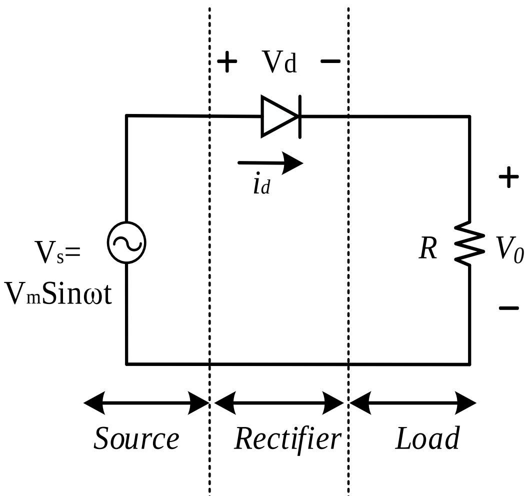
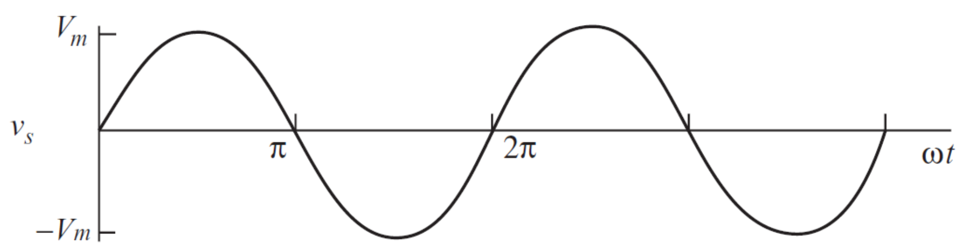


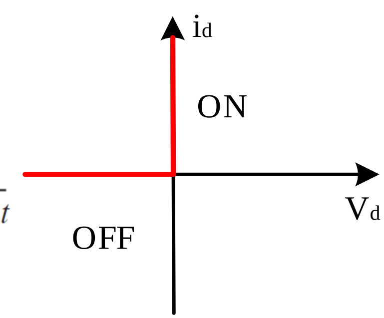
Figure 3-1

- Vo is purely DC or has a specified DC component
- AC source could be sinusoidal, could be a voltage source

A. The Uncontrolled Half-Wave Rectifiers with resistive load:



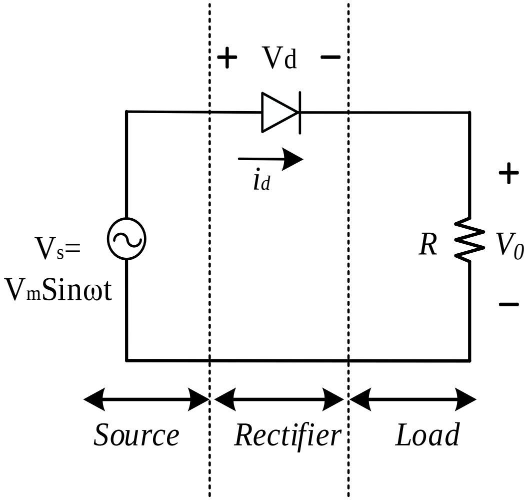


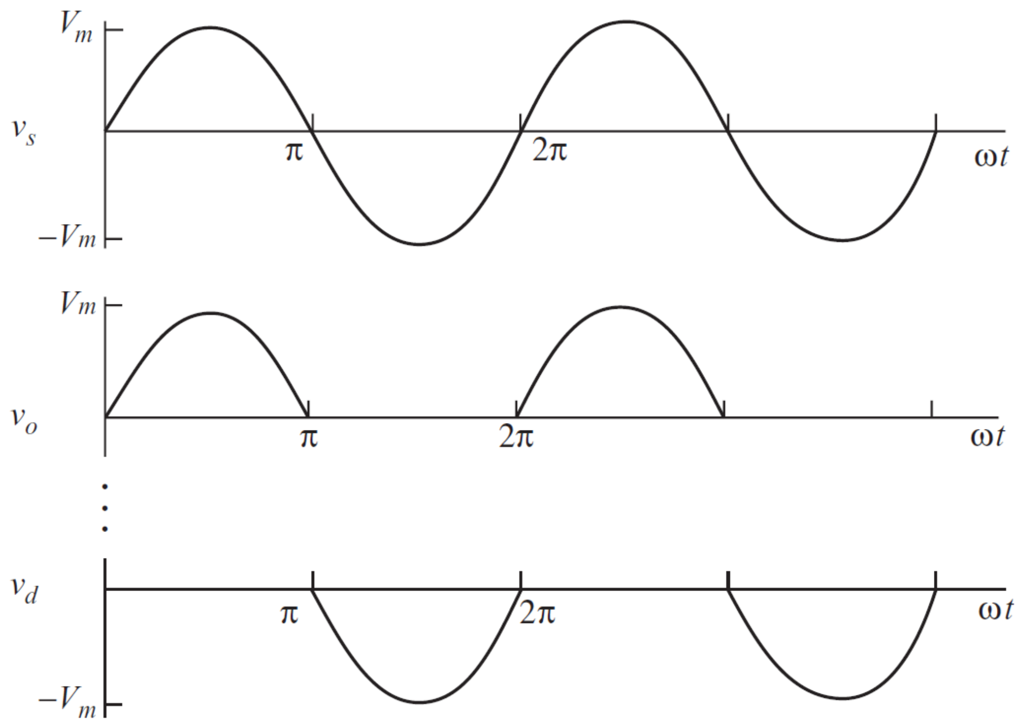


$$\begin{cases} 0 < \omega t < \pi \quad \text{assume} \quad \text{D is OFF} \Rightarrow i_d = 0 \rightarrow V_0 = 0 \rightarrow V_s = V_d > 0 \\ 0 < \omega t < \pi \quad \text{assume} \quad \text{D is ON} \Rightarrow V_d = 0 \rightarrow V_0 = V_s \rightarrow i_d = \dfrac{V_s}{R} > 0 \end{cases}$$

×Wrong assumption

√ Right assumption





$$\begin{cases} \pi < \omega t < 2\pi \quad \text{assume} \quad \text{D is ON} \\ \Rightarrow \; V_d = 0 \;\rightarrow\; V_0 = V_{in} \;\rightarrow\; i_d = \dfrac{V_{in}}{R} < 0 \end{cases}$$

×Wrong assumption

$$\begin{cases} \pi < \omega t < 2\pi \quad \text{assume} \quad \text{D is OFF} \\ \Rightarrow \; i_d = 0 \;\rightarrow\; V_0 = 0 \;\rightarrow\; V_d = V_{in} < 0 \end{cases}$$

√ Right assumption

$$< V_o > = V_{avr} = \frac{1}{2\pi}\int_0^{2\pi} V_0 \; d(\omega t) = \frac{1}{2\pi}\int_0^{\pi} V_m \,\mathrm{Sin}(\omega t)\; d(\omega t) + \overbrace{\frac{1}{2\pi}\int_{\pi}^{2\pi} 0 \; d(\omega t)}^{0} = \frac{V_m}{\pi}$$

← The average value is not zero

$$< i_d > = \frac{V_m}{R\pi}$$

$$V_{orms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_0^2 \, d\omega t} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} [V_m \,\mathrm{Sin}(\omega t)]^2 \, d(\omega t)} = \sqrt{\frac{1}{2\pi}\frac{V_m^2 \pi}{2}} = \frac{V_m}{2}$$

$$P_{out} = \frac{V_{o-rms}^2}{R}$$

$$I_{in-rms} = I_{R-rms} = I_{d-rms} = I_{o-rms} = \frac{V_{orms}}{R} = \frac{V_m}{2R}$$

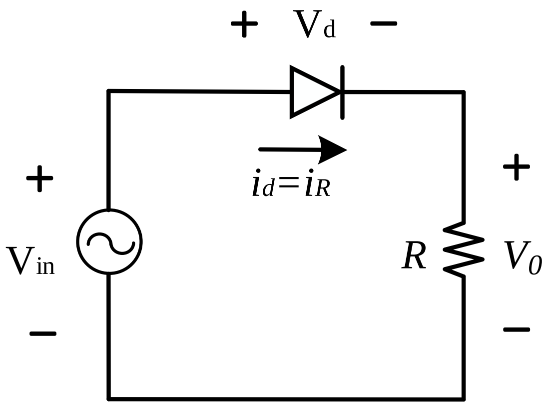
Example: If $V_{in}(t)$=170Sin (366t) and R=12 Ω, determine:

(a) The average value of the load current

(b) The rms value of the load current

(c) The apparent power supplied by the source

(d) Power factor of the circuit



Solution:

(a) $$< i_o > = < i_R > = < i_d > = \frac{V_m}{R\pi} = \frac{170}{\pi \times 12} = 4.51 \quad \text{(A)}$$

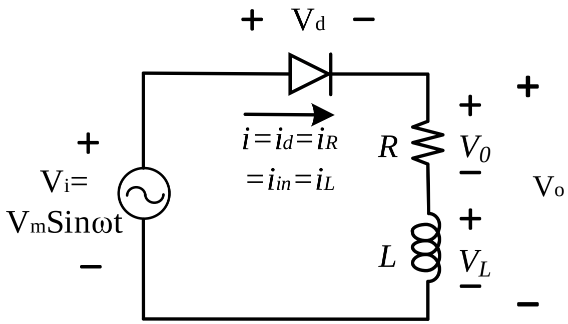
(b) $$I_{in-rms} = I_{R-rms} = I_{d-rms} = I_{o-rms} = \frac{V_m}{2R} = \frac{170}{2 \times 12} = 7.08 \quad \text{(A)}$$

(c) $$\text{apparent power} = V_{in-rms}.I_{in-rms} = \frac{170}{\sqrt{2}} \times 7.08 = 851.47$$

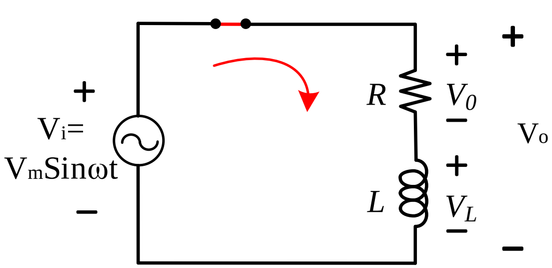
(d) $$\text{pf} = \frac{\text{average power}}{\text{apparent power}} = \frac{P_{out}}{V_{in-rms}.I_{in-rms}} = \frac{R \times I_{o-rms}^2}{V_{in-rms}.I_{in-rms}} = \frac{12 \times (7.08)^2}{\frac{170}{\sqrt{2}} \times 7.08} = \frac{\sqrt{2}}{2} = 0.707$$

B. The Uncontrolled Half-Wave Rectifiers with R-L load:

Example: Providing power to the field winding of a DC machine or armature winding of a DC machine (winding has internal resistance R and self-inductance L).



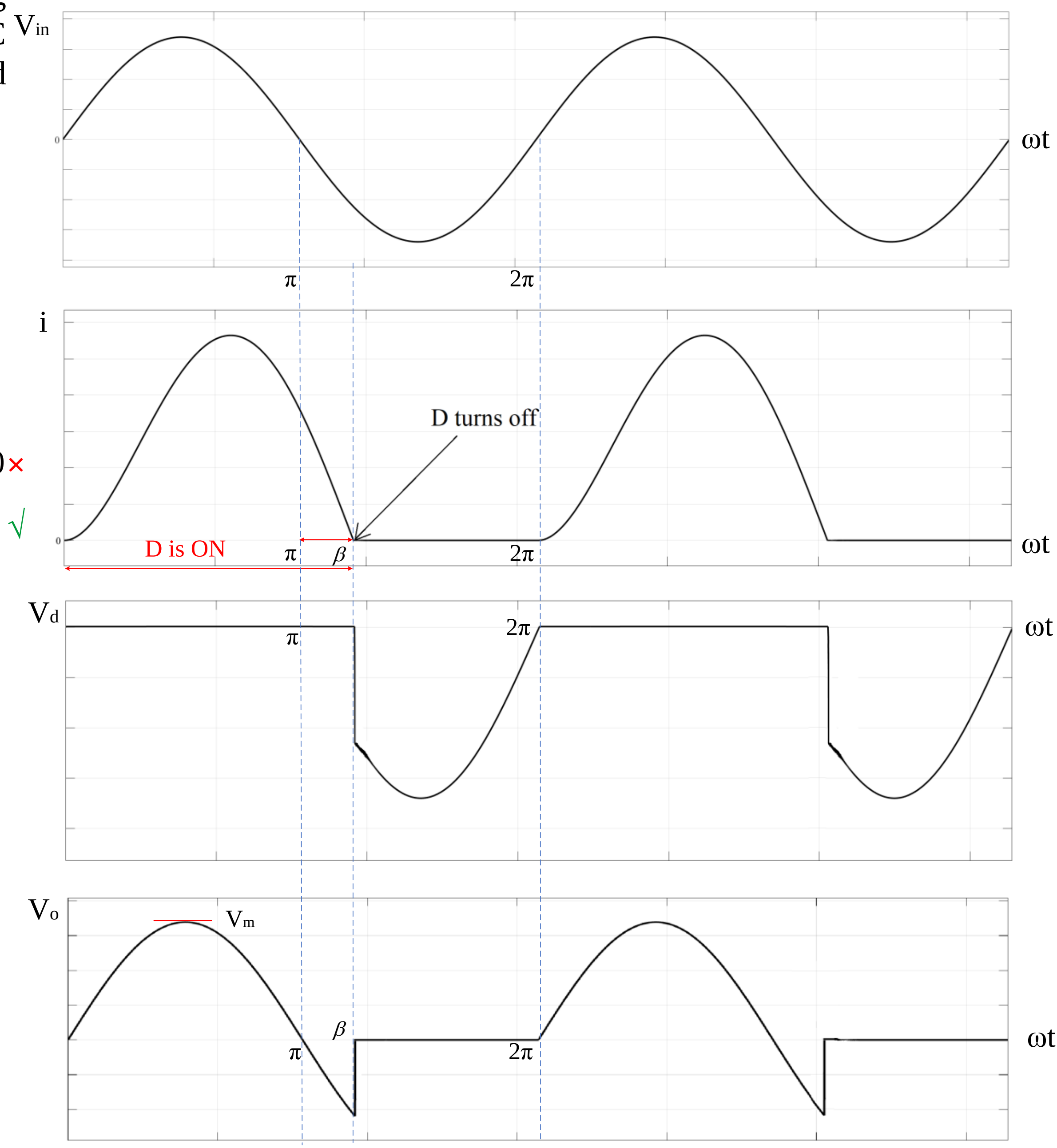
$$\begin{cases} 0 < \omega t < \pi \quad \text{assume} \quad \text{D is OFF} \quad \Rightarrow V_{in} = V_d > 0 \; \times \\ 0 < \omega t < \pi \quad \text{assume D is ON} \Rightarrow V_d = 0 \rightarrow V_0 = V_{in} \; \surd \end{cases}$$

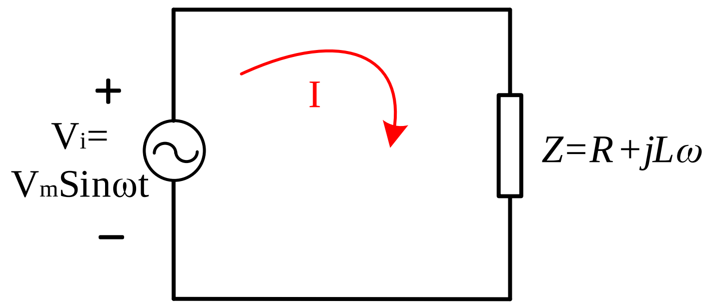


$$V_{in} = V_R + V_L \Rightarrow V_m \,\text{Sin}\, \omega t = Ri + L\frac{di}{dt}$$

$$\Rightarrow i(t) = \underbrace{\frac{V_m}{\sqrt{R^2 + (L\omega)^2}} \text{Sin}(\omega t - \theta)}_{steady-state\ response} + \underbrace{Ae^{-t/\frac{L}{R}}}_{natural\ response}$$

$$\theta = \tan^{-1}\left(\frac{L\omega}{R}\right) > 0$$





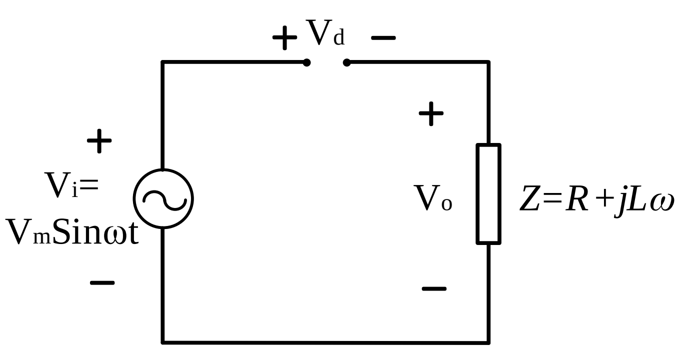
$$0 = Ri + L\frac{di}{dt} \longrightarrow i(t) = Ae^{-t/\frac{L}{R}}$$

$$I = \frac{V_m}{|Z|}\text{Sin}(\omega t - \measuredangle Z)$$

$A = ?$ initial condition $i(0) = 0 \Rightarrow 0 = \frac{V_m}{\sqrt{R^2 + (L\omega)^2}}\text{Sin}(-\theta) + A \Rightarrow A = \frac{V_m \text{Sin}\,\theta}{\sqrt{R^2 + (L\omega)^2}}$

$$i(t) = \frac{V_m}{\sqrt{R^2 + (L\omega)^2}}\left[\text{Sin}(\omega t - \theta) + \text{Sin}\,\theta e^{-t/\frac{L}{R}}\right]$$

$\begin{cases} \text{D is ON for } 0 < \omega t < \beta \\ \text{D is OFF for } \beta < \omega t < 2\pi \end{cases}$ $\beta > \pi$ because $i(t)$ tends to become negative



D is OFF

D is OFF $\Rightarrow i_d = 0 \Rightarrow V_R = V_L = V_o = 0 \Rightarrow V_{in} = V_d$

when ωt=$\beta \Rightarrow i(\omega t = \beta) = 0$

$$i(\beta) = \frac{V_m}{\sqrt{R^2 + (L\omega)^2}}\left[\text{Sin}(\beta - \theta) + \text{Sin}\,\theta e^{-\beta/\frac{L\omega}{R}}\right] = 0$$

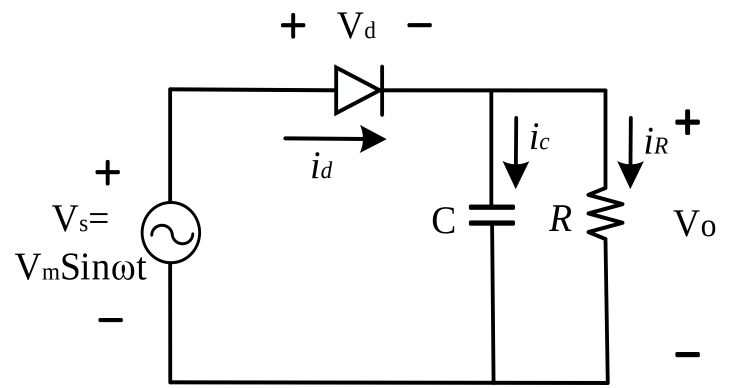
* There is no closed-form sloution to find the β and it must be calculated by software programing like MATLAB but the value of β is always as $\pi \le \beta \le 2\pi$

* β is called as conduction angle or turn off angle of diode.

C. The Uncontrolled Half-Wave Rectifiers with R-C load:

$$D \text{ is ON} \Rightarrow i_d > 0 \text{ or } i_d = i_C + i_R > 0$$

$$\text{if } D \text{ is ON} \Rightarrow V_{in} = V_C = V_o = V_m \text{ Sin}(\omega t)$$



Half-wave rectifier with RC load

$$i_C + i_R > 0$$

$$C\frac{dV_C}{dt} + \frac{V_0}{R} > 0$$

$$C\frac{d(V_m \text{Sin}(\omega t))}{dt} + \frac{V_m \text{Sin}(\omega t)}{R} > 0 \rightarrow C\omega V_m \text{Cos}(\omega t) + \frac{V_m}{R}\text{Sin}(\omega t) > 0$$

θ is the time at which the diode is going to turn off because its current turns to get negative.

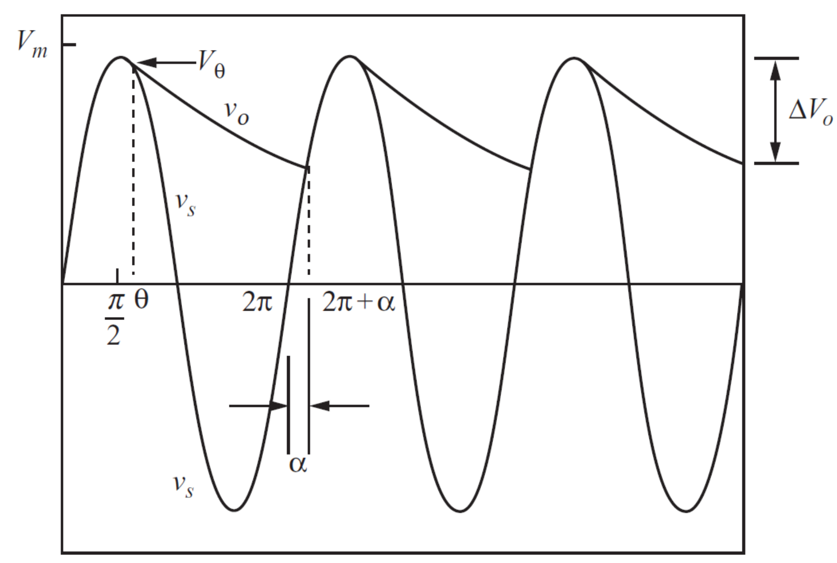
Until ωt=π/2 both terms are positive. After ωt>π/2 the first term get negative and D turns off at ωt=*Θ*. So, the capacitor is getting discharged through the resistor.

$$C\omega V_m \text{Cos}\,\theta + \frac{V_m}{R}\text{Sin}\,\theta = 0$$

$$\tan\theta = \text{-}RC\omega \rightarrow \theta = \tan^{-1}(-RC\omega) = \pi - \tan^{-1}(RC\omega)$$

$$RC\omega > 0 \rightarrow 0 < \tan^{-1}(RC\omega) < \frac{\pi}{2} \Rightarrow \frac{\pi}{2} < \theta < \pi$$

$$\text{if } RC\omega >> 0 \Rightarrow \tan^{-1}(RC\omega) \rightarrow \frac{\pi^-}{2} \Rightarrow \theta \rightarrow \frac{\pi^+}{2}$$



Input and output voltages

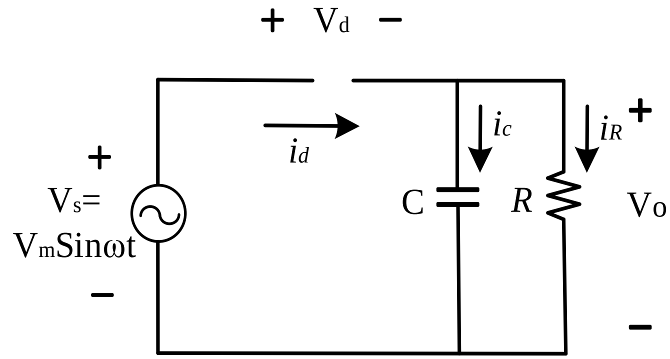
In practical circuits where the time constant is large,

$$\theta \approx \frac{\pi}{2} \text{ and } V_m \text{ Sin}\,\theta \approx V_m$$

D turns OFF:

In this mode capacitor C discharges through resistor R.

if D is OFF: $i_C + i_R = 0 \rightarrow C\frac{dV_C}{dt} + \frac{V_C}{R} = 0$



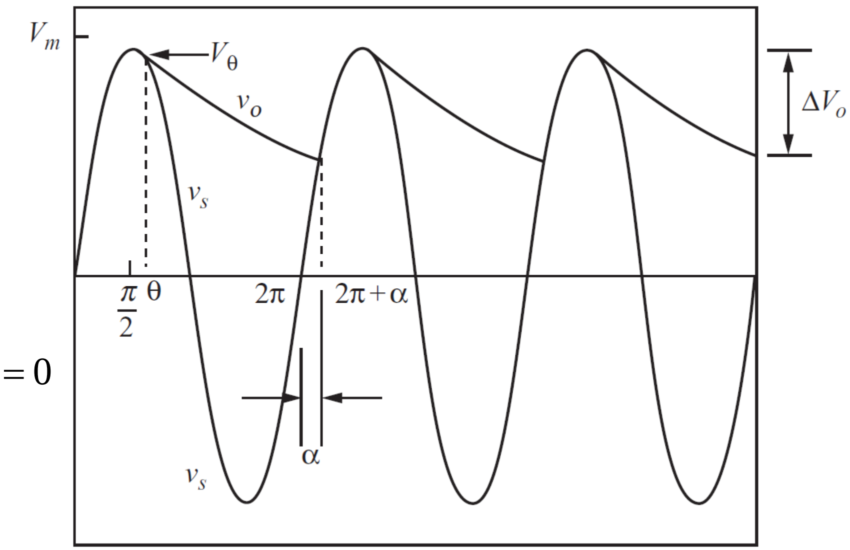
Half-wave rectifier with RC load

$$\frac{dV_C}{V_C} + \frac{dt}{RC} = 0 \rightarrow \frac{dV_C}{V_C} = -\frac{dt}{RC}$$

$$\int_{V_\theta}^{V_C} \frac{dV_C}{Vc} = -\int_{\theta}^{\omega t} \frac{dt}{RC} \rightarrow \int_{V_m \sin\theta}^{V_C} \frac{dV_C}{Vc} = -\frac{1}{\omega}\int_{\theta}^{\omega t} \frac{d\omega t}{RC} \rightarrow Ln\frac{V_C}{V_\theta} = -\left(\frac{\omega t - \theta}{\omega RC}\right)$$

$$V_C = v_o = V_m \sin\theta e^{-(\omega t - \theta)/\omega RC}$$

for $\omega t = 2\pi + \alpha \rightarrow V_C = V_m \sin\theta e^{-(2\pi + \alpha - \theta)/\omega RC}$



if D turns ON at $\omega t = 2\pi + \alpha \Rightarrow V_m \sin(2\pi + \alpha) - V_m \sin\theta e^{-(2\pi + \alpha - \theta)/\omega RC} = 0$

$$\underbrace{{}^{*}\sin\alpha - \sin\theta e^{-(2\pi + \alpha - \theta)/\omega RC} = 0}_{\text{No closed form solution for } \alpha \text{ must be solved numerically}}$$

Input and output voltages

$$V_o(\omega t) = V_C(\omega t) = \begin{cases} V_m \sin\theta e^{-(\omega t - \theta)/\omega RC} & \theta \le \omega t \le 2\pi + \alpha \text{ (diode is OFF)} \\ V_m \sin\omega t & 2\pi + \alpha \le \omega t \le 2\pi + \theta \text{ (diode is ON)} \end{cases}$$

$$i_C = C\frac{dV_C}{dt} = \begin{cases} -\dfrac{V_m \sin\theta e^{-(\omega t - \theta)/\omega RC}}{R} & \text{diode is OFF} \\ C\omega V_m Cos\,\omega t & \text{diode is ON} \end{cases}$$

where $\theta = \pi - \tan^{-1}(RC\omega)$ and α is the numerical solution of $*$.

What is the peak to peak ripple of output voltage (ΔVo)?

The maximum output voltage is V_m. The minimum output voltage occurs at $\omega t = 2\pi + \alpha$, which can be computed from $V_m \sin\alpha$. Then, the peak-to-peak ripple of output voltage can be expressed as:

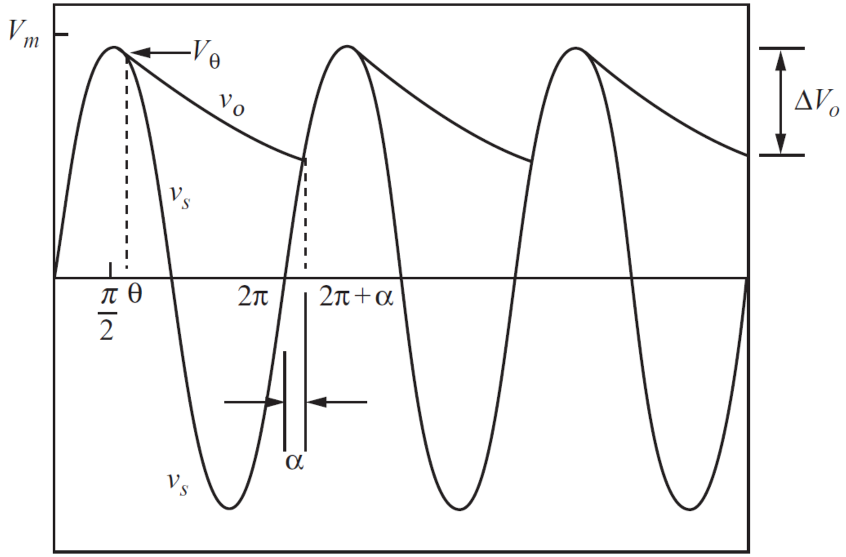
$$\Delta V_0 = V_m - V_m \sin\alpha = V_m (1 - \sin\alpha)$$

It is not easy to use this equation if is not existed. For this reason we need to consider some approximations.

The peak-to-peak ripple is approximately:

$$\Delta V_o \approx V_m \left(\frac{2\pi}{\omega RC}\right) = \left(\frac{V_m}{fRC}\right)$$

*This approximation is valid if: $\begin{cases} \omega RC \gg 1 \\ or \\ RC \gg T \end{cases}$



Example: The half-wave rectifier with RC load has a 120-V rms source at 60 Hz, R= 500 (Ω), and C=100 (μF). Determine:

a. an expression for output voltage

b. the peak-to-peak voltage variation on the output

c. an expression for capacitor current

d. the peak diode current

e. the value of C such that ΔV_o is 1 percent of V_m.

Solution:

From the parameters given,

$$V_m = 120\sqrt{2} = 169.7\ V$$

$$\omega RC = (2\pi \times 60)(500)(10)^{-6} = 18.85 \text{ rad}$$

The angle θ is determined as $\rightarrow \theta = \pi - \tan^{-1}(RC\omega) = \pi - \tan^{-1}(18.85) = 1.62 \text{ rad} = 93^o$

The angle α is determined as $\rightarrow \sin\alpha - \sin\theta e^{-(2\pi+\alpha-\theta)/\omega RC} = 0 \rightarrow \sin\alpha - \sin(1.62)e^{-(2\pi+\alpha-1.62)/18.85} = 0$

yielding: $\alpha = 0.843 \text{ rad} = 48^o$

a. Output voltage is expressed as:

$$V_o(\omega t) = \begin{cases} V_m \sin\theta e^{-(\omega t-\theta)/\omega RC} & \theta \le \omega t \le 2\pi+\alpha \\ V_m \sin\omega t & 2\pi+\alpha \le \omega t \le 2\pi+\theta \end{cases} \Rightarrow \begin{cases} 169.7 e^{-(\omega t-1.62)/18.85} & \theta \le \omega t \le 2\pi+\alpha \\ 169.7\sin\omega t & 2\pi+\alpha \le \omega t \le 2\pi+\theta \end{cases}$$

b. Peak-to-peak output voltage can be expressed as:

$$\Delta V_0 = V_m(1-\sin\alpha) = 169.7(1-\sin 0.843) = 43\ V$$

c. The capacitor current is determined as:

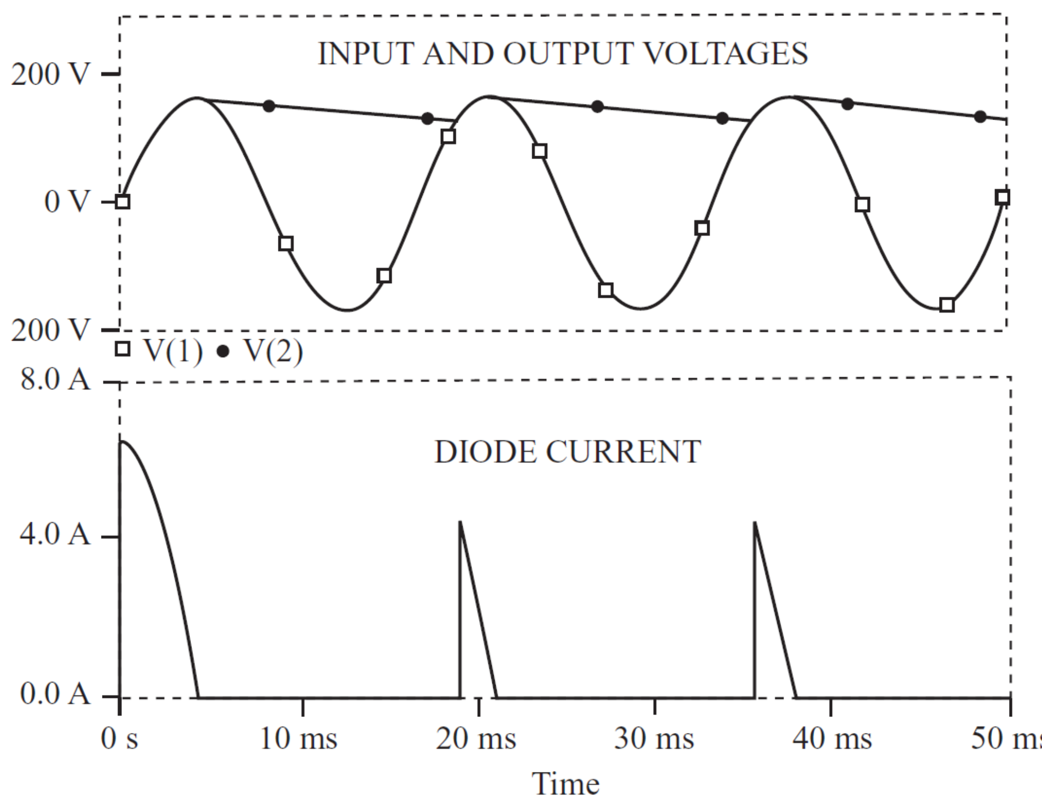
$$i_C(\omega t)=\begin{cases}-\dfrac{V_m\sin\theta e^{-(\omega t-\theta)/\omega RC}}{R} & \theta\le\omega t\le 2\pi+\alpha\\ C\omega V_m Cos\,\omega t & 2\pi+\alpha\le\omega t\le 2\pi+\theta\end{cases}\Rightarrow\begin{cases}-0.339e^{-(\omega t-1.62)/18.85}\ \text{A} & \theta\le\omega t\le 2\pi+\alpha\\ 6.4Cos(\omega t)\ \text{A} & 2\pi+\alpha\le\omega t\le 2\pi+\theta\end{cases}$$

d. Peak diode current is determined as:

$$\mathrm{i_{d,peak}}=\mathrm{V_m}(\mathrm{C}\omega\cos\alpha+\frac{\sin\alpha}{R})=\sqrt{2}(120)\left[377(10)^{-4}\cos 0.843+\frac{\sin 0.843}{500}\right]=4.26+0.34=4.50\ \text{A}$$

e. For $\Delta V_o = 0.01\ V_m$, C can be calculated as:

$$C\approx(\frac{V_m}{fR(\Delta V_o)})=(\frac{V_m}{60\times 500\times 0.01V_m})=\frac{1}{300}F=3333\mu F$$

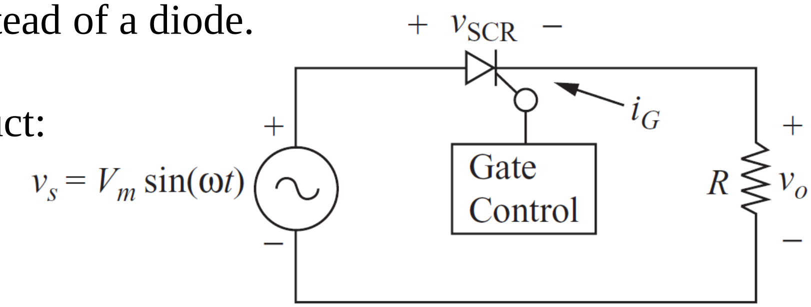


2. The Controlled Half-Wave Rectifier:

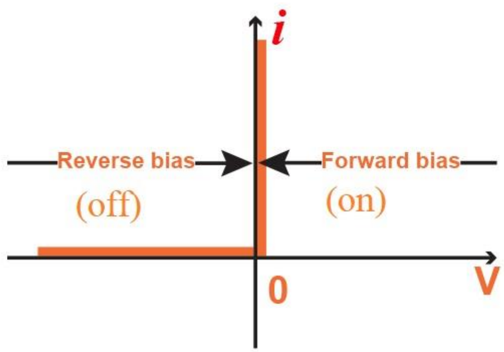
The half-wave rectifiers analyzed previously are classified as uncontrolled rectifiers. Away to control the output of a half-wave rectifier is to use an SCR instead of a diode.

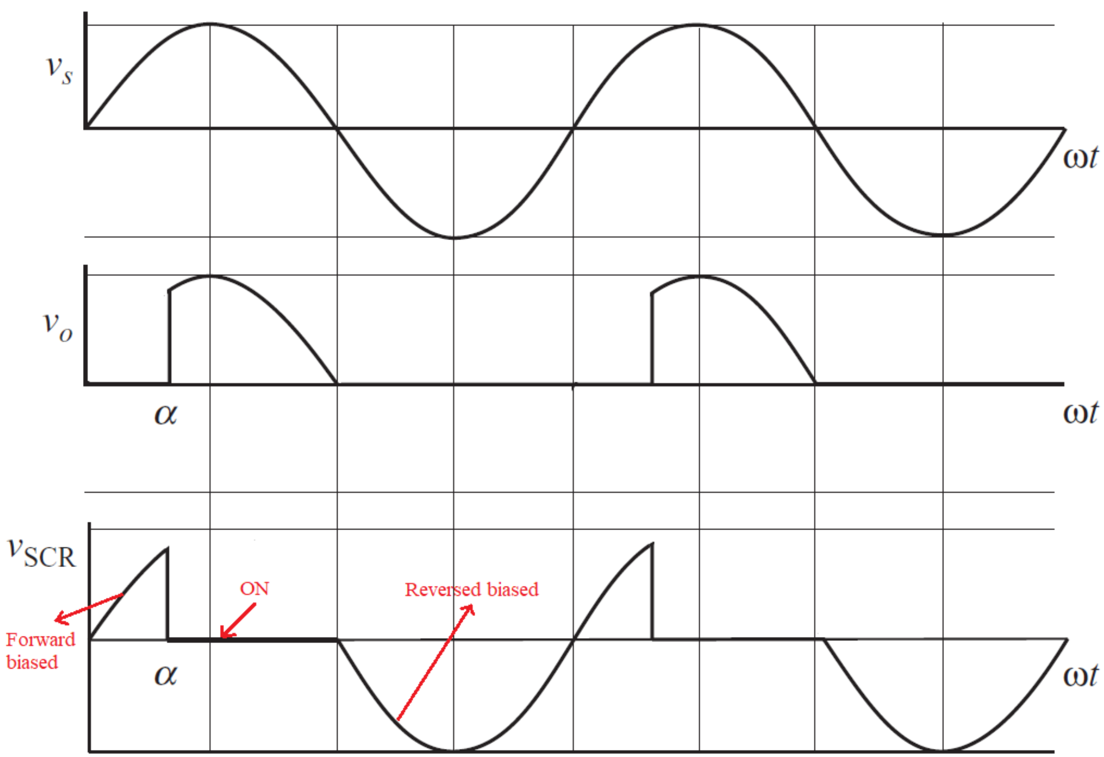
Two conditions must be met before the SCR can conduct:

1. The SCR must be forward-biased ($v_{SCR} > 0$).
2. A current must be applied to the gate of the SCR.



A basic controlled rectifier





Voltage waveforms

$0 < \omega t < \alpha$, SCR is forward blocking (OFF)

$\alpha < \omega t < \pi$, SCR is ON

$\pi < \omega t < 2\pi$, SCR is reversed biased

@ $\omega t = \alpha$, the SCR is triggered and starts conducting

@ $\omega t = \pi$, the SCR current reduces to zero and it stops conducting

α is called the delay angle

The average (DC) voltage across the load resistor can be calculated as:

$$V_o = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{V_m}{2\pi}(1 + \cos\alpha)$$

The power absorbed by the resistor is V_{rms}^2 / R, where the rms voltage across the resistor is computed from

$$V_{o,rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}\left[V_o(\omega t)\right]^2 d(\omega t)} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\pi} V_m^2 \,\mathrm{Sin}^2(\omega t)\, d(\omega t)} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\pi} V_m^2 \times \left(\frac{1-\mathrm{Cos}\,2\omega t}{2}\right) d(\omega t)}$$

$$= \frac{V_m}{2}\sqrt{\left(\frac{\pi-\alpha}{\pi}\right) + \left(\frac{\mathrm{Sin}\,2\alpha - \mathrm{Sin}\,2\pi}{2\pi}\right)} = \frac{V_m}{2}\sqrt{\left(1-\frac{\alpha}{\pi}\right) + \left(\frac{\mathrm{Sin}\,2\alpha}{2\pi}\right)} \qquad \alpha \text{ in rad}$$

$$\text{Power Factor (pf)} = \frac{\text{average power}}{\text{apparent input power}} = \frac{P_{in} = P_{out}}{V_{in,rms} \times I_{rms}} = \frac{V_{rms}^2 / R}{V_{in,rms} \times V_{o,rms}/R} = \frac{V_{o,rms}}{V_{in,rms}}$$

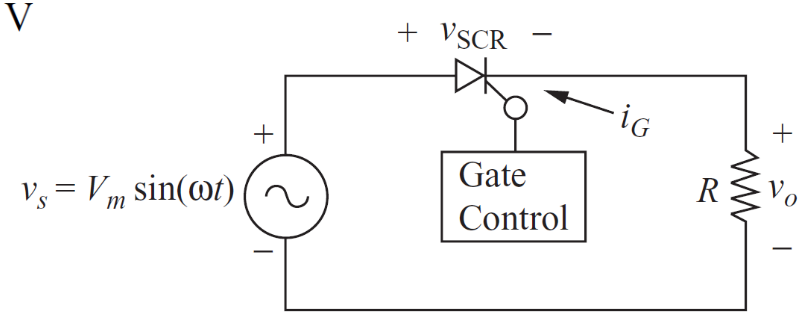
$$= \frac{\frac{Vm}{2}\sqrt{1-\frac{\alpha}{\pi}+\frac{\mathrm{Sin}\,2\alpha}{2\pi}}}{\frac{Vm}{\sqrt{2}}} = \sqrt{\frac{1}{2}-\frac{\alpha}{2\pi}+\frac{\mathrm{Sin}\,2\alpha}{4\pi}} \qquad \alpha \text{ in rad}$$

Example: Design a circuit to produce an average voltage of 40 V across a 100-Ω load resistor from a 120-V rms 60-Hz ac source. Determine the power absorbed by the resistance and the power factor.

Solution: In an uncontrolled half-wave rectifier, the average voltage will be $V_m/\pi=(120\sqrt{2})/\pi$ =54 V.

$$\alpha = \cos^{-1}\left[V_o\left(\frac{2\pi}{V_m}\right) - 1\right] = \cos^{-1}\left\{40\left[\frac{2\pi}{\sqrt{2}(120)}\right] - 1\right\} = 61.2° = 1.07 \text{ rad}$$

$$V_{\text{rms}} = \frac{\sqrt{2}(120)}{2}\sqrt{1 - \frac{1.07}{\pi} + \frac{\sin[2(1.07)]}{2\pi}} = 75.6 \text{ V}$$



Load power is:

$$P_R = \frac{V_{\text{rms}}^2}{R} = \frac{(75.6)^2}{100} = 57.1 \text{ W}$$

The power factor of the circuit is:

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{S,\text{rms}} I_{\text{rms}}} = \frac{57.1}{(120)(75.6/100)} = 0.63$$

Questions and comments are most welcome!