Principles of Economics I
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Problem Set 5 (Due October 13, 2023)

1. A chemical plant (C) is located on a river upstream from a fishery (F). The chemical plant can direct its waste into the river at cost 30 per year or process its waste on dry land at 80 per year. The fishery makes 200 per year if the river is clean (i.e. the waste is processed on dry land) and 130 if the waste ends in the river.
   (a) Suppose that no monetary transfers are possible between (F) and (C). Find Pareto-efficient outcomes for waste processing.
   (b) Suppose next that the fishery can make a monetary transfer T to the chemical plant. Assuming that both F and C care only about the monetary consequences (i.e. adding up the profit and transfer), find the Pareto efficient waste processing decisions at different levels of T.
   (c) If C has property rights to the river, it can pollute the river if it wants. Let C make the proposal to F as follows: if you accept and pay a price P, then we treat the waste on dry land. If you reject, then we put the waste in the river. Solve this game by backward induction (i.e. find the best response for C as a function of P and determine optimal P).
   (d) Suppose next that F has property rights to the river, i.e. it can charge a pollution fee for putting the waste in the river. In particular, F gets to set a price P for pollution and C either pays the price and pollutes or does not pay the price and treats its waste on dry land. Solve the game by backward induction.

2. A yoga studio opens a site in a small town. As the only yoga studio in its market, it has market power, i.e. to sell larger quantities, it has to set lower prices.
   (a) The demand curve for memberships at the studio is $P = 45 - \frac{1}{10}Q$ for $Q \leq 300$ and $P = 0$ for $Q > 300$ where Q denotes number of memberships and P the price. Draw the demand curve for memberships at the in the (Q, P) co-ordinates (Q on horizontal axis). How many memberships can be sold at $P = 20$?
   (b) The fixed cost of running the studio is 700. A yoga instructor costs 400 to hire and each instructor can serve up to 50 members. Draw the cost curve for the studio and draw also the marginal

cost and the average cost curve for the studio. Is it possible to operate the studio profitably?

(c) What is the marginal cost and marginal revenue from adding one more instructor?

(d) How many instructors should be hired and what is the optimal membership price?

3. Consider a competitive market where firms can produce quantity q using a technology whose cost function is given by $C(q) = F + cq + bq^2$, for $q > 0$, $\quad C(0) = 0$.

    (a) What is the fixed cost of the technology, what is the marginal cost?

    (b) What is the efficient scale of production (i.e. the quantity minimizing the average cost) for each firm?

    (c) The demand curve for the market is given by $P = 300-2Q$, where Q denotes the total quantity demanded. Set $F = 64$, $c = 12$ and $b = 16$ What is the long-run equilibrium price for the market? How many firms enter?

    (d) How does the number of entering firms depend on F, b and c?

4. A competitive market for electricity production operates using fossil fuels. The market consists of small plants that differ from each other in terms of their marginal cost (perhaps due to technologies of different vintage). The supply curve of the industry is given by $S(Q) = \frac{1}{10}Q$. The demand for electricity is given by $P(Q) = 100 - \frac{1}{10}Q$.

    (a) Compute the equilibrium price and quantity for the market.

    (b) A new nuclear plant that produces a quantity $Q^N = 200$ at zero marginal cost. Find the new industry supply curve and compute the new equilibrium price and quantity.

    (c) The market for hybrid cars expands and as a result, the demand for electricity grows by 200 whenever the price of electricity falls below 70 (the price of the equivalent amount of gasoline). Draw the new demand curve.

    (d) Find the new equilibrium price and quantity for the market when the supply is as in part b. and the demand is as in part c.

5. Consider a model of trade between two countries, Finland (F) and Sweden (S). In F, there are 80 competitive firms operating with marginal cost $MC^F(q) = 5q$ at individual production of q. In S, there are 120 competitive firms with a lower marginal cost $MC^S(q) = 3q$.

(a) Draw the supply curves for the two countries in a diagram with the coordinate system where total production Q is on the horizontal axis and price P is on the vertical axis.

(b) Suppose that the demand function in both countries is given by $P = 3 - \frac{Q}{100}$. Find the equilibrium prices and quantities (P^F, Q^F) and (P^S, Q^S).

(c) Suppose that the two countries start trading so that there will be a single equilibrium price for the two countries such that total demand equals total supply. Draw the total supply for the market.

(d) Solve for the equilibrium price and quantity. How does the free trade equilibrium quantity compare with the combined quantity $Q^F + Q^S$ in the absence of trade?

6. There are two types of individuals: students and trust fund kids. Students work hard at their studies and earn 200 in period 2 when they are old. Unfortunately since they study in period 1, they have no income in that period. Trust fund kids get an inheritance of 300 and they conclude that they do not have to study. As a result, they have no labor income in period 2. Denote consumptions in the two periods by c_1 and c_2.

   (a) Since both types of individuals like to consume on both periods, they realize that a market for borrowing and loans might be a good idea. Suppose that there is a market rate for lending and borrowing at r so students can borrow c_1 for consumption when young in exchange of paying $(1+r)c_1$ back when old. We require that $c_2 = 200-(1+r)c_1 \geq 0$ so that any amount borrowed can be paid back. Similarly the trust fund kids may save s when young to get back $(1+r)s$ for consumption when old. Draw the budget sets for the two types of individuals.

   (b) Assume that the two types have same indifference curves and that their $MRS = \frac{c_2}{c_1}$. Use the budget constraint and the requirement that MRS = MRT to solve algebraically the optimal savings and borrowings.

   (c) Determine the effect of an increase in r on the optimal savings and borrowings graphically. Show the income and substitution effects in the graphs.

   (d) Extra Bonus: At equilibrium interest rates, total borrowing equals total savings. Find the equilibrium interest rate when there are twice as many students as trust fund kids.