

# **Power Electronics**

## **ELEC-E8412 Power Electronics, 5 ECTS**

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# Course Objectives

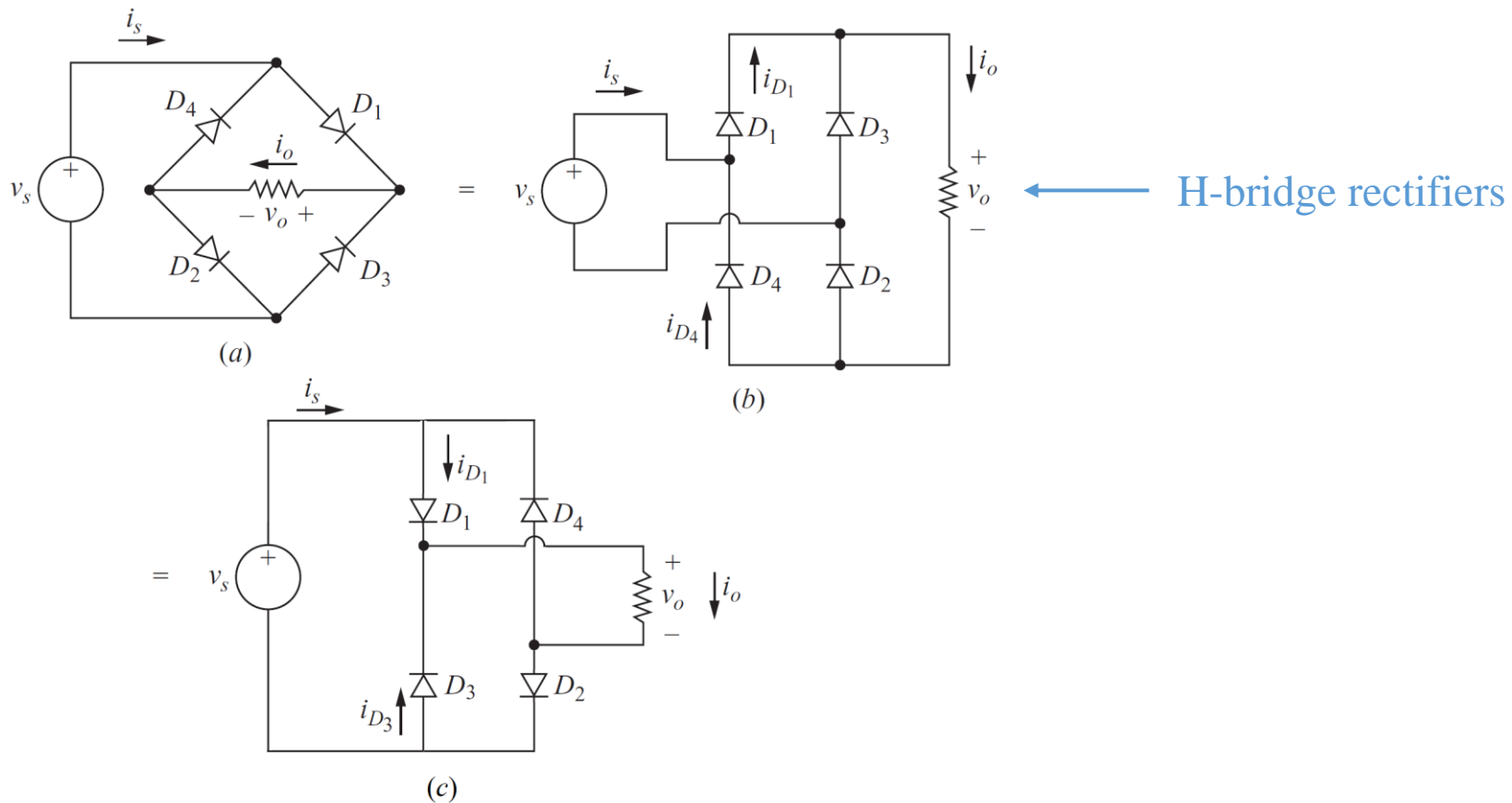
At the end of this course, you will be able to:

generally analyse the uncontrolled and controlled full-wave rectifiers with different loads and apply the power computation concepts from the previous chapter to these circuits.

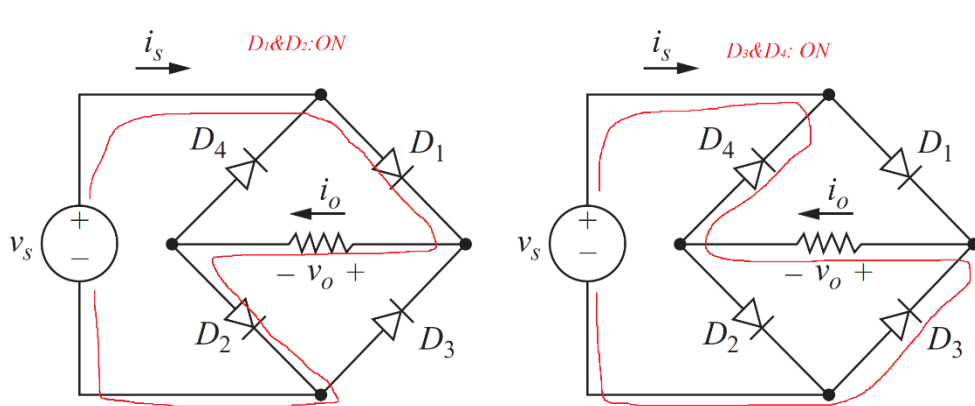
# Full-Wave Rectifiers

## Single-phase Full-Wave Rectifier:

- The objective of a full-wave rectifier is to produce a voltage or current that is **purely dc** or has **some specified dc component**.
- The source is utilized in **entire cycle** (not half a cycle as used in half-wave rectifiers).



**Figure 4-1:** Full-wave bridge rectifier. (a) Circuit diagram. (b&c) Alternative representation.



**Figure 4-2:** Voltages and currents of full-wave bridge rectifier.

1. Diodes  $D_1$  and  $D_2$  conduct together, and  $D_3$  and  $D_4$  conduct together. Kirchhoff's voltage shows that  $D_1$  and  $D_3$  cannot be ON at the same time (similarly,  $D_2$  and  $D_4$ ). The load current can be positive or zero but can never be negative.

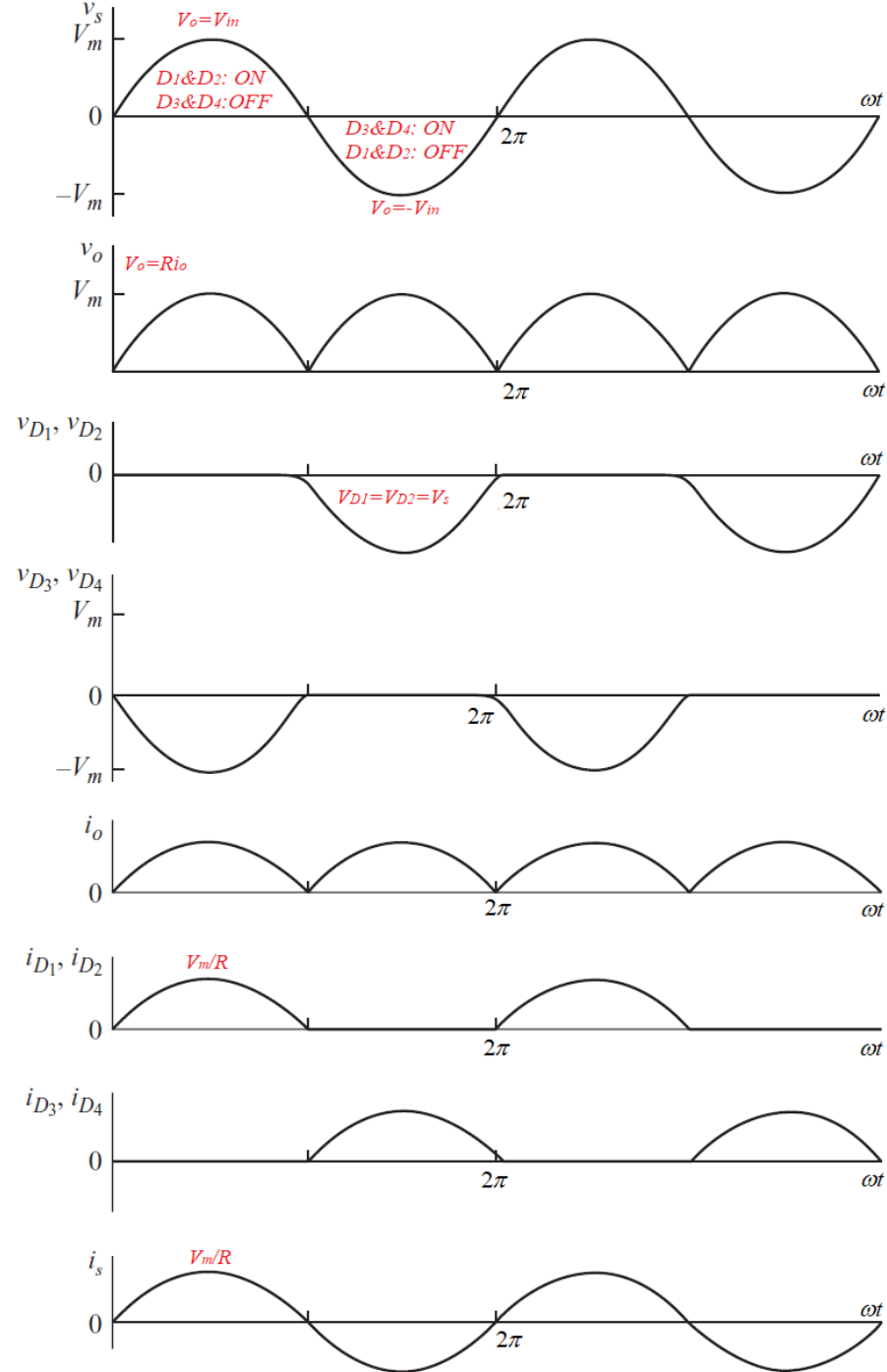
2. The voltage across the load is  $+v_s$  when  $D_1$  and  $D_2$  are ON. The voltage across the load is  $-v_s$  when  $D_3$  and  $D_4$  are ON.

3. The maximum voltage across a reverse-biased diode is the peak value of the source. This can be obtained by KVL around the loop containing the source,  $D_1$ , and  $D_3$ . With  $D_1$  ON, the voltage across  $D_3$  is  $-v_s$ .

4. The current entering the bridge from the source is  $i_{D1} - i_{D4}$ , which is symmetric about zero. Therefore, the average source current is zero.

5. The rms source current is the same as the rms load current. The source current is the same as the load current for one-half of the source period and is the negative of the load current for the other half. The squares of the load and source currents are the same, so the rms currents are equal.

6. The fundamental frequency of the output voltage is  $2\omega$  where  $\omega$  is the frequency of the ac input since two periods of the output occur for every period of the input.



$$V_o(\omega t) = |V_{in}(\omega t)|$$

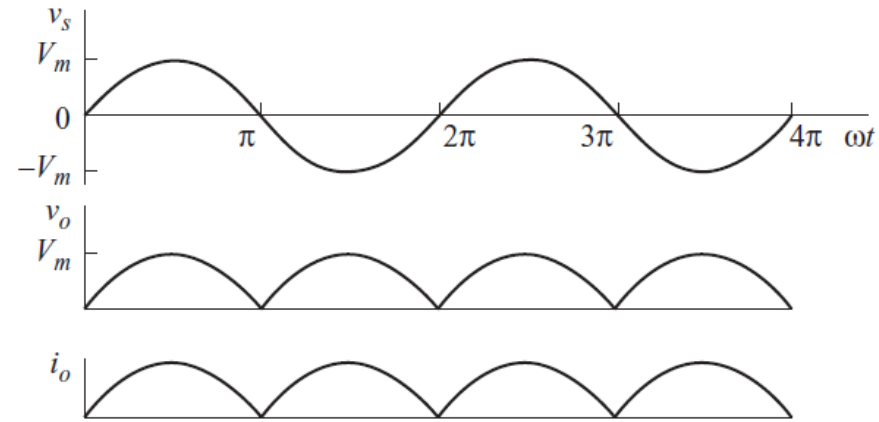
$$\langle V_o(\omega t) \rangle = \frac{1}{2\pi} \int_0^{2\pi} |V_m \sin(\omega t)| d(\omega t) = \frac{2}{2\pi} \int_0^{\pi} V_m \sin(\omega t) d(\omega t) = \frac{2V_m}{\pi}$$

$$\langle i_o(\omega t) \rangle = \frac{\langle V_o(\omega t) \rangle}{R} = \frac{2V_m}{\pi R}$$

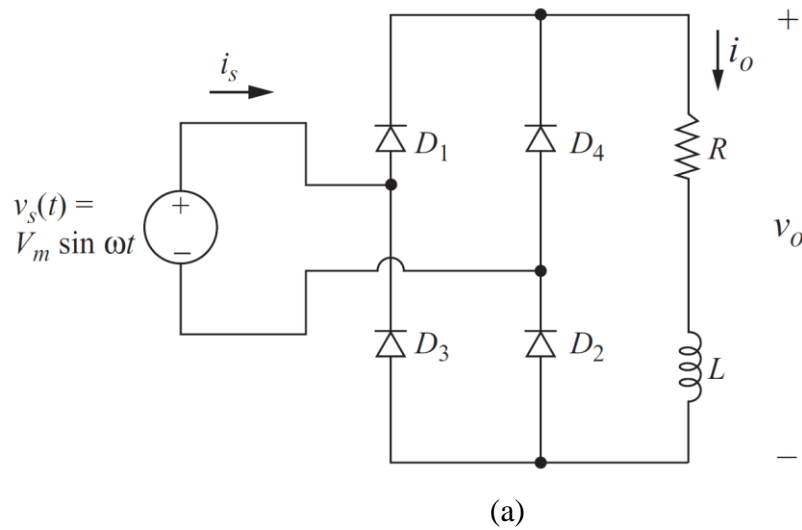
$$V_{o,rms} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} |V_m \sin(\omega t)|^2 d(\omega t)} = \sqrt{\frac{1}{2\pi} \times \frac{V_m^2}{2} \times 2\pi} = \frac{V_m}{\sqrt{2}}$$

$$I_{o,rms} = \frac{V_{o,rms}}{R} = \frac{V_m}{\sqrt{2}R} = \frac{I_m}{\sqrt{2}}$$

$$pf = \frac{\text{average output power}}{\text{apparent input power}} = \frac{R \times I_{o,rms}^2}{V_{in,rms} \times I_{in,rms}} = \frac{R \times \left(\frac{V_m}{\sqrt{2}R}\right)^2}{\frac{V_m}{\sqrt{2}} \times \frac{V_m}{\sqrt{2}R}} = 1$$



## Full-Bridge Rectifier with R-L load:

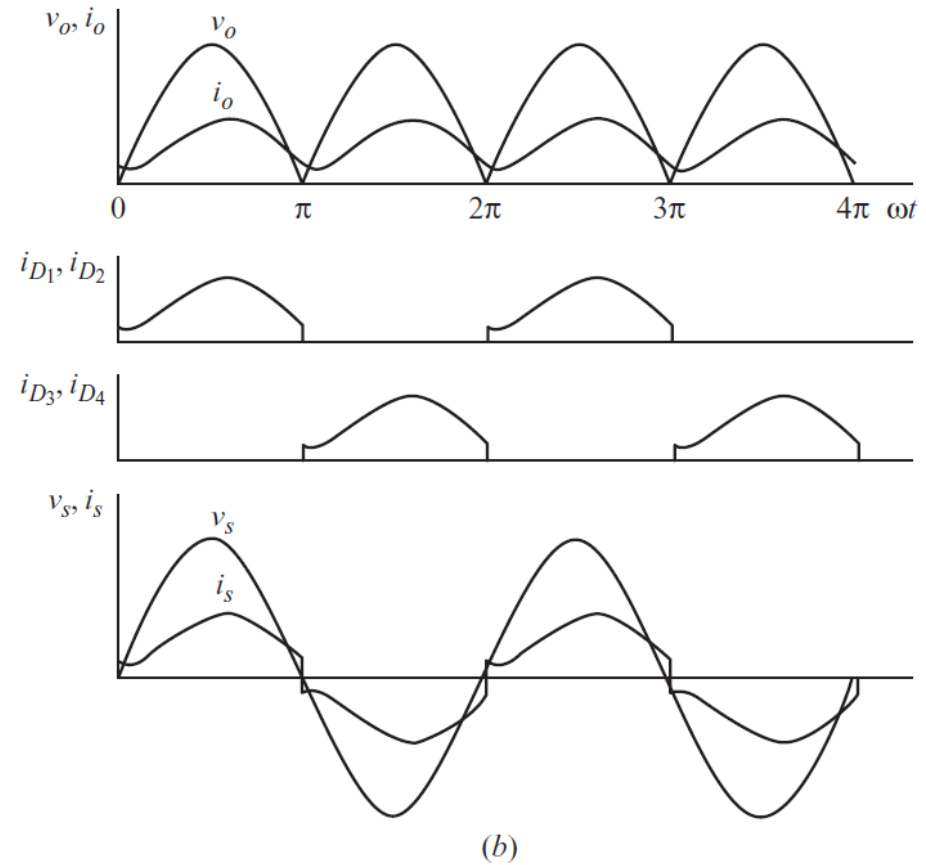


$$\text{if } v_s > 0 \rightarrow \begin{cases} D_1 \& D_2 \text{ are ON} \\ D_3 \& D_4 \text{ are OFF} \end{cases}$$

$$\text{if } v_s < 0 \rightarrow \begin{cases} D_1 \& D_2 \text{ are OFF} \\ D_3 \& D_4 \text{ are ON} \end{cases}$$

The  $v_o$  will be absolute value of  $v_s$  ( $v_o = |v_s|$ )

$$v_o = L \frac{di_o}{dt} + Ri_o \rightarrow |V_m \sin \omega t| = L \frac{di_o}{dt} + Ri_o$$

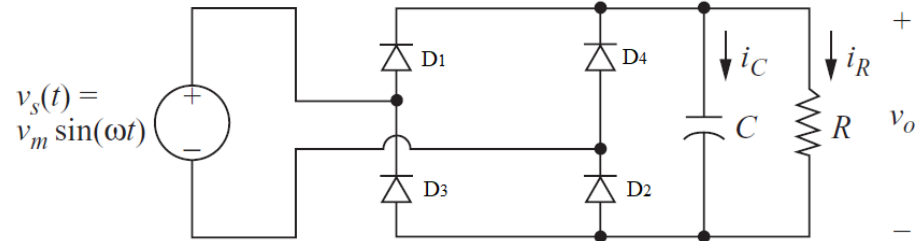


**Figure 4-3** (a) Bridge rectifier with an RL load; and (b) Voltages and currents.

## Full-Bridge Rectifier with R-C load:

$$\text{if } v_s > 0 \rightarrow \begin{cases} D_1 \& D_2 \text{ are ON} \\ D_3 \& D_4 \text{ are OFF} \end{cases}$$

$$\text{if } v_s < 0 \rightarrow \begin{cases} D_1 \& D_2 \text{ are OFF} \\ D_3 \& D_4 \text{ are ON} \end{cases}$$



The analysis proceeds exactly as for the half-wave rectifier.

$$v_o(\omega t) = \begin{cases} |V_m \sin \omega t| & \text{one diode pair on} \\ (V_m \sin \theta) e^{-(\omega t - \theta)/\omega RC} & \text{diodes off} \end{cases}$$

$\theta$  is the angle where the diodes become reverse biased

$$\theta = \tan^{-1}(-\omega RC) = -\tan^{-1}(\omega RC) + \pi$$

At that boundary point:

$$(V_m \sin \theta) e^{-(\pi + \alpha - \theta)/\omega RC} = -V_m \sin(\pi + \alpha)$$

or

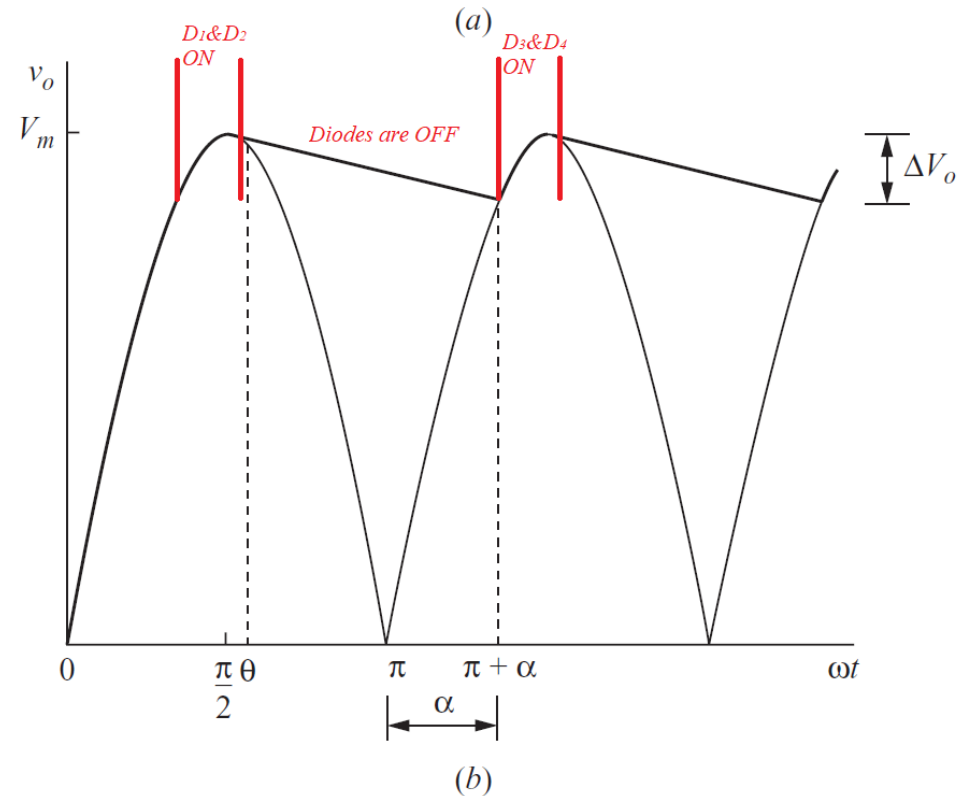
$$(\sin \theta) e^{-(\pi + \alpha - \theta)/\omega RC} - \sin \alpha = 0$$

The peak-to-peak voltage variation, or ripple is:

$$\Delta V_o = V_m - |V_m \sin(\pi + \alpha)| = V_m(1 - \sin \alpha)$$

In practical circuits where  $RC\omega \gg \pi$ :

$$\Delta V_o \approx \frac{V_m \pi}{\omega RC} = \frac{V_m}{2fRC}$$



**Figure 4-4** (a) Full-wave rectifier with capacitance filter; (b) Source and output voltage.

**Note that** the approximate peak-to-peak ripple voltage for the full-wave rectifier is one-half that of the half-wave rectifier.

**Example:** The full-wave rectifier of Fig. 4-4(a) has a 120 V source at 60 Hz,  $R=500 \Omega$ , and  $C=100 \mu\text{F}$ . (a) Determine the peak-to-peak voltage variation of the output. (b) Determine the value of capacitance that would reduce the output voltage ripple to 1 percent of the dc value.

**Solution**

$$V_m = 120\sqrt{2} = 169.7 \text{ V}$$

$$\omega RC = (2\pi \times 60)(500)(10)^{-6} = 18.85$$

The angle  $\theta$  is determined as:  $\theta = -\tan^{-1}(18.85) + \pi = 1.62 \text{ rad} = 93^\circ$

$$V_m \sin \theta = 169.7 \times \sin(93^\circ) = 169.5 \text{ V}$$

The angle  $\alpha$  is determined by the numerical solution of following equation:

$$(\sin \theta)e^{-(\pi+\alpha-\theta)/\omega RC} - \sin \alpha = 0 \Rightarrow \sin(1.62)e^{-(\pi+\alpha-1.62)/18.85} - \sin \alpha = 0$$

$$\alpha = 1.06 \text{ rad} = 60.6^\circ$$

(a) Peak-to-peak output voltage is described as:

$$\Delta V_o = V_m(1 - \sin \alpha) = 169.7[1 - \sin(1.06)] = 22 \text{ V}$$

Note that this ripple value is half of ripple value in a half-wave rectifier.

(b) With the ripple limited to 1 percent, the output voltage will be held close to  $V_m$ .

$$\Delta V_o \approx \frac{V_m}{2fRC} \Rightarrow \frac{\Delta V_o}{V_m} = 0.01 \approx \frac{1}{2fRC} \Rightarrow C \approx \frac{1}{2fR(\frac{\Delta V_o}{V_m})} = \frac{1}{2(60)(500)(0.01)} = 1670 \mu\text{F}$$



## Controlled Full-Bridge Rectifier:

Controlling is possible by substituting controlled switches such as thyristors (SCRs) for the diodes.

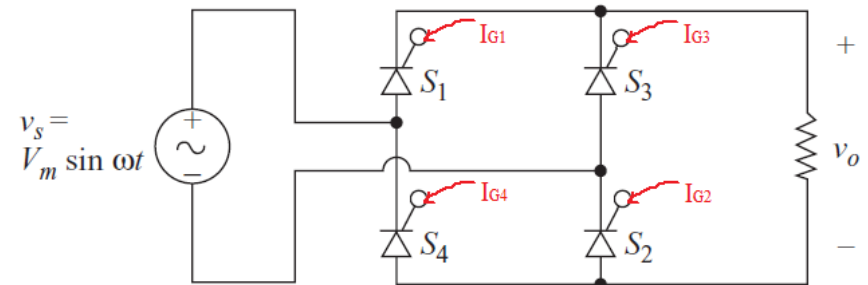
$I_{G1}$  and  $I_{G2}$  are sent to  $S_1$  and  $S_2$  and turn them ON at the same time. With 180 degree phase shift triggering happening for  $I_{G3}$  and  $I_{G4}$ .

Average output voltage is:

$$V_o = \frac{1}{\pi} \int_{\alpha}^{\pi} V_m \sin(\omega t) d(\omega t) = \frac{V_m}{\pi} (1 + \cos \alpha)$$

Average output current is then:

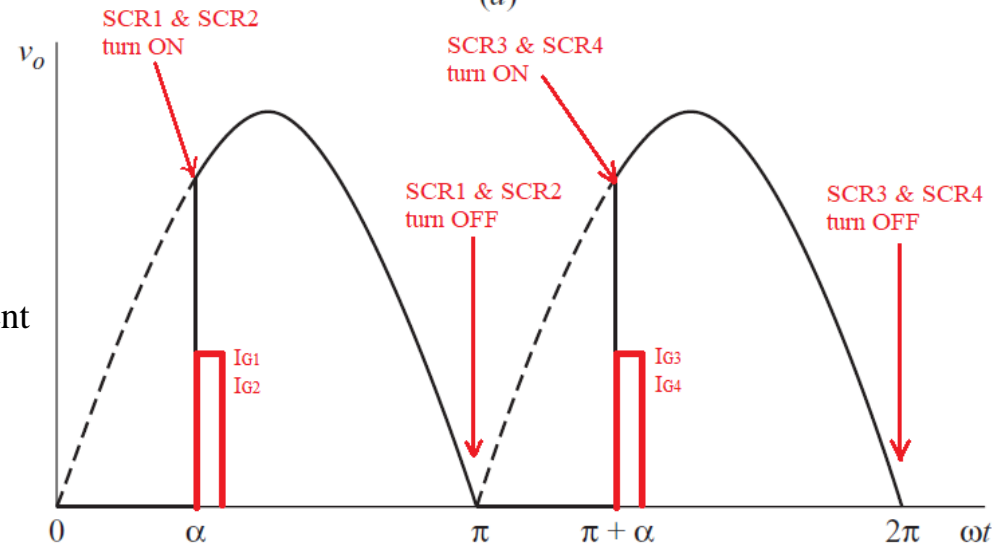
$$I_o = \frac{V_o}{R} = \frac{V_m}{\pi R} (1 + \cos \alpha)$$



(a)

$$I_{\text{rms}} = \sqrt{\frac{1}{\pi} \int_{\alpha}^{\pi} \left( \frac{V_m}{R} \sin \omega t \right)^2 d(\omega t)} = \frac{V_m}{R} \sqrt{\frac{1}{2} - \frac{\alpha}{2\pi} + \frac{\sin(2\alpha)}{4\pi}}$$

The rms current in the source is the same as the rms current in the load.



(b)

**Figure 4-5** (a) Controlled full-wave bridge rectifier; (b) Output for a resistive load.

**Example:** The full-wave controlled bridge rectifier of Fig. 4-5 (a) has an ac input of 120 V rms at 60 Hz and a 20-Ω load resistor. The delay angle is 40 degree. Determine the average current in the load, the power absorbed by the load, and the source voltamperes

**Solution:**

The average output voltage is determined from

$$V_o = \frac{V_m}{\pi} (1 + \cos \alpha) = \frac{\sqrt{2}(120)}{\pi} (1 + \cos 40^\circ) = 95.4 \text{ V}$$

Average load current is

$$I_o = \frac{V_o}{R} = \frac{95.4}{20} = 4.77 \text{ A}$$

Power absorbed by the load is determined from the rms current

$$I_{\text{rms}} = \frac{\sqrt{2}(120)}{20} \sqrt{\frac{1}{2} - \frac{0.698}{2\pi} + \frac{\sin[2(0.698)]}{4\pi}} = 5.80 \text{ A}$$

$$P = I_{\text{rms}}^2 R = (5.80)^2 (20) = 673 \text{ W}$$

The rms current in the source is also 5.80 A, and the apparent power of the source is

$$S = V_{\text{rms}} I_{\text{rms}} = (120)(5.80) = 696 \text{ VA}$$

Power factor is

$$\text{pf} = \frac{P}{S} = \frac{672}{696} = 0.967$$

## Controlled Full-Bridge Rectifier R-L load:

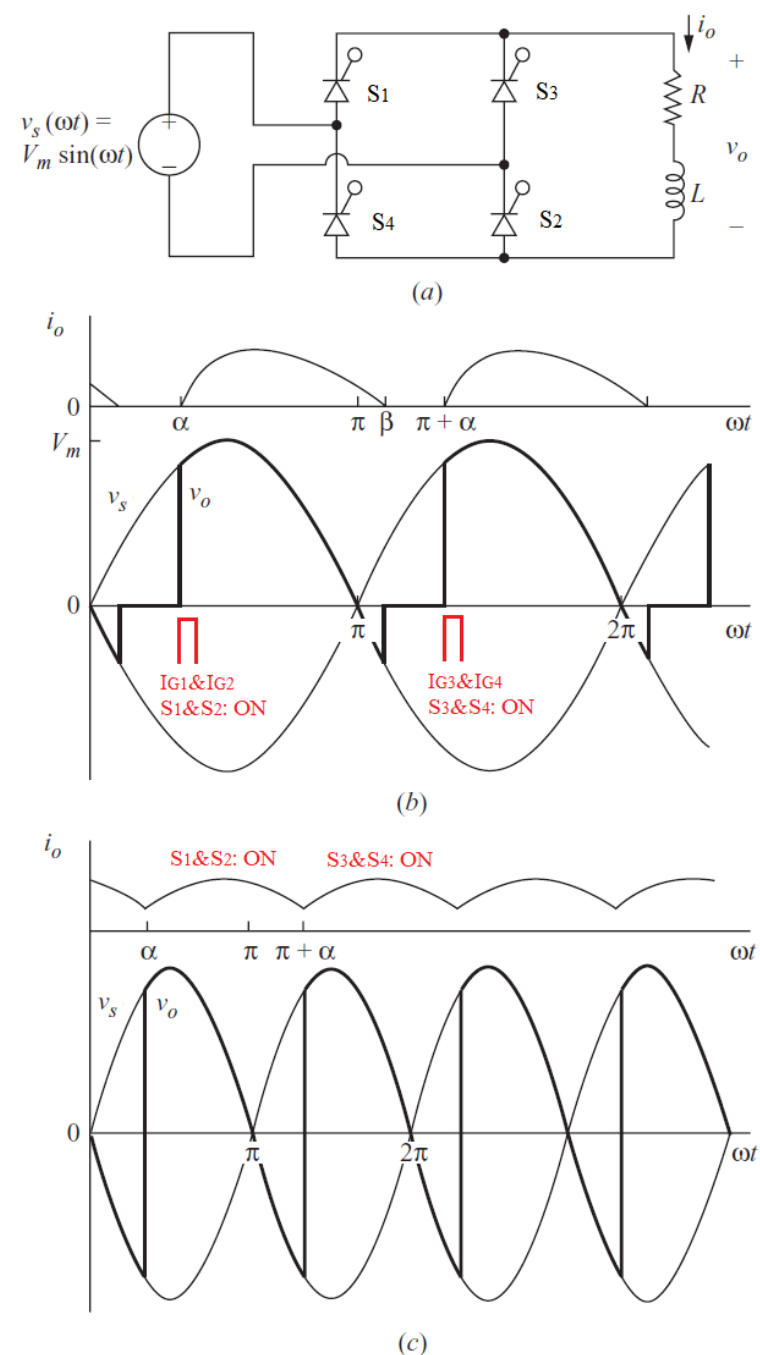
**Case 1:** Inductor is small or firing angle  $\alpha$  is large, so the output current is discontinuous. It means that it starts from zero, goes somewhere up and comes back to zero and remains there for the next half of the cycle to repeat.

**Case 2:** Inductor is large or firing angle  $\alpha$  is small, so the output current is continuous and current continuously flow through in load (before the current reaches zero, the next triggering starts).

How do we understand whether we are in continuous mode or in discontinuous mode?

if  $\alpha < \tan^{-1}\left(\frac{L\omega}{R}\right) \Rightarrow$  Continuous Current Happens

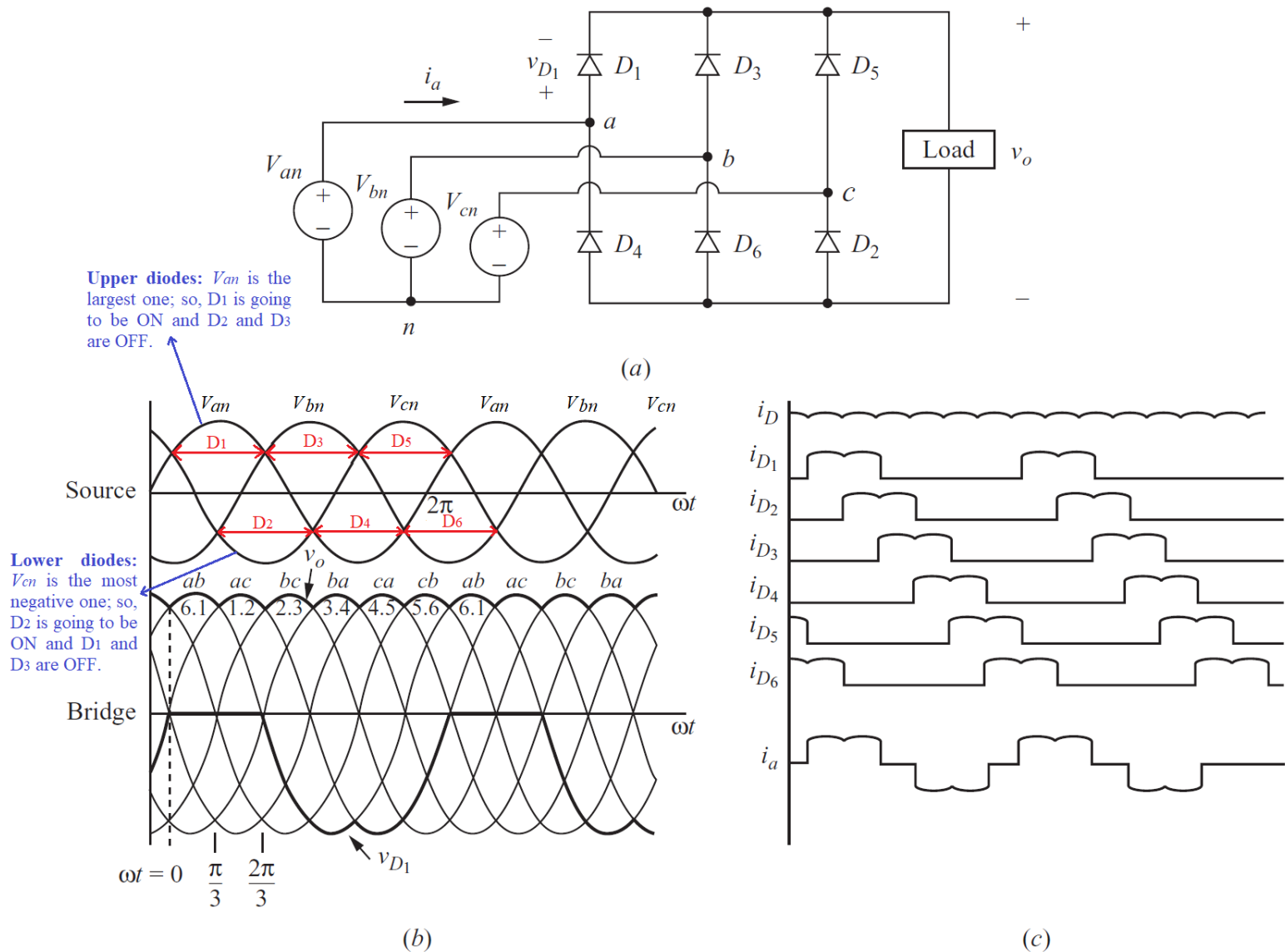
otherwise: Discontinuous Current Happens



**Figure 4-6** (a) Controlled rectifier with  $RL$  load; (b) Discontinuous current; (c) Continuous current.

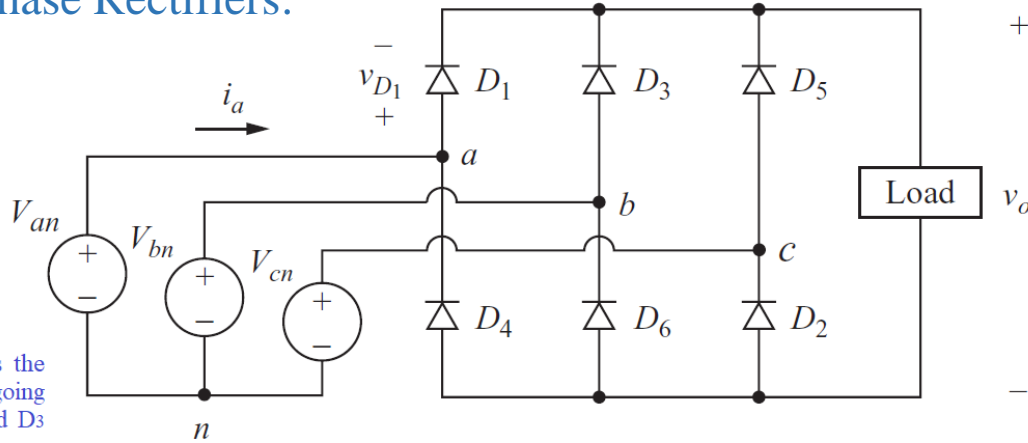
## Three-Phase Rectifiers:

Three-phase rectifiers are commonly used in industry to produce a dc voltage and current for large loads.

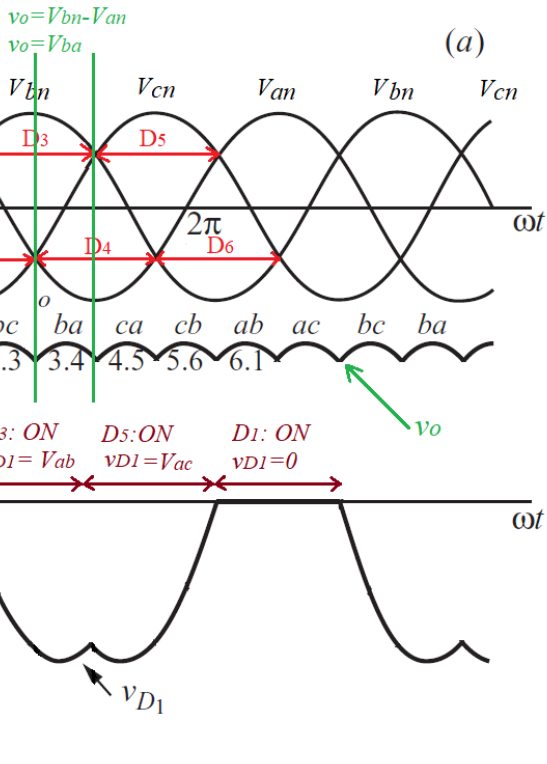


**Figure 4-7** (a) Three-phase full-bridge rectifier; (b) Source and output voltages; (c) Currents for a resistive load.

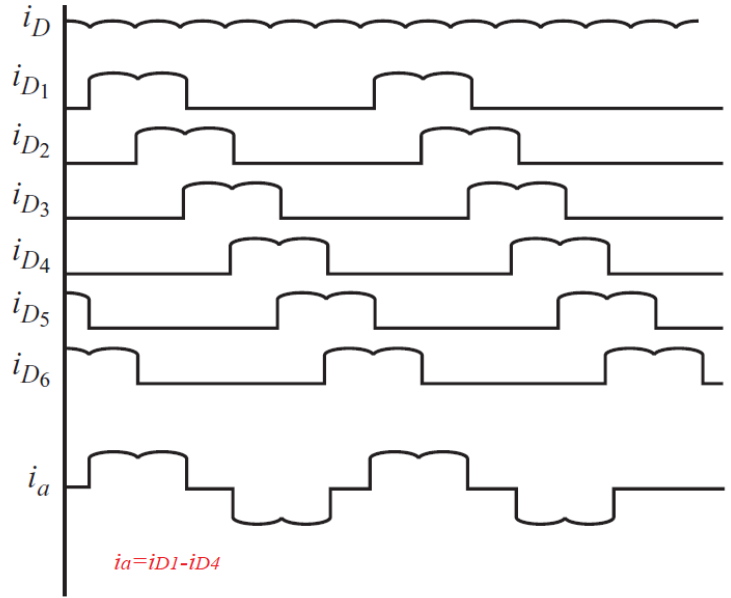
# Three-Phase Rectifiers:



**Upper diodes:**  $V_{an}$  is the largest one; so,  $D_1$  is going to be ON and  $D_2$  and  $D_3$  are OFF.



**Lower diodes:**  $V_{cn}$  is the most negative one; so,  $D_2$  is going to be ON and  $D_1$  and  $D_3$  are OFF.



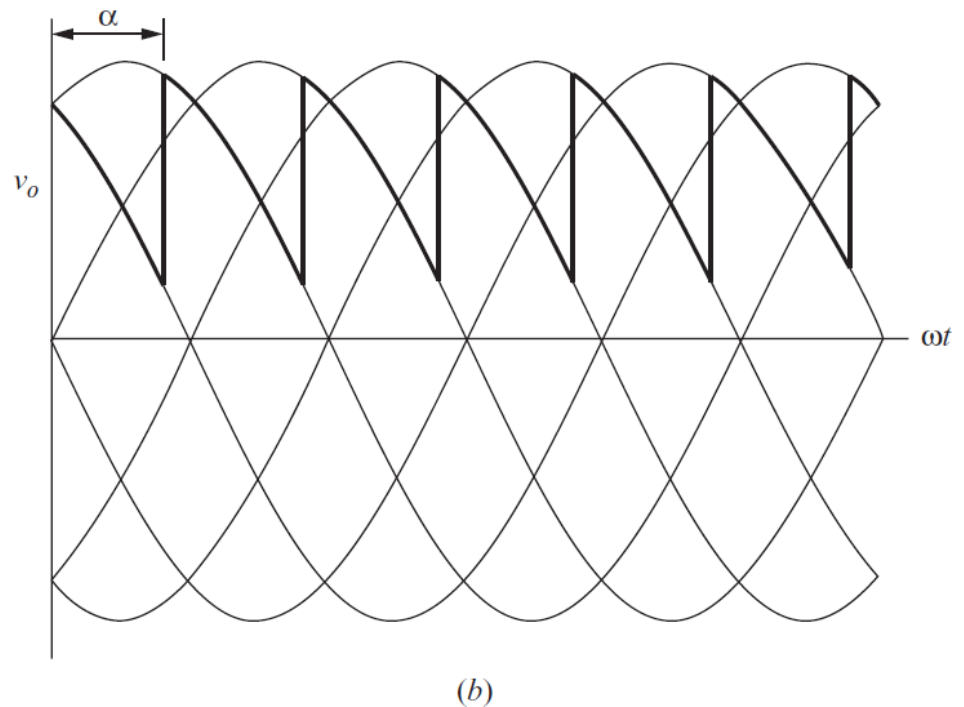
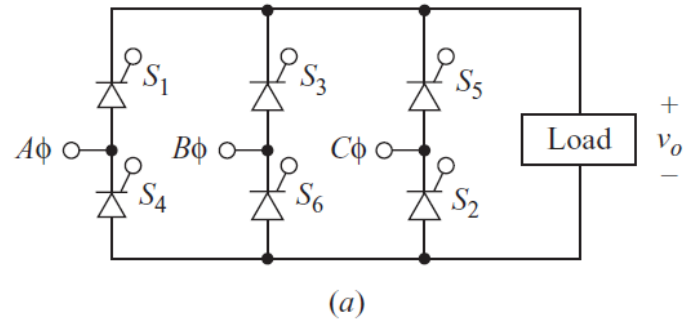
(b)

(c)

**Figure 4-8** (a) Three-phase full-bridge rectifier; (b) Source and output voltages; (c) Currents for a resistive load.

## Controlled Three-Phase Rectifier:

The output of the three-phase rectifier can be controlled by substituting SCRs for diodes.



**Figure 4-9** (a) A controlled three-phase rectifier; (b) Output voltage for  $\alpha = 45^\circ$ .

**Questions and comments are  
most welcome!**

