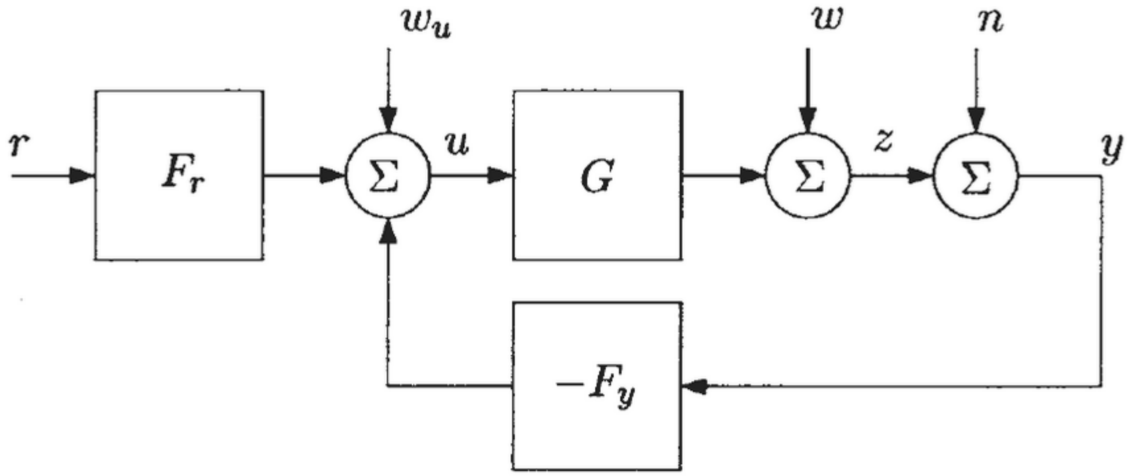
Exercise 1.



Assume that Fy = Fr = K

$$\begin{cases} y = z + n \\ z = Gu + w \\ u = Kr - Ky + w_u \end{cases}$$

$$u = Kr - Kz - Kn + w_u$$

$$z = GKr - GKz - GKn + Gw_u + w$$

$$z(I + GK) = GKr - GKn + Gw_u + w$$

$$z = \underbrace{(I+GK)^{-1}GK}_{G_c}\, r - \underbrace{(I+GK)^{-1}GK}_{T}\, n + \underset{L}{(I+GK)^{-1}G}\, w_u + \underbrace{(I+GK)^{-1}}_{S}\, w$$

Exericise 2

The transfer functions from the disturbance signals to the outputs y and u.

$$\begin{cases} e = r - y = -y \\ u = Ke + d_u = -Ky + d_u \\ y = Gu + d_y = G(-Ky + d_u) + d_y = -GKy + Gd_u + d_y \end{cases}$$

$$\Leftrightarrow y + GKy = Gd_u + d_y$$

$$\Leftrightarrow y(I + GK) = Gd_u + d_y$$

$$\Leftrightarrow y = (I + GK)^{-1} Gd_u + (I + GK)^{-1} d_y = G(I + KG)^{-1} d_u + (I + GK)^{-1} d_y$$

similarly for u

$$u = -Ky + d_u = -K(Gu + d_y) + d_u = -KGu - Kd_y + d_u$$

$$\Leftrightarrow u + KGu = -Kd_y + d_u$$

$$\Leftrightarrow u(I + KG) = -Kd_y + d_u$$

$$\Leftrightarrow u = -(I + KG)^{-1} Kd_y + (I + KG)^{-1} d_u = (I + KG)^{-1} d_u - K(I + GK)^{-1} d_y$$

Hence

$$u = (I + KG)^{-1} d_u - K(I + GK)^{-1} d_y$$

$$y = G(I + KG)^{-1} d_u + (I + GK)^{-1} d_y$$

Exercise 3

Transfer function of the system

$$C_v s \delta T(s) = \delta Q(s) + \alpha \delta T_0(s) - \alpha \delta T(s)$$

$$C_v s \delta T(s) + \alpha \delta T(s) = \delta Q(s) + \alpha \delta T_0(s)$$

$$\delta T(s)(C_v s + \alpha) = \delta Q(s) + \alpha \delta T_0(s)$$

$$\delta T(s) \frac{(C_v s + \alpha)}{\alpha} = \frac{\delta Q(s) + \alpha \delta T_0(s)}{\alpha}$$

$$\delta T(s) \frac{(C_v s + \alpha)}{\alpha} = \frac{1}{\alpha} \delta Q(s) + \delta T_0(s)$$

$$\delta T(s) \left(\underbrace{\frac{C_v}{\alpha}}_{\tau} s + 1 \right) = \left[\frac{1}{\alpha} \delta Q(s) + \delta T_0(s) \right]$$

$$\delta T(s) = \frac{1}{\tau s + 1} \left[\frac{1}{\alpha} \delta Q(s) + \delta T_0(s) \right]$$

The scaled form of the process is

$$y(s) = G(s)u(s) + G_d(s)d(s)$$

Transfer function in expanded form

$$\delta T(s) = \frac{1}{\tau s+1}\frac{1}{\alpha}\delta Q(s) + \frac{1}{\tau s+1}\delta T_0(s)$$

By obtaining transfer functions with respect to Q and T0, we can calculate the scaled transfer functions

$$G_Q(s) = \frac{\delta T(s)}{\delta Q(s)} = \frac{1}{\tau s+1}\frac{1}{\alpha}$$

$$\text{if } y(s) = \frac{\delta T(s)}{\delta T_{max}} \text{ and } u(s) = \frac{\delta Q(s)}{\delta Q_{max}}$$

$$\text{then } G(s) = \frac{y(s)}{u(s)} = \frac{\dfrac{\delta T(s)}{\delta T_{max}}}{\dfrac{\delta Q(s)}{\delta Q_{max}}} = \frac{\delta T(s)}{\delta T_{max}}\frac{\delta Q_{max}}{\delta Q(s)} = \underbrace{\frac{\delta T(s)}{\delta Q(s)}}_{G_Q(s)}\frac{\delta Q_{max}}{\delta T_{max}} = \frac{1}{\tau s+1}\frac{\delta Q_{max}}{\delta T_{max}}\frac{1}{\alpha}$$

$$G_{T_0}(s) = \frac{\delta T(s)}{\delta T_0(s)} = \frac{1}{\tau s+1}$$

$$\text{if } y(s) = \frac{\delta T(s)}{\delta T_{max}} \text{ and } d(s) = \frac{\delta T_0(s)}{\delta T_{0,max}}$$

$$\text{then } G_d(s) = \frac{y(s)}{d(s)} = \frac{\dfrac{\delta T(s)}{\delta T_{max}}}{\dfrac{\delta T_0(s)}{\delta T_{0,max}}} = \frac{\delta T(s)}{\delta T_{max}}\frac{\delta T_{0,max}}{\delta T_0(s)} = \underbrace{\frac{\delta T(s)}{\delta T_0(s)}}_{G_{T_0}(s)}\frac{\delta T_{0,max}}{\delta T_{max}} = \frac{1}{\tau s+1}\frac{\delta T_{0,max}}{\delta T_{max}}$$