31E11100 - Microeconomics: Pricing
Re-take Exam, December 12, 2022
Pauli Murto
Please answer the questions below. Answer shortly but justify your answers and explain accurately what you are doing. If you are confused about some question statement, please explain clearly what you assume when answering.

The maximum total points is 40 (which makes $40 \%$ of the total grade of the course). There is no need for a calculator.

1. (12 points) A monopolist produces a good with constant marginal cost $c$, where $0<c<1$. There is a population of consumers of size 1 .
(a) Assume that all the consumers have identical demand $Q(p)=1-p$ and the monopolist can only charge a uniform price per unit (i.e. a linear price). Find the profit maximizing price and compute the profit of the monopolist.
(b) Suppose that the monopolist can charge a two-part tariff $(f, p)$, where $f$ is the fixed fee and $p$ is the price per unit. Find the twopart tariff that maximizes the profit and compute the profit of the monopolist. Discuss the welfare effects of the two-part tariff (i.e. compare the profits and consumer surplus between a) and b)).
(c) Assume now that there are two types of consumers, but the monopolist cannot discriminate among the consumers. The consumers of type 1 have demand $Q_{1}(p)=1-p$ and consumers of type 2 have demand $Q_{2}(p)=1-\frac{p}{2}$. There are equally many consumers of both types. Assume here that marginal cost is $c=\frac{1}{2}$. Find the two-part tariff that maximizes the profit of the monopolist and discuss the properties of the solution.
2. (12 points) This question is about behavior based price discrimination. A monopolist sells a product repeatedly to a unit mass of heterogenous
consumers. There are two periods, $t=1,2$. Each consumer $i$ has the same value $v_{i}$ in both periods, but this value is private information of the consumer, and is uniformly distributed across the consumers, i.e. $v_{i} \sim U[0,1]$. In each period, each consumer $i$ may either buy or not buy the product, enjoying consumption utility $v_{i}$ at each period when she buys. The seller maximizes the sum of profits over the two periods and consumers maximize the sum of their consumer surplus over the two periods.
(a) Assume first that the seller cannot identify the tastes of individuals and must set a uniform price to all consumers in both periods. Compute the optimal pricing strategy and profit of the seller.
(b) Assume next that the seller cannot identify the tastes of individuals in the first period, but in the second period the firm can tell apart those consumers who bought in the first period from those who did not buy in the first period. Therefore, the pricing strategy of the seller consists of setting a uniform price $p_{1}$ to all consumer in the first period, and setting two different prices $p_{2}^{+}$and $p_{2}^{-}$in the second period, where $p_{2}^{+}$is the price set to consumers who bought in the first period, and $p_{2}^{-}$is the price set to those consumers who did not buy in the first period. Assume further that the consumers are naive in the sense that they make their purchase decision in the first period without considering the consequences of their decision for the price that they face in the second period. Compute the optimal pricing strategy and profit of the seller, and compare to case a). Are the consumers on average better or worse off in b) compared to a)?
(c) Suppose that the monopolist tries to implement the pricing strategy that you derived in b), but consumers are forward looking and understand the implication of their first-period consumption choice for the next period. How would consumers react to the
pricing structure in b)? Is the seller better off here than in a)?
3. (16 points) A monopolist firm produces a homogenous good at constant marginal cost $c=1$, so that the production cost function is given by

$$
c(q)=q, q \geq 0 .
$$

The utility of a consumer of type $\theta$, who consumes quantity $q$ and pays tranfer $t$, is given by:

$$
u(\theta, q, t)=\theta \sqrt{q}-t .
$$

(a) Find the first-best level of $q$ as a function of $\theta$, where $\theta>0$. (i.e. find the level of $q$ that maximizes the total surplus)
(b) Suppose there are two types of buyers: $\theta^{H}=4, \theta^{L}=2$, and the fraction of high type buyers is $\lambda$. If the monopolist can perfectly identify each consumer's type, what would be the optimal quantity-price pair $\left(q^{H}, t^{H}\right)$ for consumer type $\theta^{H}$, and correspondingly to $\theta^{L}$ ?
(c) Suppose now that the seller cannot identify individual consumers' types, but can offer a menu $\left\{\left(q^{H}, t^{H}\right),\left(q^{L}, t^{L}\right)\right\}$ consisting of two quantity-price pairs and lets buyers select from those. Write down the IC and IR constraints for both types of buyers and discuss verbally which of those should be binding when the monopolist chooses the menu $\left\{\left(q^{H}, t^{H}\right),\left(q^{L}, t^{L}\right)\right\}$ optimally.
(d) Explain how the optimal menu can be solved, and solve it explicitly if you can. Describe qualitatively the optimal solution and the resulting distortions. What is the nature of the solution if $\lambda$ is large (i.e. close to one)?

