31E11100 - Microeconomics: Pricing Re-take exam, December 20, 2018 Pauli Murto

Please answer the questions below. Answer shortly but justify your answers and explain accurately what you are doing. If you are confused about some question statement, please explain clearly what you assume when answering.

The maximum total points is 60. There is no need for a calculator or a dictionary.

- 1. (15 points) Answer the following questions verbally. Give short answers and try to be as accurate as possible.
  - (a) Discuss the different forms of price discrimination. What are their informational requirements? What kind of effects do they have for profits, total surplus, and consumer surplus?
  - (b) Explain what is meant by winner's curse. In what kind of auctions can it arise? How should bidders take it into account in their bidding behavior?
  - (c) Discuss the problem of a monopolist that produces a durable good over time. What makes dynamic price discrimination difficult for the monopolist? What kind of a commitment problem is there, and how could the monopolist solve it?
- 2. (15 points) A firm can produce two versions of a product, standard and premium, where  $q \in \{q^s, q^p\}$  denotes the fixed quality level of standard and premium product, respectively. The production cost is the same for both versions, and for simplicity assume that this cost is zero. There are two types of consumers, low type (L) and high type (H). The consumers have a unit demand (i.e. they buy at most one product), and their valuation for a product depends both on the product version and consumer's type in the following way. The utility for the standard

quality product is 4 for the low type and 5 for the high type, i.e.  $\theta_s^L = 4$ and  $\theta_s^H = 5$ . The utility for the premium quality product is 6 for the low type and 8 for the high type, i.e.  $\theta_p^L = 6$  and  $\theta_p^H = 8$ . Let  $\lambda$  denote the fraction of high type consumers in the population.

- (a) Suppose that the firm can perfectly discriminate the two types of buyers. Which product version would she offer to each type of consumer, and at what price?
- (b) Suppose that the firm cannot identify the types of buyers. What happens if she tries to implement the same outcome as in a), i.e. offer the two product versions at the prices that you derived in a) and let each consumer choose which of the two versions to buy?
- (c) Derive the optimal pricing strategy of the monopolist when she cannot identify the types of individual buyers. Discuss the solution and explain how it depends on  $\lambda$  (to get started, you may first consider optimal solution in the cases  $\lambda = 0$  and  $\lambda = 1$ ). For what values of  $\lambda$  is it optimal to sell both versions?
- 3. (15 points) A monopolist produces two products, A and B, at zero cost. There is a unit mass of consumers. Each consumer is identified by type  $(\theta_A, \theta_B)$ , where  $\theta_A$  and  $\theta_B$  are the consumer's respective valuations for goods A and B. Assume that consumer valuations for the two products are uniformly distributed over the unit square. This means that the valuations  $\theta_A$  and  $\theta_B$  for an individual consumer are independently and uniformly distributed over [0, 1]. Each consumer can buy either no product, one product, or both products.
  - (a) Suppose that the monopolist sells the two products separately, i.e. sets separate prices  $p_A$  and  $p_B$  for the two products. Describe the consumer types that buy product A only, product B only, or both products (you may find it useful to use a drawing on a unit square). Derive the demand for each product for the seller (i.e.

quantities of A and B sold at prices  $p_A$  and  $p_B$ ). What are the optimal prices and what is the resulting total profit?

- (b) Suppose that the monopolist bundles the two goods together, and sells them together at a single price  $p_{AB}$ . Describe the consumer types that buy the bundle. Derive the demand for the bundle and solve for the optimal bundle price and resulting profit. Compare to the result in a) and discuss.
- (c) Suppose that the monopolist sells products A and B separately at prices  $p_A$  and  $p_B$ , respectively, and in addition offers a bundle consisting of both products at price  $p_{AB}$ . You can assume that the monopolist always wants to set  $p_A = p_B \equiv p_S$ . Derive the demand for products A and B sold separately and for the bundle consisting of both products (again, use drawing). Argue that by choosing  $p_S$  and  $p_{AB}$  optimally, the monopolist can get a higher profit than in a) or b). Solve the model as far as possibe and discuss the result.
- 4. (15 points) A monopoly firm operates in a market with a sesonally varying demand. The inverse demand function in season  $i \in \{s, w\}$  is given by:

$$p^i = \alpha^i - \beta^i q^i,$$

where  $\alpha^s = 18$ ,  $\alpha^w = 14$ ,  $\beta^s = 2$ ,  $\beta^w = 3$ . There is a constant marginal cost of production so that producing  $q^i$  units in season *i* costs  $cq^i$ , where c = 2.

- (a) Assume first that there is no capacity constraint for the firm. What is the optimal production level and corresponding price for each season?
- (b) Keep on assuming that there is no capacity constraint for the firm. If the firm must use the same price in both seasons, what is the

total quantity sold over the two seasons for given price p? What is the optimal price p?

- (c) Assume next that there is a fixed capacity level k. The capacity sets the maximum output that the firm can produce in either season:  $q^i \leq k$  for  $i \in \{s, w\}$ . What is the optimal production level and corresponding price for each season as a function of k?
- (d) Assume now that the firm must choose its production capacity to serve the market, and it costs fk to build k units of capacity. The firm maximizes its total profits over the two seasons, and can choose different prices in the two seasons. For what values of f is the capacity constraint binding in the high season only?
- (e) Solve the optimal capacity and supply levels for the two seasons both in the case where the capacity constraint binds only in one season and where it binds in both seasons. Discuss the solution.