31E11100 - Microeconomics: Pricing

Exam, October 22, 2018

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Please answer the questions below. Answer shortly but justify your answers and explain accurately what you are doing. If you are confused about some question statement, please explain clearly what you assume when answering.

The maximum total points is 60. There is no need for a calculator or a dictionary.

1. (15 points) Answer the following questions verbally. Give short answers and try to be as accurate as possible.

   (a) We know from empirical studies that there is substantial price dispersion for homogenous products in many markets. How can such findings be explained? Does price dispersion necessarily imply that some firms are making higher profits than others? Why/why not?

   (b) What is the Revenue Equivalence Theorem? Give an example where it applies, and explain what it implies in your example.

   (c) Discuss the problem of a monopolist that produces a durable good over time. What makes dynamic price discrimination difficult for the monopolist? What kind of a commitment problem is there, and how could the monopolist solve it?

2. (15 points) A monopolist produces a homogeneous good at zero marginal cost. Consider the optimal pricing of the monopolist in the following situations.

   (a) There is a unit mass of consumers indexed by parameter θ, where θ is uniformly distributed on $[0, 1]$. The consumer of type θ has a unit demand with reservation value θ. In other words, if the monopolist sets price p, then all consumers with $\theta \geq p$ buy. Derive the aggregate demand $Q(p)$, i.e. the total quantity bought at price p.

(b) If the monopolist must set a uniform linear price to all consumers, what is the optimal price? What is the profit and aggregate consumer surplus in the market?

(c) If the monopolist can use first-degree price discrimination, what will she do? What is the total profit and aggregate consumer surplus in the market?

(d) Assume now that there is a unit mass of *identical* consumers, where each consumer has an individual demand function given by

$$q(p) = 2 - p.$$

What is the optimal linear price by the profit-maximizing monopolist? Give a utility function $v(q)$ of the consumer that is consistent with this demand.

(e) Continue with the demand of part (d). Suppose that the monopolist can use a two-part tariff: a fixed fee plus a price per unit. What would be the optimal two-part tariff? What is the profit and consumer surplus?

3. (15 points) A monopolist firm produces a homogenous good at constant marginal cost $c = 1$, so that the production cost function is given by

$$c(q) = q, \ \ q \geq 0.$$

The utility of a consumer of type θ, who consumes quantity q and pays tranfer t, is given by:

$$u(\theta, q, t) = \theta\sqrt{q} - t.$$

(a) Find the first-best level of q as a function of θ, where $\theta > 0$. (i.e. find the level of q that maximizes the total surplus)

(b) Suppose there are two types of buyers: $\theta^H = 4$, $\theta^L = 2$, and the fraction of high type buyers is λ. If the monopolist can perfectly identify each consumer's type, what would be the optimal quantity-price pair (q^H, t^H) for consumer type θ^H, and correspondingly to θ^L?

(c) Suppose now that the seller cannot identify individual consumers' types. Suppose she offers a menu $\{(q^H, t^H), (q^L, t^L)\}$ consisting of the two quantity-price pairs that you derived in (b) and lets buyers select. What happens? What is the revenue for the seller? What is the rent to the buyers?

(d) Write down the IC and IR constraints for both types of buyers, and discuss verbally which of those should be binding when the monopolist chooses the menu $\{(q^H, t^H), (q^L, t^L)\}$ optimally.

(e) Explain how the optimal menu can be solved, and solve it explicitly if you can. Describe qualitatively the optimal solution and the resulting distortions. What is the nature of the solution if λ is large (i.e. close to one)?

4. (15 points) A seller has a single indivisible item to sell. The buyers have a unit demand for the item and their valuations are independently drawn from the uniform distribution on $[0, 1]$. We consider here alternative mechanisms that the seller can use to sell the object.

(a) To begin, suppose that there is just one buyer, and the seller must use a posted-price mechanism. This means that the seller fixes the price, and the buyer then either buys or not. What is the optimal posted price and what is the expected revenue for the seller?

(b) From now on, assume that there are two buyers with indepenently drawn valuations. What is the optimal posted price for the seller in this case? (if both buyers have a valuation above the posted price, then the item is allocated randomly to one of the buyers at the posted price).

(c) Suppose the seller uses a standard second price auction. What do buyers bid in equilibrium? What is the resulting expected revenue for the seller?

(d) Can the seller increase profits by adding a reserve price in the auction? Discuss why this is the case. How does the optimal reserve price compare to the optimal posted price that you have derived in (b)?

(e) Suppose that the seller approaches the two buyers sequentially with posted price offers. In other words, first she approaches the first buyer with a take-it-or-leave it offer at price p_1. If the first buyer accepts, then the item is sold at that price. If the first buyer rejects, then the seller approaches the second buyer with a take-it-or-leave-it offer at price p_2. If this is accepted the item is sold, otherwise the item is left unsold. What is the optimal pair (p_1, p_2) of posted prices in this case, and what is the expected revenue of the seller? Discuss the intuition for your finding.