31E11100 - Microeconomics: Pricing
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Problem Set 1 (for the exercise session on September 24)

1. An individual consumer has quasi-linear preferences with utility of consumption $v(q)$ given by

$$v(q) = \begin{cases} q - \frac{q^2}{2} \text{ for } 0 \leq q \leq 1 \\ \frac{1}{2} \text{ for } q > 1 \end{cases}.$$

   (a) Derive the individual demand function $q(p)$ of this consumer.

   (b) Derive the optimal linear pricing strategy of a monopolist who faces such a consumer, and who has cost function

$$c(q) = cq,$$

   where $0 < c < 1$.

   (c) Suppose the monopolist can use a two-part tariff, i.e. a fixed fee plus a linear price component. Derive the optimal two-part tariff.

2. A buyer has a unit demand and valuation $v = 1$ for a product. There is a large number of sellers in the market, and it is assumed that the prices offered by the sellers are independently distributed according to a uniform distribution in $[0, 1]$. However, to get a price quote from a seller, the buyer must incur a search cost $c > 0$ (per price quote). Given multiple price quotes, the buyer chooses the lowest price.

   (a) Suppose that the buyer chooses to get only 1 price quote. What is the expected price that she pays, and what is her expected total payoff? What is the expected payoff if the buyer chooses to get 2 quotes? What if she asks n quotes, where n is just some number?

(b) Formulate this "fixed sample search" problem of the buyer, i.e. write the optimization problem of the buyer who chooses n to maximize her expected payoff. How do you expect the optimal value of n to vary in c? If you can, solve the problem explicitly.

(c) Suppose now that the buyer searches sequentially. Conside first the simplest case, i.e. suppose that the buyer has already received one price quote, say p, and has a chance to ask for one more quote at cost c. For which values of p should the buyer ask for another quote?

(d) Suppose that the buyer searches sequentially as long as she wishes, i.e. she asks for one quote at the time, and decides after each quote whether to buy at the lowest offer so far or whether to continue asking for another quote. Formulate the problem of the buyer, and discuss the nature of the solution. How do you expect the solution to vary in c. Again, if you can, solve the problem explicitly.

(e) Is the buyer better off with fixed sample or sequential search (i.e. one in b. or in d.)? Why? How would you modify the model to make this question more interesting?

(f) Discuss economic situations where one or the other form of search (fixed sample/sequential) might be more appropriate.

3. A municipality wants to procure a service. There are $N = 2$ identical firms, who decide simultaneously whether or not to make a price quote for the service. Preparing the price quote costs $c > 0$, but there are no production costs. The municipality has a reservation value $R > 0$ for the service. The firm that offers the lowest price p gets a deal and makes profit $p - c$ (as long as $p \leq R$; in case there is a tie with m firms offering the lowest price, each of them makes profit $p/m - c$). Those who offer a price that is not the lowest, get no deal and make profit $-c$, and those firms that do not offer anything get 0.

(a) Formalize this as a simultanous move game between the firms.

What is the set of strategies available to each firm?

(b) Assume first that $c = 0$. Find a Nash equilibrium of the game.

(c) Assume that $c > 0$. Does the game have a Nash equilibrium in pure strategies?

(d) Derive a symmetric mixed strategy equilibrium with an atomless price distribution with support $[c, R]$. What is the expected profit of each firm in this equilibrium? What is the probability of an individual firm making a positive price offer? What is the equilibrium price distribution?

(e) Does this model have any other symmetric equilibria?

(f) (bonus: harder) Can you generalize your answer in d) to any $N \geq 2$? What is the probability distribution of the number of firms that make a price offer for a fixed N? What is the limiting probability distribution of the number of firms that make an offer in the limit where $N \to \infty$?

4. A pharmaceutical company sells a given drug in two geographically separated markets, denoted A and B. The demands are given by $Q_A(p_A) = 1 - p_A$ and $Q_B(p) = \frac{1}{2} - p_B$. For simplicity, the transport and production costs are assumed to be zero.

   (a) Suppose that the firm sets a uniform price across the two markets. What is the profit-maximizing uniform price, and what are the quantities sold at that price in the two markets?

   (b) Suppose that the firm can set different prices in the two markets. What are the profit-maximizing prices and what are the quantities sold in the two markets?

   (c) Compute the producer's and consumers' surpluses under a uniform price and under geographical price discrimination. Compare the two situations and discuss.

   (d) Do the insights of c) hold generally? What if the demand in market B is changed to $Q_B(p) = \frac{1}{3} - p_B$?