31E11100 - Microeconomics: Pricing
Autumn 2021
Pauli Murto and Riku Buri

Problem Set 2 (for the exercise session on October 8)

1. A firm sells a product in a market where there are two types of consumers, $\theta \in \{\theta^H, \theta^L\}$. Assume that the mass of consumers is normalized to one, and there are equally many of both types of consumers. All consumers have a unit demand, and θ denotes the reservation value of consumer type θ. Let $\theta^H = 12$ and $\theta^L = 5$. The marginal cost of production is $c = 1$.

   (a) Find the profit maximizing price for the firm.

   (b) The firm then considers producing an additional, lower quality version of the good (a "damaged good"). The damaged version of the good can be produced at constant marginal cost $c = 1.5$, and the reservation value of that good is 4 for the high type, and 3 for the low type. Find the optimal prices for the two versions of the product for the firm that wants to maximize its profit.

   (c) Should the firm introduce the damaged version of the good? Why? What are the welfare implications of introducing the damaged version?

2. Amazon.com has a single cover price for the books that it sells, but it has a menu of different delivery options ranging from 1-2 days to two weeks. Let's have a model of second-degree price discrimination to explain this.

   (a) Assume that buyers have different valuations for fast delivery. This is captured by a parameter θ, where value $\theta = \theta^H$ captures consumers with high valuation for fast delivery and $\theta = \theta^L$ captures consumers with low valuation for fast delivery. Let s denote the actual delivery time and assume the following payoff function:

$$v(\theta, s) = \begin{cases} \theta(1-s), & \text{if } s \leq 1 \\ 0 \text{ otherwise.} \end{cases}$$

Interpret this payoff function (i.e. invent a story that rationalizes the function).

(b) Let the cost of delivery at time s be $c(s) = 2(1-s)^2$ for $0 \leq s \leq 1$ and $c(s) = 0$ for $s > 1$. Does this function make sense?

(c) What are the first best delivery times for the two types of buyers? I.e. how would the seller choose $\widehat{s}^H$ and $\widehat{s}^L$ for θ^H and θ^L respectively if she could see the type of the buyer? What would the corresponding prices $\widehat{t}^H$ and $\widehat{t}^L$ be for those delivery times?

(d) Would the menu $\{(\widehat{s}^H, \widehat{t}^H), (\widehat{s}^L, \widehat{t}^L)\}$ be incentive compatible if the seller does not see θ?

(e) Suppose that fraction λ of the buyers are of type θ^H and $(1-\lambda)$ are of type θ^L. Solve for the profit maximizing incentive compatible menu of delivery times and prices for the seller. For what parameter values should the seller offer two different delivery times?

3. A monopolist sells two products $i \in \{1, 2\}$. There is a unit mass (continuum) of consumers, who each have independent valuations for the two goods. Assume that the valuations are uniformly distributed over the unit interval, i.e. $v_i \sim U[0,1]$, $i = 1, 2$. The production cost is assumed to be zero for the seller.

(a) Suppose the monopolist sells the two products only separately, i.e. sets separate prices p_1 and p_2 for the two products, and lets each consumer decide which product(s) to buy. Compute optimal separate prices and the corresponding profit.

(b) Suppose next that the monopolist sells the two products as a bundle only (pure bundling). What is the demand function for the bundle, i.e. the total quantity bought at a given bundle price p_b? To derive that demand, it is helpful to draw a unit square with axes v_1 and v_2 that represents the set of possible player types. For a given bundle price p_b, what is the region in that figure representing those consumers that buy? The demand is then just the area of that region. What would be the profit of the seller if she chooses bundle price $p_b = p_1 + p_2$, where p_1 and p_2 are the ones you derived in a)? Would buyers be better or worse off?

(c) What is the optimal bundle price p_b and the associated profit for the seller? (in the case where the good is sold only as a bundle)

(d) Finally, consider mixed bundling, where the seller offers prices $(\overline{p}_1, \overline{p}_2, \overline{p}_b)$, i.e. $\overline{p}_1$ the price of good 1 separately, $\overline{p}_1$ the price of good 2 separately, and $\overline{p}_b$ the price of the bundle containing both goods. Derive the demands for products 1 and 2 and for the bundle for the given $(\overline{p}_1, \overline{p}_2, \overline{p}_b)$. Again, utilize the graphical representation suggested in (b).

(e) You can take it as given that the seller will in such a case always choose $\overline{p}_1 = \overline{p}_2 := \overline{p}_s$. What would be the optimal prices $(\overline{p}_s, \overline{p}_b)$ in the case of mixed bundling?

(f) Which mode of pricing (separate, pure bundling, mixed bundling) is best for the seller?

4. A monopoly firm operates in a market with a sesonally varying demand. The inverse demand function in season $i \in \{s, w\}$ is given by:

$$p^i = \alpha^i - \beta^i q^i,$$

where $\alpha^s = 10$, $\alpha^w = 8$, $\beta^s = 1$, $\beta^w = 2$. There is a constant marginal cost of production so that producing q^i units in season i costs cq^i, where $c = 1$.

(a) Assume first that there is no capacity constraint for the firm. What is the optimal production level and correponding price for each season?

(b) Keep on assuming that there is no capacity constraint for the firm. If the firm must use the same price in both seasons, what is the total quantity sold over the two seasons for given price p? What is the optimal price p?

(c) Assume next that there is a fixed capacity level k. The capacity sets the maximum output that the firm can produce in either season: $q^i \leq k$ for $i \in \{s, w\}$. What is the optimal production level and correponding price for each season as a function of k?

(d) Assume now that the firm must choose its production capacity to serve the market, and it costs fk to build k units of capacity. The firm maximizes its total profits over the two seasons, and can choose different prices in the two seasons. For what values of f is the capacity constraint binding in the high season only? Solve the optimal capacity and supply levels for the two seasons both in the case where the capacity constraint binds only in one season and where it binds in both seasons.