31E11100 - Microeconomics: Pricing
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Problem Set 3 (for the exercise session on October 22)

1. We consider here pricing of an experience good. There is a unit mass of consumers with unit demand and reservation valuation uniformly distributed between 1 and 5, i.e. $v \sim U[1,5]$. However, being an experience good, the consumers do not know their individual valuations before consuming (so, before consuming, each individual just knows that her value is drawn from $v \sim U[1,5]$). After consuming once, a consumer learns perfectly her reservation valuation. There are two periods of consumption, and no discounting between the periods. The production cost is zero for the firm.

   (a) Let us consider the second period first. What is the optimal price that the monopolist charges in the second period if no consumer consumed in the first period (i.e. all consumers are still uninformed about their valuations)?

   (b) What would be the optimal price that the monopolist charges in the second period if all consumers consumed in the first period (i.e. all consumers know their valuations)?

   (c) Suppose that the monopolist charges in the second period the price that you derived in b). What is the maximum price that the consumers would then be willing to pay in the first period? (note: to compute this, you need to compute the value of information for the consumers)

   (d) Using your findings above, characterize an equilibrium, where the monopolist chooses prices optimally in both periods, and the consumers choose optimally whether or not to buy, given the prices.

2. In this problem we continue with the example of Lecture 10, where we compared different procedures that a seller might use to sell an object.

The seller has a single object to sell and there are just two potential buyers, whose valuations for the object are drawn independently from uniform distribution so that $v_i \sim U[0,1]$, $i = 1, 2$.

(a) Review the case, where the seller uses a poster price mechanism: the seller sets price p and buyers either accept or reject the offer. If one buyer accepts, she gets the object and pays p; if both buyers accept, the object is allocated randomly to one buyer who pays p; if neither accepts, the objects is not sold and no payments are made. What is the expected revenue to the seller for given p? Find the price that maximizes the expected revenue and compute that revenue.

(b) Suppose the seller approaches the two buyers sequentially. First, she approaches buyer 1 and offers to sell at posted price p_1. If buyer 1 accepts, the object is sold at price p_1 and the game is over. If rejected, the seller approaches buyer 2 and offers to sell at posted price p_2. If buyer 2 accepts, the object is sold at price p_2; if rejected, the good is not sold. What is the expected revenue to the seller if she sets $p_1 = p_2 = p$ (for an arbitrary p)?

(c) What is the expected revenue in (b) if the seller sets p_1 and p_2 optimally? Compare to a) and discuss.

(d) Suppose that the seller uses a second-price auction to allocate the object and she set a reserve price $r > 0$. Is bidding one's own value a dominant strategy for the buyers?

(e) Compute the expected revenue to the seller for an arbitrary r (Hint: think about the three different cases in the slides for Lecture 10, and utilize them in computing the expectation).

(f) Show that $r = 1/2$ is the optimal reserve price and compute the expected revenue at that reserve price.

3. Here we consider a second-price sealed bid auction (SPA) where N bidders have independent valuations drawn from some distribution F.

(a) Formulate this auction as a Bayesian game.

(b) Is bidding one's own valuation a dominant strategy in this game.

(c) Suppose that instead of a SPA the seller uses a third-price auction, where the highest bidder wins but pays the third-highest bid. Is bidding one's own valuation a dominant strategy? If not, do you expect bidders to bid higher or lower than their own valuation?

(d) Consider again the SPA and assume that all the players are using their dominant strategies. Suppose from now on that valuations are from uniform distribution $v_i \sim U[0,1]$. If player i has valuation v_i, what is her chance of winning the auction?

(e) Compute the expected revenue for the seller (Hint: the density function of the second-highest valuation is $g(v) = N(N-1)(1-v)v^{N-2}$. How do you derive this? How do you use this to compute the expectation of the second highest valuation?)

4. Consider a second-price auction where the seller sets a participation fee for the bidders. Assume that each of two potential bidders has a privately known private valuation v_i that is uniformly distributed on $[0,1]$. The auctioneer may charge a fee f to each bidder for participating in the auction. If only one bidder participates, she pays the participation fee f but gets the good for free. If nobody participates, no payments are made. If both participate, then both bidders pay f and the good is allocated according to a second price auction. Keep in mind that we have a second-price auction, and therefore, in the event where both bidders participate, they bid according to $b_i = v_i$, $i = 1, 2$.

(a) If $f = 0$, do you expect both bidders to participate? Compute the expected value from participating in the auction for a bidder with type v_i.

(b) Let $f > 0$. For each $v_i \in [0,1]$, compute the value of participating in the auction for i if she believes that j participates if and only if $v_j \geq v_i$.

(c) Argue that for any $f > 0$, there is an equilibrium where v_i participates in the auction (and pays f) if and only if $v_i \geq \sqrt{f}$.

(d) (Harder) Compute the expected revenue to the seller and solve for the optimal participation fee f. Can you use the revenue equivalence theorem to explain the relationship of this result to the result in 2(f)?