Exercise Session 6 (PS5 Solutions)

Principles of Economics I

Aalto University
School of Business

Hung Le

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Q1

Note:

- To prove an allocation is not Pareto-efficient: show ***1*** alternative where at least 1 player is better off, and others are not worse off.

Exists ***1*** Pareto improvement

- To prove an allocation is Pareto-efficient: show that for ***all*** alternatives, at least 1 player is worse off.

No Pareto improvement

Q1

a. An allocation is Pareto-efficient if there is no other allocation to make one better off without making the other worse off.

Denote the payoffs of the chemical plant and the fishery (c, f)

In the case of no waste processing, the payoffs are $(-30,130)$.

In the case of waste processing, the payoffs are $(-80,200)$.

Without transfer, there are only 2 alternatives. **Both allocations are Pareto-efficient**. It is impossible to switch to the other alternative without harming one player.

Q1

b. Because the fishery is better off with no waste processing, it will only have incentives to make a monetary transfer in the case of waste processing.

Denote the payoffs of the chemical plant and the fishery (c, f)

In the case of no waste processing, the payoffs are $(-30,130)$.

In the case of waste processing, the payoffs are $(-80 + t, 200 - t)$.

Short answer: waste processing with any transfer t is Pareto-efficient while no waste processing is never Pareto-efficient.

Q1

b. Having waste processing $(-80+t, 200-t)$ is always Pareto-efficient because

- No waste processing $(-30,130)$ is worse off for at least 1 player
  - C is worse off under no waste processing if $-80+t > -30$
  - F is worse off under no waste processing if $200-t > 130$
  - At least 1 player is worse off under no waste processing if $-80+t > -30$ or $200-t > 130$
  - At least 1 player is worse off under no waste processing if $t > 50$ or $70 > t$ → true for all t
- Any extra transfer x between 2 players $(-80+t+x, 200-t-x)$ will make one player worse off from $(-80+t, 200-t)$. x can be positive or negative.

Q1

b. No waste processing $(-30,130)$ is never Pareto-efficient because

Having waste processing$(-80+t, 200-t)$ with a transfer t $(50 \leq t \leq 70)$ is better off for at least one player while not harming the other:

- C is better off or equally good under waste processing if $-80+t \geq -30$
- F is better off or equally good under waste processing if $200-t \geq 130$
- At least 1 player is better off while the other is not harmed under waste processing if $-80+t \geq -30$ and $200-t \geq 130$
- At least 1 player is better off while the other is not harmed under waste processing if $t \geq 50$ and $70 \geq t$.
- Pareto improvement: waste processing$(-80+t, 200-t)$ with $50 \leq t \leq 70$

Q1

c. When C can propose F to pay P for waste processing:

In the case of no waste processing, the payoffs are $(-30,130)$.

In the case of waste processing, the payoffs are $(-80+P, 200-P)$.

For any $P > 70$, F will choose “no waste processing”, and C has -30.

For any $P < 70$, F will choose “waste processing”, and C has $-80+P$. The higher P is, the better for C.

To maximize its profits, C should propose $\boldsymbol{P = 70 - \varepsilon}$ (ε is infinitely-small). In that case, there will be waste processing, and C gets $-10+\varepsilon$.

Accept other answers such as P=70, P=69.

Q1

d. When F can charge C a price P for waste processing:

In the case of no waste processing, the payoffs are $(-30 - P, 130 + P)$.

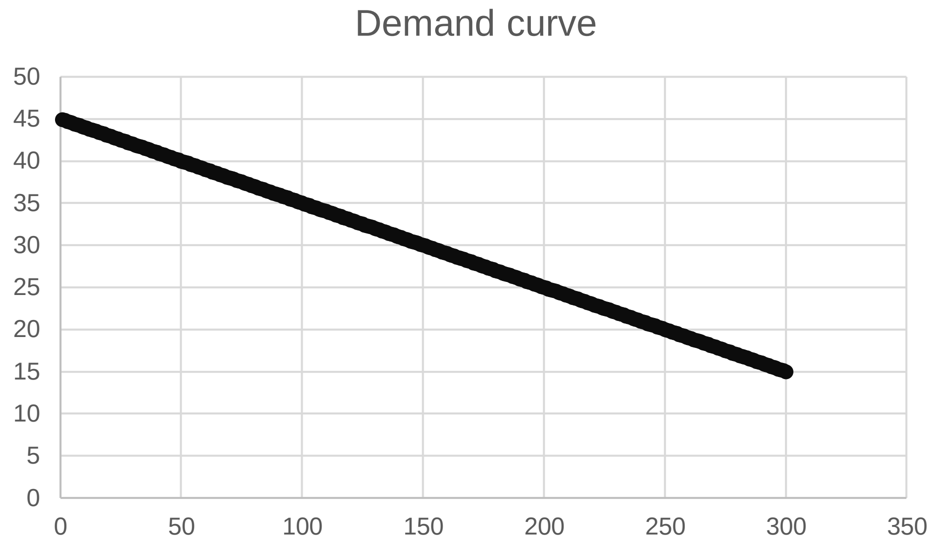
In the case of waste processing, the payoffs are $(-80{,}200)$.

For any $P > 50$, C would choose “waste processing”, and F has 200.

For any $P < 50$, C would choose “no waste processing”, and F has $130 + P < 180 < 200$.

The maximum payoff F can get is 200 when it sets $P > 50$. In that case, there will be waste processing, and the payoffs are $(-80{,}200)$.

Q2



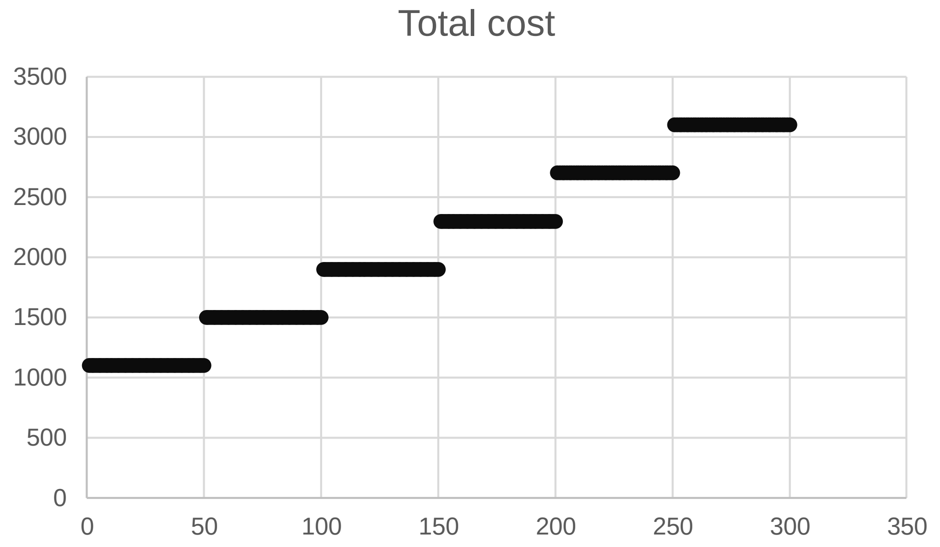
a.

$$P = 45 - \frac{Q}{10}$$

If $P = 20$

$$20 = 45 - \frac{1}{10}Q$$
$$Q = 250$$

Q2



b. (only graphs are enough)

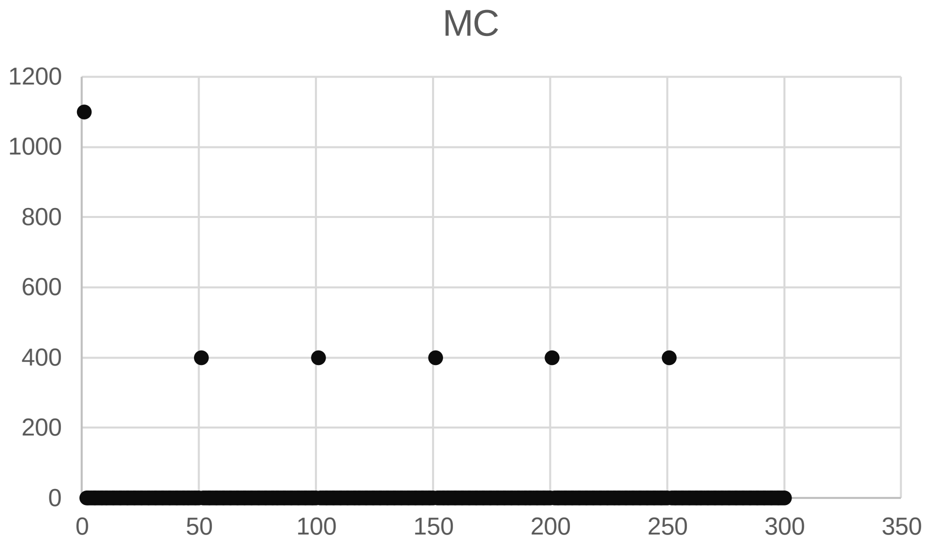
The total cost function:

$$TC = \begin{cases} 700 + 400 * int\left(\frac{Q}{50}\right) if\ mod(Q, 50) \neq 0 \\ 700 + 400 * \frac{Q}{50} if\ mod(Q, 50) = 0 \end{cases}$$

$int(x)$ gives the highest integer that is less than $x.$ For example, $int(7.2) = 7$

$mod(x, y)$ gives the remainder when we divide x by y. For example, $int(7,2) = 1$

Q2



b. (only graphs are enough)

$$MC = \begin{cases} 0 \; if \; mod(Q, 50) \neq 0 \\ 400 \; if \; mod(Q, 50) = 0 \end{cases}$$

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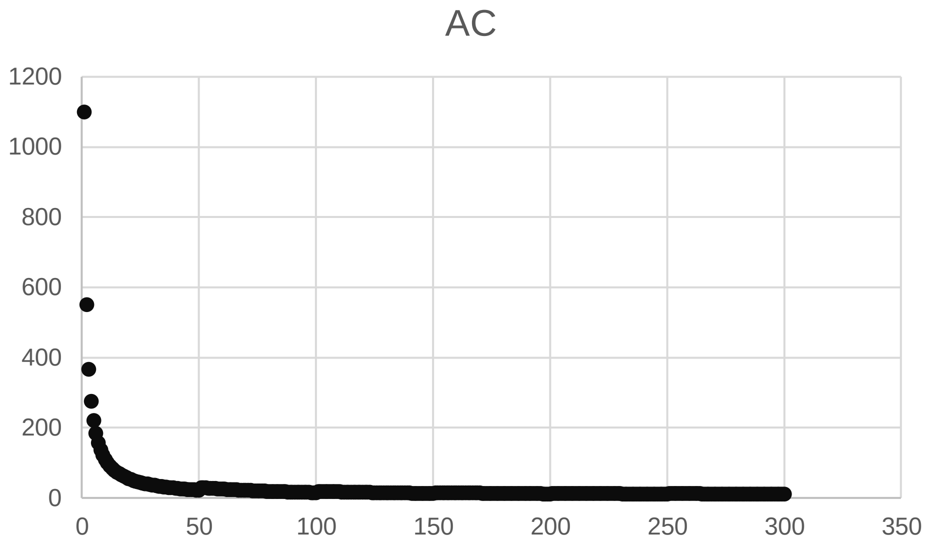
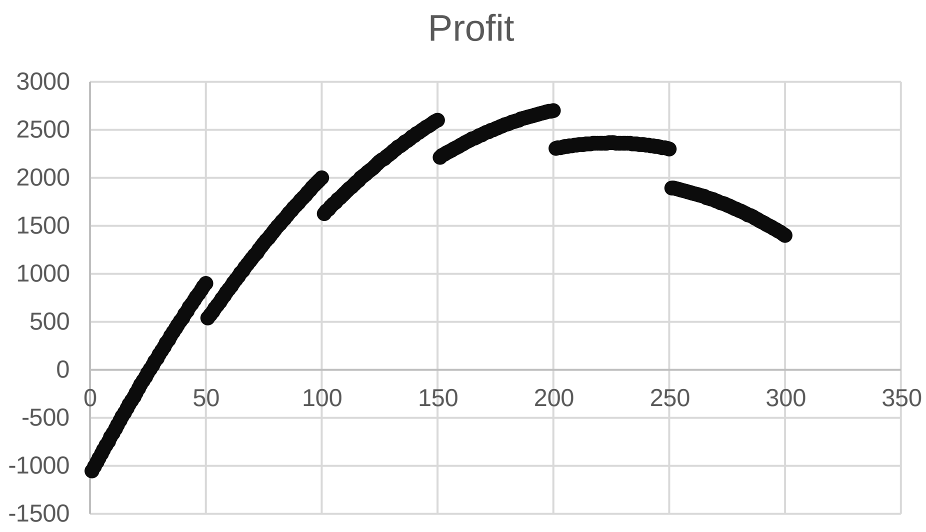
Q2

b. (only graphs are enough)



$$AC = \begin{cases} \dfrac{700 + 400 * int\left(\frac{Q}{50}\right)}{Q} & if\ mod(Q,50) \neq 0 \\ \dfrac{700 + 400 * Q/50}{Q} & if\ mod(Q,50) = 0 \end{cases}$$

Q2

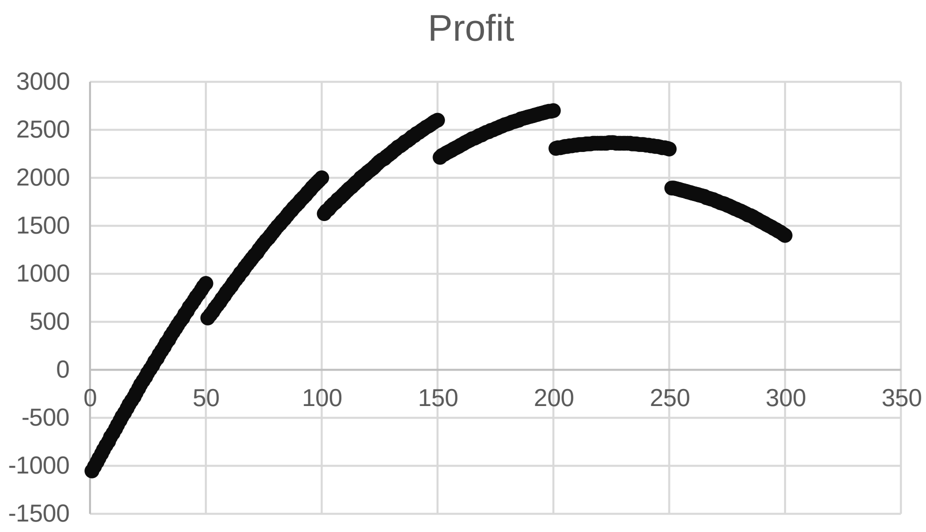


b.

$$\Pi(Q) = P * Q - TC$$

$$\Pi(Q) = \begin{cases} \left(45 - \frac{Q}{10}\right)Q - 700 - 400 * int\left(\frac{Q}{50}\right) if\ mod(Q,50) \neq 0 \\ \left(45 - \frac{Q}{10}\right)Q - 700 - 400 * \frac{Q}{50} if\ mod(Q,50) = 0 \end{cases}$$

Q2



b.

Accepted answer:

It is profitable when $\Pi(Q) > 0$. From the total revenue curve, it is profitable when $26 \leq Q \leq 300$ $(15 \leq P \leq 42.4)$

Accepted answer:

An example of Q to show it is possible to operate the studio profitably.

Q2

b. Detailed answer:

With the same number of instructors, having 1 more consumer increases profits if:

$\frac{d\Pi(Q)}{dQ} > 0$ or $\frac{dMR(Q)}{dQ} > 0$ (because $MC = 0$ in this case)

$$45 - \frac{Q}{5} > 0$$

$$Q < 225$$

Q2

b.

We check the lowest point of profit in each segment to see whether it has positive profit or not. If the lowest point has positive profits, the rest of the segment does too.

When the segment goes up, we check the first point. When the segment goes down, we check the last point. When the segment goes up, then down, we check both first and last points.

We need to check profits at the following points $Q = 1$, $Q = 51$, $Q = 101$, $Q = 151$, $Q = 201$, $Q = 250$, $Q = 300$ because the profit curve's slope becomes negative after $Q = 225$.

Q2

b.

$\Pi(51) > 0$. Therefore, for any $51 \leq Q \leq 100$, $\Pi(Q) > 0$

$\Pi(101) > 0$. Therefore, for any $101 \leq Q \leq 150$, $\Pi(Q) > 0$

$\Pi(151) > 0$. Therefore, for any $151 \leq Q \leq 200$, $\Pi(Q) > 0$

$\Pi(201) > 0$ *and* $\Pi(250) > 0$. Therefore, for any $201 \leq Q \leq 250$, $\Pi(Q) > 0$

$\Pi(300) > 0$. Therefore, for any $251 \leq Q \leq 300$, $\Pi(Q) > 0$

Q2

b. For $1 \leq Q \leq 50$, $\Pi(1) < 0$. For this segment, profit is increasing in Q. We check when it changes sign from negative to positive.

$$\Pi(Q) > 0$$

$$\left(45 - \frac{Q}{10}\right)Q - 700 - 400 > 0$$

$$Q^2 - 450Q + 1100 < 0$$

$$5\left(45 - \sqrt{1585}\right) < Q < 5\left(45 + \sqrt{1585}\right)$$

$$25.9 < Q < 424.1$$

For any $26 \leq Q \leq 50$, $\Pi(Q) > 0$

Q2

c. The marginal cost of adding one more instructor is 400.

We assume the studio hires an additional instructor only if the number of customer exceeds the capacity of the current instructors. The marginal revenue of adding one more instructor is:

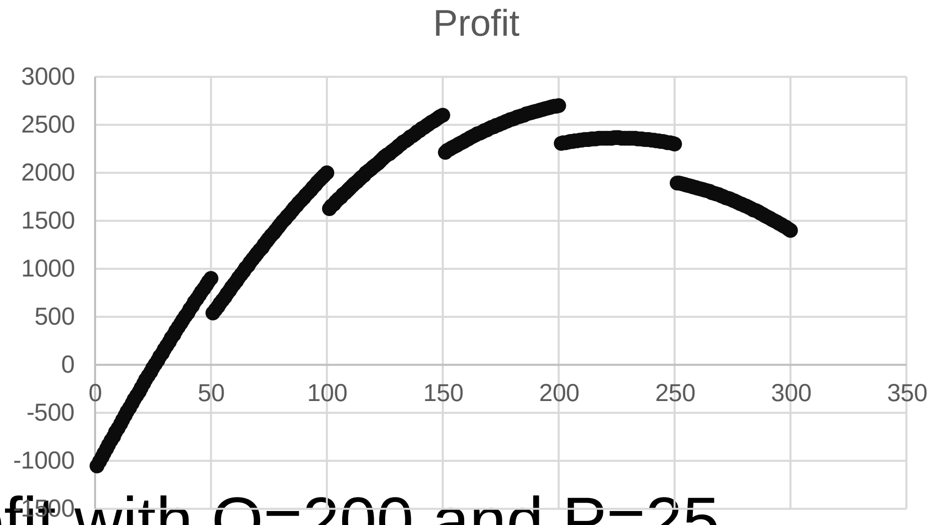
No. of instructors	Optimal Q within each range	Revenue	MR of instructor
1	50	40*50=2000	2000
2	100	35*100=3500	1500
3	150	30*150=4500	1000
4	200	25*200=5000	500
5	225	22.5*225=5062.5	62.5
6	251	19.9*251=4994.9	-67.6

Q2

c. Notice that with 5 instructors, it's optimal to have Q=225 instead of 250 because MR(Q)=0 at Q=225. Increasing number of customers when Q>=225 will decrease profits.

For the same reason, with 6 instructors, it's optimal to have Q=251 because the profit function is decreasing in Q.

Q2



d. Accepted answer:

From the profit curve, the studio has the highest profit with Q=200 and P=25. The studio should hire 4 instructors.

Notice from the profit curve that because of the demand curve we have, it is not always optimal to fill up a class of 50 members for each hired instructor (when we have 5 or 6 instructors).

Q2

d.

From part c, we have MR of adding the 5^{th} instructor (62.5) is lower than the MC (400). Therefore, it's optimal to have 4 instructors. The optimal Q with 4 instructors is 200, the optimal price is 25.

Q3

a. Fixed cost: F

Marginal cost: c+2bq

Q3

b. Efficient scale of production: $\mathrm{MC} = \mathrm{AC}$

$$c + 2bq^* = \frac{F + cq^* + b{q^*}^2}{q^*}$$

$$cq^* + 2b{q^*}^2 = F + cq^* + b{q^*}^2$$

$$q^* = \sqrt{\frac{F}{b}}$$

Q3

c. $\mathrm{F} = 64, \mathrm{c} = 12, \mathrm{b} = 16$

$$q^* = \sqrt{\frac{F}{b}} = 2$$

Long-run equilibrium price: $P^* = \mathrm{AC} = \mathrm{MC}$

$$P^* = c + 2bq^* = 12 + 2 * 16 * 2 = 76$$

Long-run number of firms:

$$P^* = 300 - 2Q^* = 76$$

$$300 - 2n^*q^* = 76$$

$$n^* = \frac{300 - 76}{2q^*} = 56$$

Q3

d. (Enough to express n^* in terms of F, b, c)

$$q = \sqrt{\frac{F}{b}}$$

$$P^* = 300 - 2Q^* = c + 2bq^*$$

$$n^* = \frac{300 - c - 2bq^*}{2q^*} = \frac{300 - c}{2q^*} - b = \frac{300 - c}{2\sqrt{\frac{F}{b}}} - b$$

Extra: n^* is decreasing in F and c.

Higher F -> lower n^*. Higher c -> lower n^*

Q3

d.

Extra:

$$n^* = \frac{300 - c}{2F^{1/2}b^{-1/2}} - b$$

$$\frac{\partial n^*}{\partial b} = \frac{(300 - c)2F^{\frac{1}{2}}b^{-\frac{3}{2}}\left(-\frac{1}{2}\right)}{4Fb^{-1}} - 1 = \frac{c - 300}{4F^{1/2}b^{1/2}} - 1$$

higher b -> higher n^* if $\frac{c-300}{4F^{1/2}b^{1/2}} - 1 > 0$

higher b -> lower n^* if $\frac{c-300}{4F^{1/2}b^{1/2}} - 1 < 0$

Q4

$$S(Q) = \frac{1}{10}Q$$

$$D(Q) = 100 - \frac{1}{10}Q$$

a) Equilibrium price and quantity

$$S(Q^*) = D(Q^*)$$

$$\frac{1}{10}Q^* = 100 - \frac{1}{10}Q^*$$

$$Q^* = 500$$

$$P^* = 100 - \frac{1}{10}Q^* = 50$$

Q4

b) Old supply

$$S(Q) = \frac{1}{10}Q$$
$$Q = 10P$$

New supply

$$Q = 10P + 200$$
$$S(Q) = \frac{1}{10}(Q - 200)$$

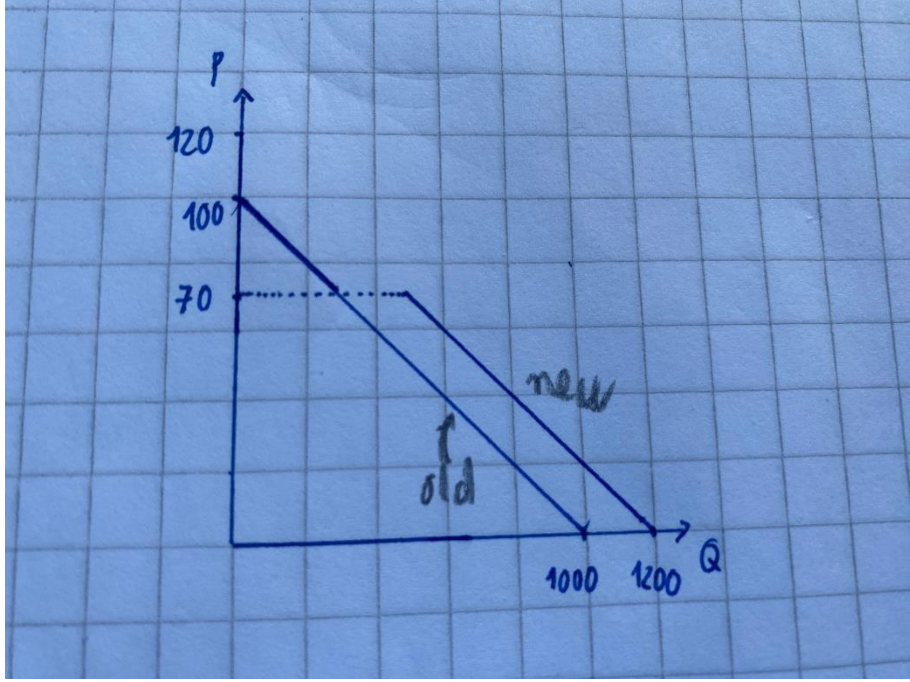
Q4

$$Q = 10P + 200$$
$$S(Q) = \frac{1}{10}(Q - 200)$$
$$D(Q) = 100 - \frac{1}{10}Q$$

b) Equilibrium price and quantity

$$S(Q^*) = D(Q^*)$$
$$\frac{1}{10}Q^* - 20 = 100 - \frac{1}{10}Q^*$$
$$Q^* = 600$$
$$P^* = 100 - \frac{1}{10}Q^* = 40$$

Q4



c)

Old demand

$$D(Q) = 100 - \frac{Q}{10}$$
$$Q = 1000 - 10P$$

New demand

$$Q = \begin{cases} 1000 - 10P \; if \; P \geq 70 \\ 1000 - 10P + 200 \; if \; P < 70 \end{cases}$$

$$D(Q) = P(Q) = \begin{cases} 100 - \frac{1}{10}Q \; if \; P \geq 70 \\ 120 - \frac{1}{10}Q \; if \; P < 70 \end{cases}$$

Q4

$$S(Q) = \frac{1}{10}(Q - 200)$$

$$D(Q) = 120 - \frac{1}{10}Q \; if \; P < 70$$

d) Equilibrium price and quantity if $P^* < 70$

$$S(Q^*) = D(Q^*)$$

$$\frac{1}{10}(Q - 200) = 120 - \frac{1}{10}Q^*$$

$$Q^* \; = 700$$

$$P^* = 120 - \frac{1}{10}Q^* = 50 \; (this \; satisfies \; P < 70)$$

Q4

$$S(Q) = \frac{1}{10}(Q - 200)$$

$$D(Q) = 100 - \frac{1}{10}Q \ if \ P \geq 70$$

d) Equilibrium price and quantity if $P^* \geq 70$

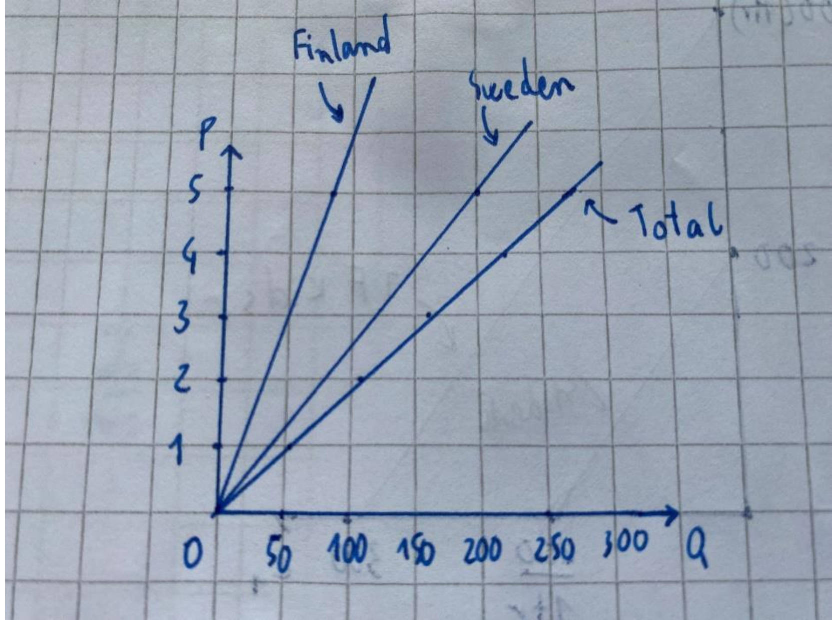
$$S(Q^*) = D(Q^*)$$

$$\frac{1}{10}(Q - 200) = 100 - \frac{1}{10}Q^*$$

$$Q^* \ = 600$$

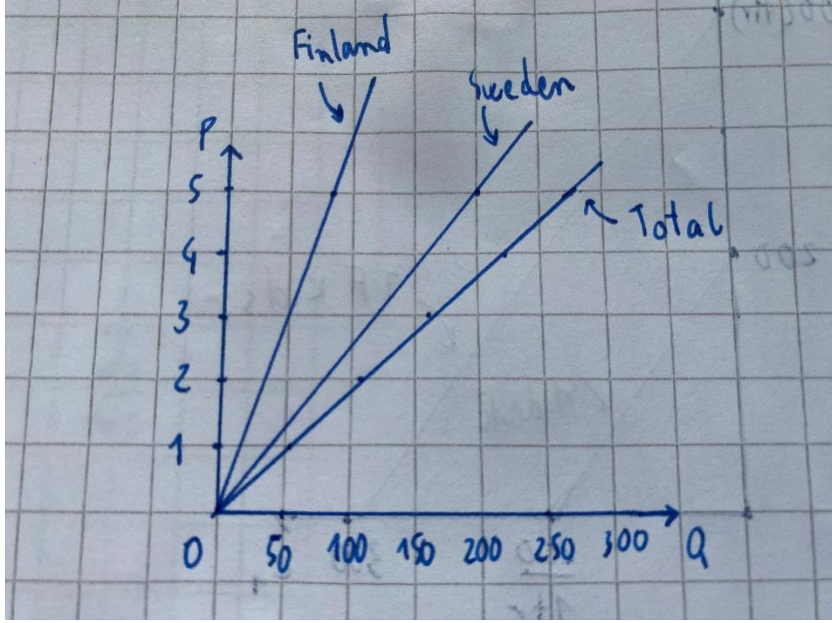
$$P^* = 100 - \frac{1}{10}Q^* = 40 \ (this \ does \ not \ satisfy \ P^* \geq 70)$$

Q5



a. In Finland, there are 80 firms. If the total market is Q^F, each firm produces $\mathrm{q^F} = \frac{Q^F}{80}, MC^F = 5q^F = 5 * \frac{Q^F}{80} = \frac{Q^F}{16}$

Q5



a. In Sweden, there are 120 firms. If the total market is Q^S, each firm produces $q^S = \frac{Q^S}{120}, MC^S = 3q^S = 3 * \frac{Q^S}{120} = \frac{Q^S}{40}$

Q5

b. In Finland,

$$5 * \frac{Q^{*F}}{80} = 3 - \frac{Q^{*F}}{100}$$
$$Q^{*F} \approx 41.38$$
$$P^{*F} \approx 2.59$$

In Sweden,

$$3 * \frac{Q^{*S}}{120} = 3 - \frac{Q^{*S}}{100}$$
$$Q^{*S} \approx 85.71$$
$$P^{*S} \approx 2.14$$

Q5

c. Let equilibrium price be P^*

Finland demand

$$P^* = 3 - \frac{D_F^*}{100}$$
$$D_F^* = 100(3 - P^*)$$

Finland supply

$$P^* = 5 * \frac{S_F^*}{80}$$
$$S_F^* = 16P^*$$

Q5

c. Let equilibrium price be P^*

Sweden demand

$$P^* = 3 - \frac{D_S^*}{100}$$

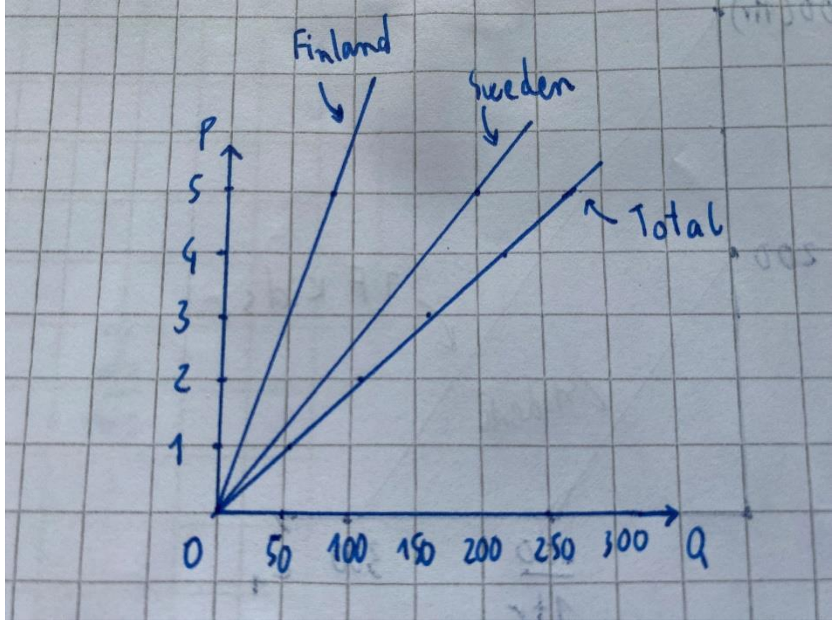
$$D_S^* = 100(3 - P^*)$$

Sweden supply

$$P^* = 3 * \frac{S_S^*}{120}$$

$$S_S^* = 40P^*$$

Q5



c,d. Let equilibrium price be P^*

Total demand: $200(3-\mathrm{P}^*)$

Total supply: $56\mathrm{P}^*$

Total demand = Total supply

$$200(3-\mathrm{P}^*) = 56\mathrm{P}^*$$

$$\mathrm{P}^* = \frac{600}{256}$$

$$\mathrm{Q}^* = 56 * \frac{600}{256} = 131.25$$

$$131.25 > 127.09, \mathrm{Q}^* > \mathrm{Q}^{*F} + \mathrm{Q}^{*S}$$

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Q6

a.

Denote the bundle (c_1, c_2) where c_1 is the consumption in period 1 and c_2 is the consumption in period 2

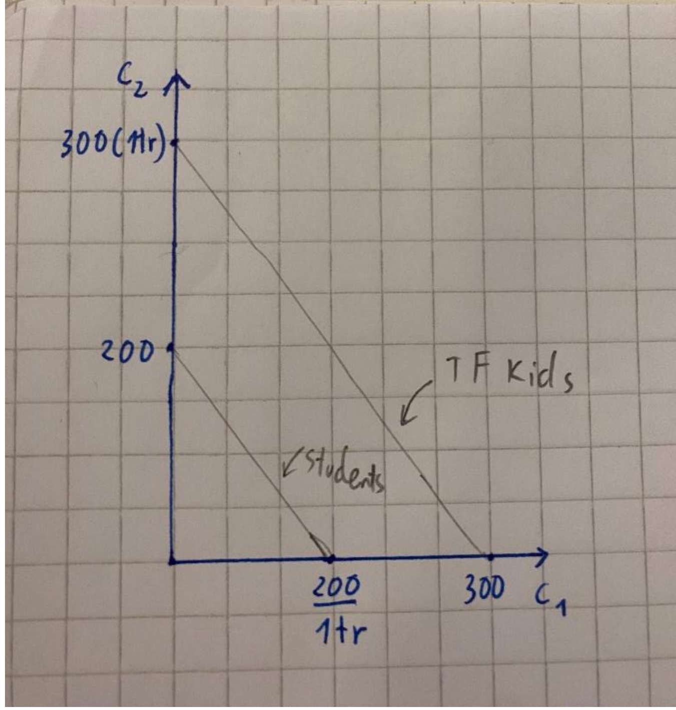
For students: If they consume everything in period 2 and nothing in period 1, they consume the bundle $(0{,}200)$.

If they consume everything in period 1 and nothing in period 2, they consume the bundle $\left(\frac{200}{1+r}, 0\right)$.

(To do so, they must borrow $\frac{200}{1+r}$ in period 1 and pay back 200 in period 2, leaving them nothing in period 2)

Q6

a.



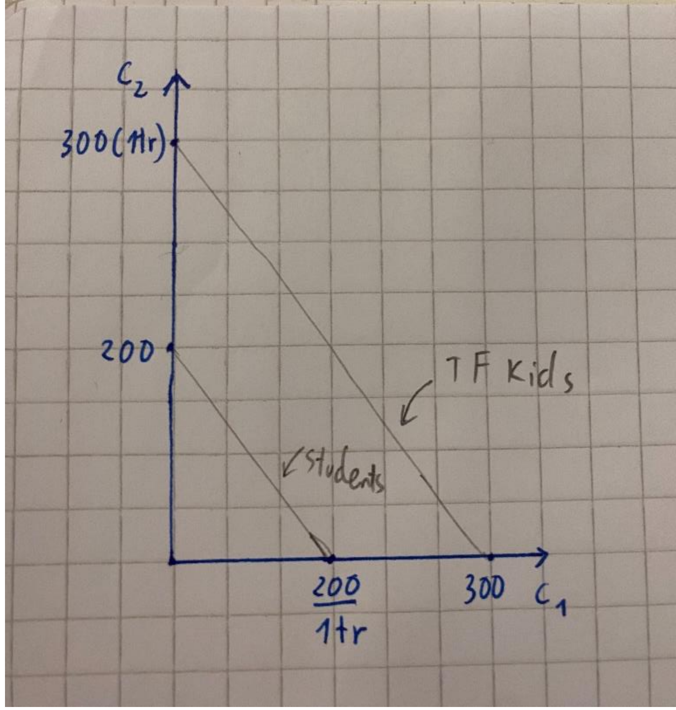
For trust fund kids:

If they consume everything in period 1 and nothing in period 2, they consume the bundle $(300,0)$.

If they consume everything in period 2 and nothing in period 1, they consume the bundle $(0,300(1+r))$.

(To do so, they must lend 300 in period 1 and receive 300(1+r) in period 2)

Q6



a.

For students: The budget constraint is

$$c_1 + \frac{c_2}{1+r} = \frac{200}{1+r}$$

(we are expressing everything in term of present value in period 1. Consumption and income in period 2 is discounted by (1+r). If we express everything in terms of value in period 2. The budget constraint is $c_1(1+r) + c_2 = 200$, which is just multiplying both sides by (1+r)

For trust fund kids: The budget constraint is

$$c_1 + \frac{c_2}{1+r} = 300$$

Q6

b. Because the interest rate is r, $\mathrm{M}RT = 1 + r$.

For both students and trust fund kids

$$\mathrm{M}RS = MRT$$

$$\frac{\mathrm{c}_2}{c_1} = 1 + r$$

$$c_1 = \frac{c_2}{1 + r}$$

Q6

b. For students: it is optimal to borrow 100 in period 1.

For trust fund kids: it is optimal to save 150 in period 1.

Q6

Note:

Here we have $\mathrm{M}RT = 1 + r$ because we plot c_2 on the vertical line and c_1 on the horizontal line. $\mathrm{M}RS = \frac{c_2}{c_1}$ makes sense with this setup. For example, the indifference curve start at a point with very high c_2 and low c_1. The slope of the indifference curve at that point is very high.

If you plot c_1 on the vertical line and c_2 on the horizontal line, you will have $\mathrm{M}RT = \frac{1}{1+r}$. $\mathrm{M}RS = \frac{c_2}{c_1}$ does not make much sense because the indifference curve starts with a very low slope and ends with a very high slope.

Give partial credits to part b, c, and d if having $\mathrm{M}RT = \frac{1}{1+r}$.

Q6

c.

It is still optimal for students to borrow $\frac{100}{1+r}$ and for TF kids to save 150.

The substitution effect makes both of them consume less in period 1 and more in period 2 because consumption in period 1 gets relatively more expensive.

The income effect makes students consume less in both periods because an increase in r makes borrowing more expensive and students are poorer.

The income effect makes TF kids consume more in both periods because an increase in r makes saving more lucrative and TK kids are richer.

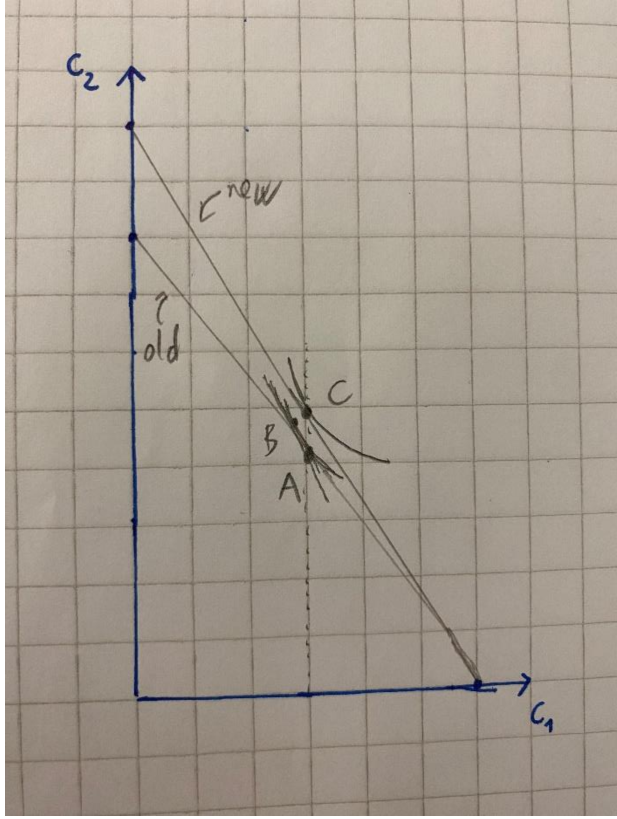
Q6

c.

With the 2 effects, it is still optimal for students to borrow $\frac{100}{1+r}$ to consume in period 1 (less than in part b) and consume 100 in period 2 (same as in part b).

With the 2 effects, it is still optimal for TF kids to save 150 and consume 150 in period 1 (same as in part b) and consume 150(1+r) in period 2 (more than in part b).

Q6



c. **TF kids**

The old optimal point is A. The new optimal point is C.

The substitution effect is from point A to point B.

TF kids consume more in period 2 and less in period 1

(consumption in period 1 is more expensive)

The income effect is from point B to point C.

TK kids consume more in both periods because they are richer (they only have income in period 1).

Q6

c. **Students**

The old optimal point is A. The new optimal point is C.

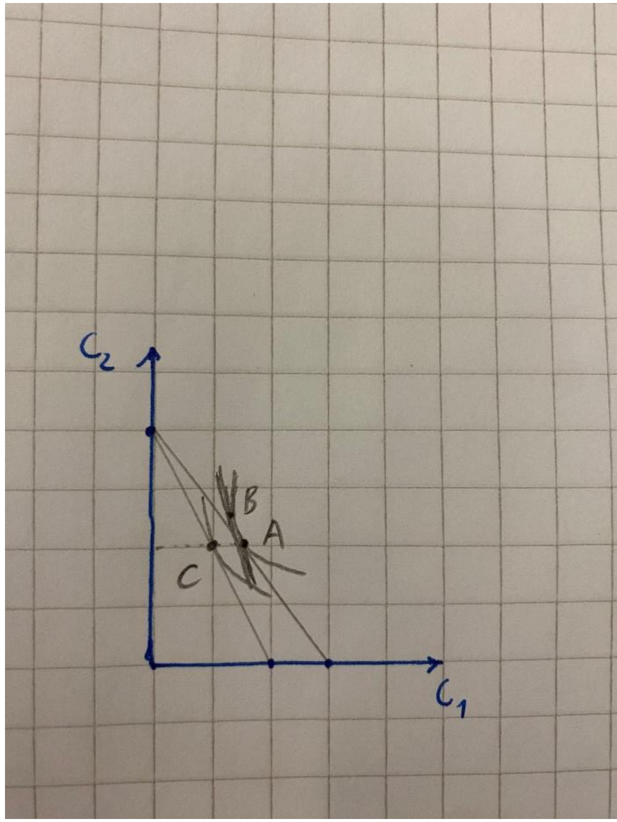
The substitution effect is from point A to point B.

Students consume more in period 2 and less in period 1

(consumption in period 1 is more expensive)

The income effect is from point B to point C.

Students consume less in both periods because they are poorer (they only have income in period 2).



Q6

d. Call the number of trust fund kids n

Total savings: $150 * n$

The number of students $2n$

Total borrowings: $\frac{100}{1+r} * 2n$

$$150 * n = \frac{100}{1+r} * 2n$$

$$1 + r = \frac{200}{150}$$

$$r = \frac{1}{3}$$