ELEC-E8116 Model-based control systems
exercise 7

1. Let the weight of the sensitivity function be given as

$$\frac{1}{W_s} = A\frac{\frac{s}{A\omega_0}+1}{\frac{s}{B\omega_0}+1}, \quad 0 < A << 1,\ B >> 1$$

Sketch a schema for the magnitude plot of the frequency response and investigate its characteristics. What is the slope in the inceasing part of the curve? What is the magnitude at frequency ω_0?

Generate a second order model, where the slope is twice as large as in the previous case. Investigate again the characteristics. What is the magnitude at frequency ω_0?

2. Consider the angular frequencies $\omega_B, \omega_c, \omega_{BT}$ which are used to define the bandwidth of a controlled system. State the definitions. Prove that when the phase margin is less than 90 degrees ($PM < \pi/2$) it holds $\omega_B < \omega_c < \omega_{BT}$. Interpretations?

3. Consider a SISO-system. The maximum values of the sensitivity and complementary functions are denoted M_S and M_T , respectively. Let the gain and phase margins of a closed-loop system be *GM* (gain margin) and *PM* (phase margin). Prove that

$$GM \geq \frac{M_S}{M_S - 1} \qquad PM \geq 2\arcsin\left(\frac{1}{2M_S}\right) \geq \frac{1}{M_S} \text{ [rad]}$$

$$GM \geq 1 + \frac{1}{M_T} \qquad PM \geq 2\arcsin\left(\frac{1}{2M_T}\right) \geq \frac{1}{M_T} \text{ [rad]}$$