

Aalto university

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Exercise sheet 1

Complex Analysis, MS-C1300.

Done during class Tuesday 24.10 or Wednesday 25.10.

- (1) (a) Assume that $z \neq 1$. Prove that

$$\sum_{j=0}^n z^j = \frac{1 - z^{n+1}}{1 - z}.$$

- (b) Find all solutions $z \in \mathbb{C}$ such that

$$z^{n-1} + z^{n-2} + \cdots + z + 1 = 0$$

where $n > 1$ is an integer.

- (2) For $z \in \mathbb{C}$ we have either $\sqrt{z^2} = z$ or $\sqrt{z^2} = -z$ (where $\sqrt{\cdot}$ is the principal square root). For which z does $\sqrt{z^2} = z$ hold? For which z does $\sqrt{z^2} = -z$ hold?
- (3) Verify that $2 \operatorname{Arg}(1 + z) = \operatorname{Arg}(z)$ when $|z| = 1$, but $z \neq -1$. (*Hint:* What is the set $K = \{1 + z; |z| = 1\}$?)
- (4) If n is a positive integer, prove that

$$1 + \cos \theta + \cos 2\theta + \cdots + \cos n\theta = \frac{1}{2} + \frac{\sin((2n+1)\theta/2)}{2 \sin(\theta/2)}$$

unless θ is a multiple of 2π .