

Aalto university

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Exercise sheet 2

Complex Analysis, MS-C1300.

Hand in exercise 1 and 2 in separate files for grading. Deadline Wednesday 25.10 at 23:59. The exercises should be uploaded to the correct folder on MyCourses as pdf-files with name and student number in the file name. **Submission via MyCourses is the only accepted way.** Done during class Thursday 26.10 or Friday 27.10.

- (1) Express the following in the form $x + iy$:
- (a) $\text{Log}(-e^2)$ (2p)
 - (b) $(-1)^i$ (2p)
 - (c) $i^{\sqrt{2}i}$ (2p)

(Remember that you should use the principal argument when calculating powers. That is, $z^w = e^{w\text{Log}(z)}$.)

- (2) Let $A = \{z; \text{Re}(z) > 0\}$ and let $f: A \rightarrow \mathbb{C}$ be given by $f(z) = \text{Log}(1 + z^2)$. Determine the range $B = f(A)$. (*Hint:* Look at f as a composition of $g(z) = z^2$, $h(z) = 1 + z$, and $k(z) = \text{Log } z$.) (6p)

- (3) Show that the locus of points in the complex plane satisfying

$$Az\bar{z} + Bz + \bar{B}\bar{z} + C = 0$$

in which A and C are real numbers satisfying

$$|B|^2 - AC > 0$$

is a circle when $A \neq 0$ and a line if $A = 0$. (The converse also holds but you don't need to show this.)

- (4) Compute:
- (a) all square roots of $-1 + i\sqrt{3}$.
 - (b) all cube roots of -8 .