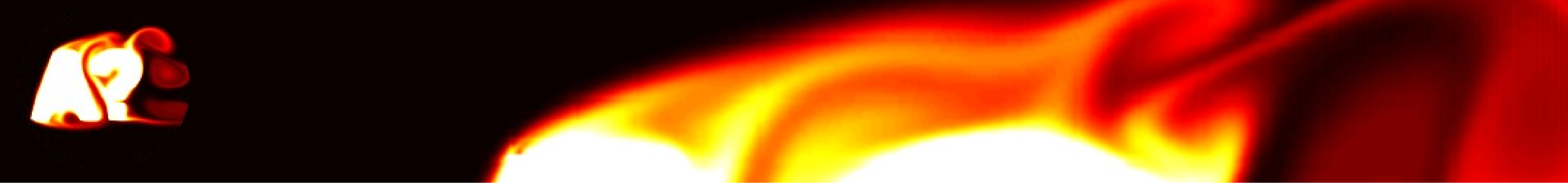
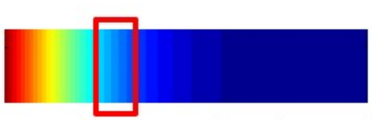


EEN-1020 Heat transfer

Week 3: Convective Heat Transfer, Internal Flow and Numerical Solution in 2d

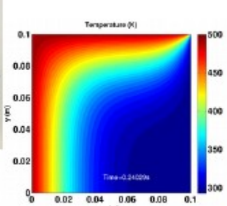
Prof. Ville Vuorinen
November 7th-8th 2023
Aalto University, School of Engineering

Week 1: Energy conservation, heat equation, conduction Fourier/Newton

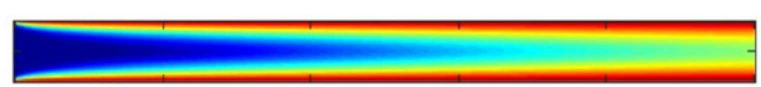


$$\frac{\partial T}{\partial t} \approx \frac{1}{\Delta x}\left(k\frac{\partial T(x+\Delta x/2,t)}{\partial x} - k\frac{\partial T(x-\Delta x/2,t)}{\partial x}\right)$$

Week 2: Fin theory, conduction, intro to convection

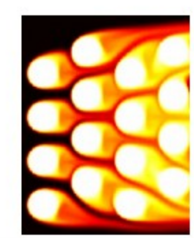


Week 3: convective heat transfer – internal flow (channel)



Week 4: convective heat transfer – external flow (fin systems)

Week 5: natural convection, boiling, correlations



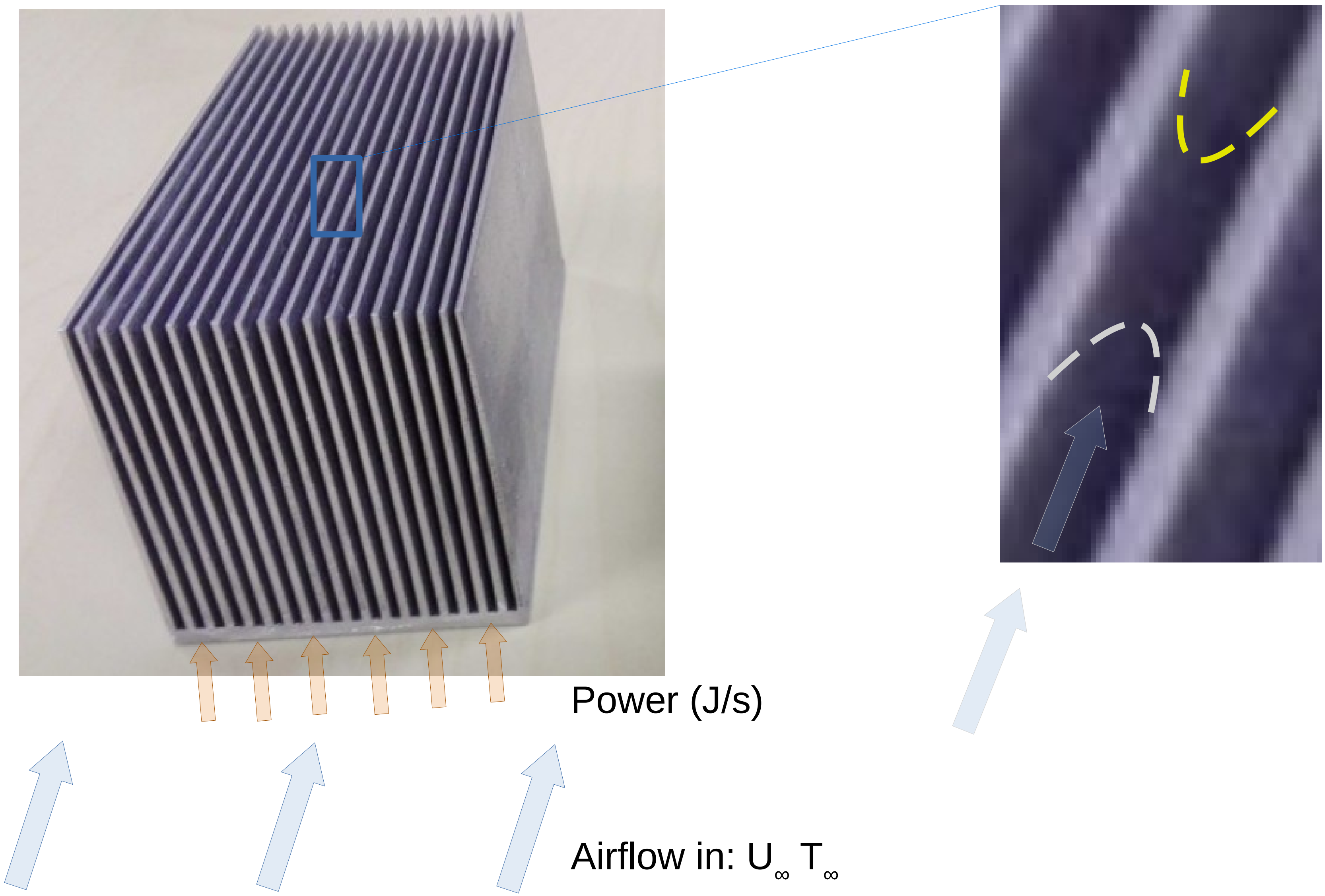
On the heat transfer course, we have "5 friends" i.e. 5 main principles that are used to explain heat transfer phenomena

1) Energy conservation: "J/s thinking"
2) Fourier's law
3) Newton's cooling law
4) Energy transport equation – convection/diffusion equation
5) Momentum transport equation – Navier-Stokes equation

Lecture 3.1 Theory: Flow through a fin system, governing equations and analysis

ILO 3: Student can write the governing equations of fluid/heat flow in a channel, estimate the energy balance and estimate temperature rise for different heating conditions. The student can confirm the channel heat transfer using generated/provided simulation data.

Air flows between the fins.
Heat transfers from the hot fin surfaces to the gas.



Airflow Temperature

Common wall boundary conditions:

Type 1: T_s = known

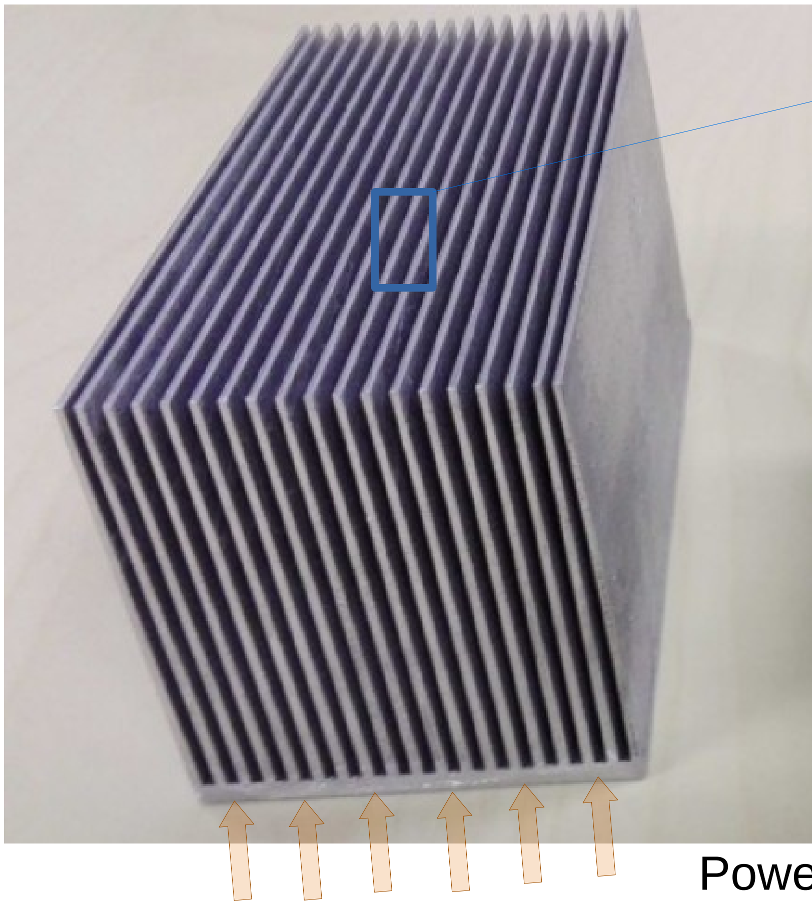
Type 2: q_s = known

Airflow velocity

Wall boundary condition:

No-slip:
U=V=W=0m/s

Air flows between the fins. Heat transfers from the hot fin surfaces to the gas.



Airflow Temperature

Common wall boundary conditions:

Type 1: T_s = known

Type 2: q_s = known

Airflow velocity

Wall boundary condition:

No-slip: U=V=W=0m/s

Power (J/s)

Airflow in: U_∞ T_∞

$$-k_f\left(\frac{\partial T}{\partial y}\right)_{y=wall} = h\left(T_s - T_{mean}\right)$$

Energy balance (J/s thinking) for gas flow when the gas is heated at power P (W). "Control volume" thinking.

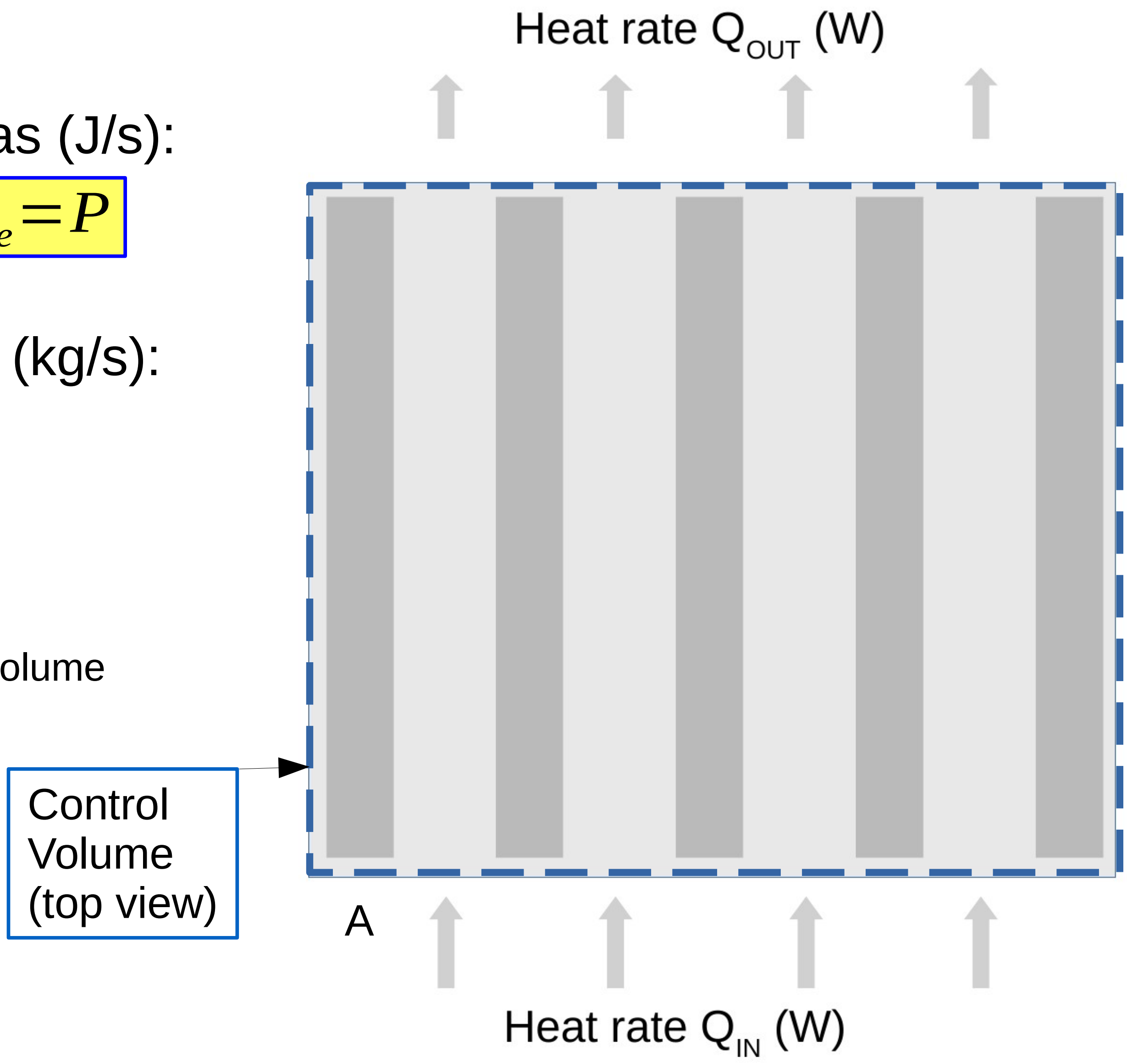
Energy balance for the gas (J/s):

$$Q_{OUT} - Q_{IN} = c_p \dot{m} \Delta T_{ave} = P$$

Mass flow rate of the gas (kg/s):

$$\dot{m} = \rho U A$$

A=cross-sectional area of control volume
U=flow average velocity



Fluid dynamics: Navier-Stokes equation for gases and liquids

Navier-Stokes equation
(conservation of momentum)

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 U}{\partial x^2}+\nu\frac{\partial^2 U}{\partial y^2}$$

$$\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\frac{\partial^2 V}{\partial x^2}+\nu\frac{\partial^2 V}{\partial y^2}$$

Time derivative | Convection terms | Pressure gradient | Diffusion terms

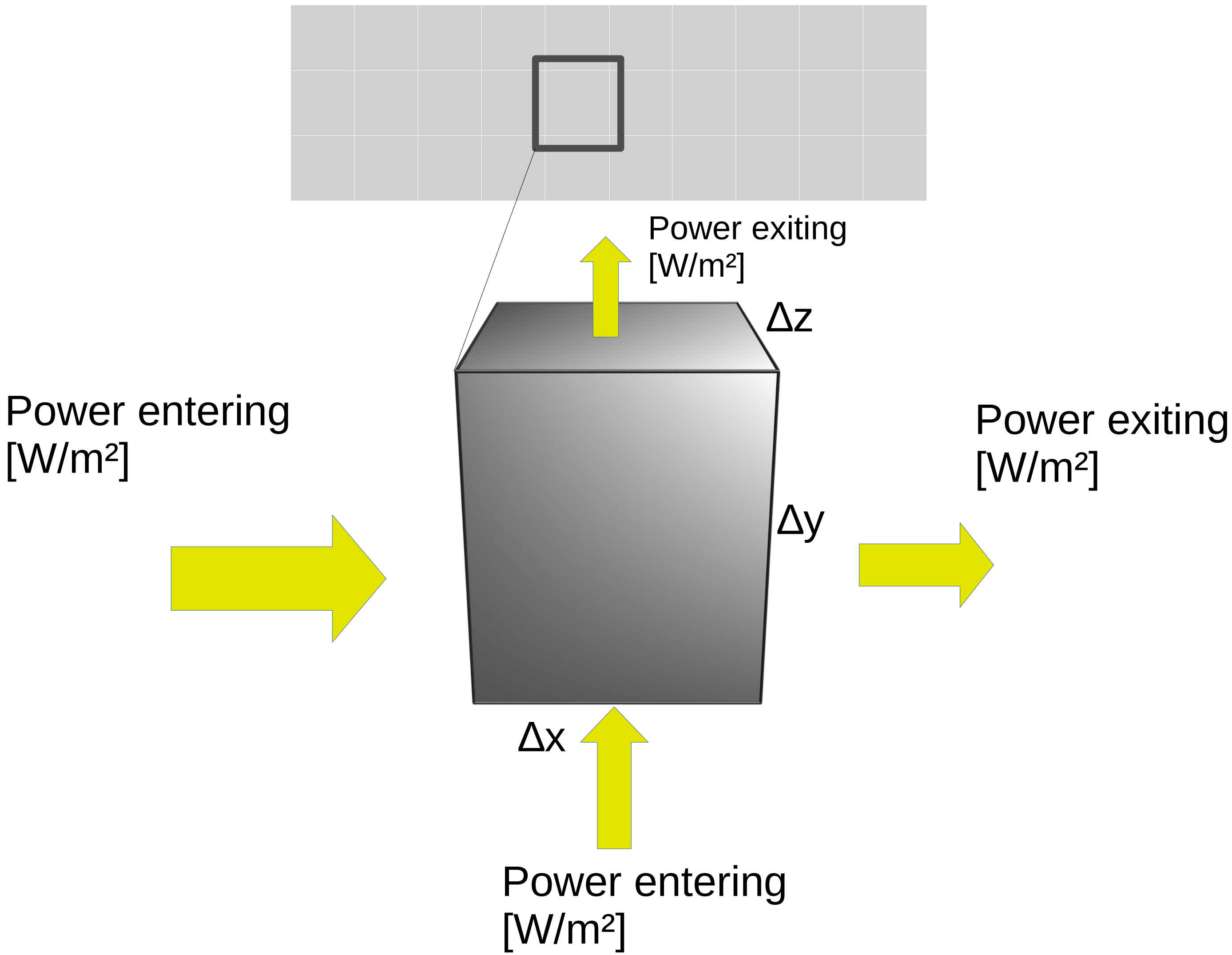
Continuity equation
(conservation of mass)

$$\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}=0$$

Kinematic viscosity: $\nu=\mu/\rho, [\nu]=m^2/s$

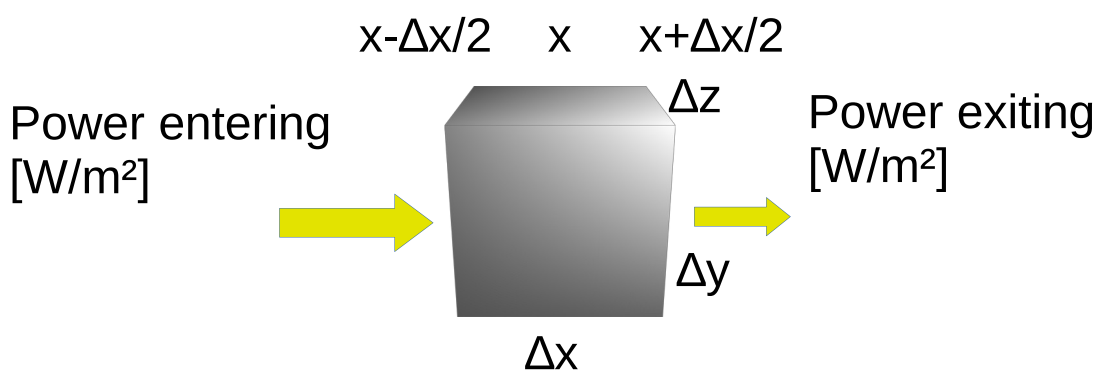
Thermodynamics&heat transfer:

Consider heat conduction and convection in 2d or 3d **fluid (gas or liquid).**
Divide the space into small (fixed) elements and carry out energy balance analysis for 1 of those elements.



Derivation of convection-diffusion equation:

Next we apply energy conservation law (“J/s thinking”) for a small infinitesimal volume assuming conduction only (e.g. gas flow between two fins)



Conduction: energy change of element due to heat fluxes in x-direction during Δt (J):

$$\Delta Q_x = [k \frac{\partial T(x+\Delta x/2, y, t)}{\partial x} - k \frac{\partial T(x-\Delta x/2, y, t)}{\partial x}] \Delta y \Delta z \Delta t$$

Convection: energy change of element due to velocity transporting heat in x-direction during Δt (J):

$$\Delta C_x = c_p \rho [-U(x+\Delta x/2, y, t) T(x+\Delta x/2, y, t) + U(x-\Delta x/2, y, t) T(x-\Delta x/2, y, t)] \Delta y \Delta z \Delta t$$

Energy change of element during Δt (J):

$$\rho c_p \Delta T(x, y, z, t) \Delta x \Delta y \Delta z = \Delta Q_x + \Delta Q_y + \Delta Q_z + \Delta C_x + \Delta C_y + \Delta C_z$$

Then: Divide both sides by $\Delta x \Delta y \Delta z \Delta t$ and take the limit when all Δ-variables → 0 → We get the convection diffusion equation.

Convection-diffusion equation

→ Thermal energy is transported by convection (flow velocity) and diffusion (conduction).
→ The convection diffusion equation for temperature is simply energy conservation law on local level of the fluid.

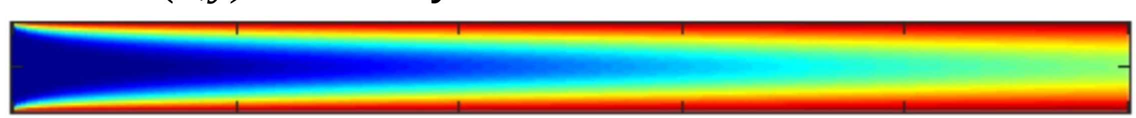
$$\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial x^2} + \alpha \frac{\partial^2 T}{\partial y^2}$$

T changes in given position in time due to convection and diffusion

T is transported by velocity field (convection)

T is transported by thermal diffusion (diffusion/conduction)

$T=T(x,y)$ in steady state 2d laminar channel flow



Air temperature distribution in a plate fin heat exchanger (cross section)

- Thermal boundary layers develop on surfaces

- Free boundary layers on outer surfaces

Focus on single gap

U_∞

T_∞

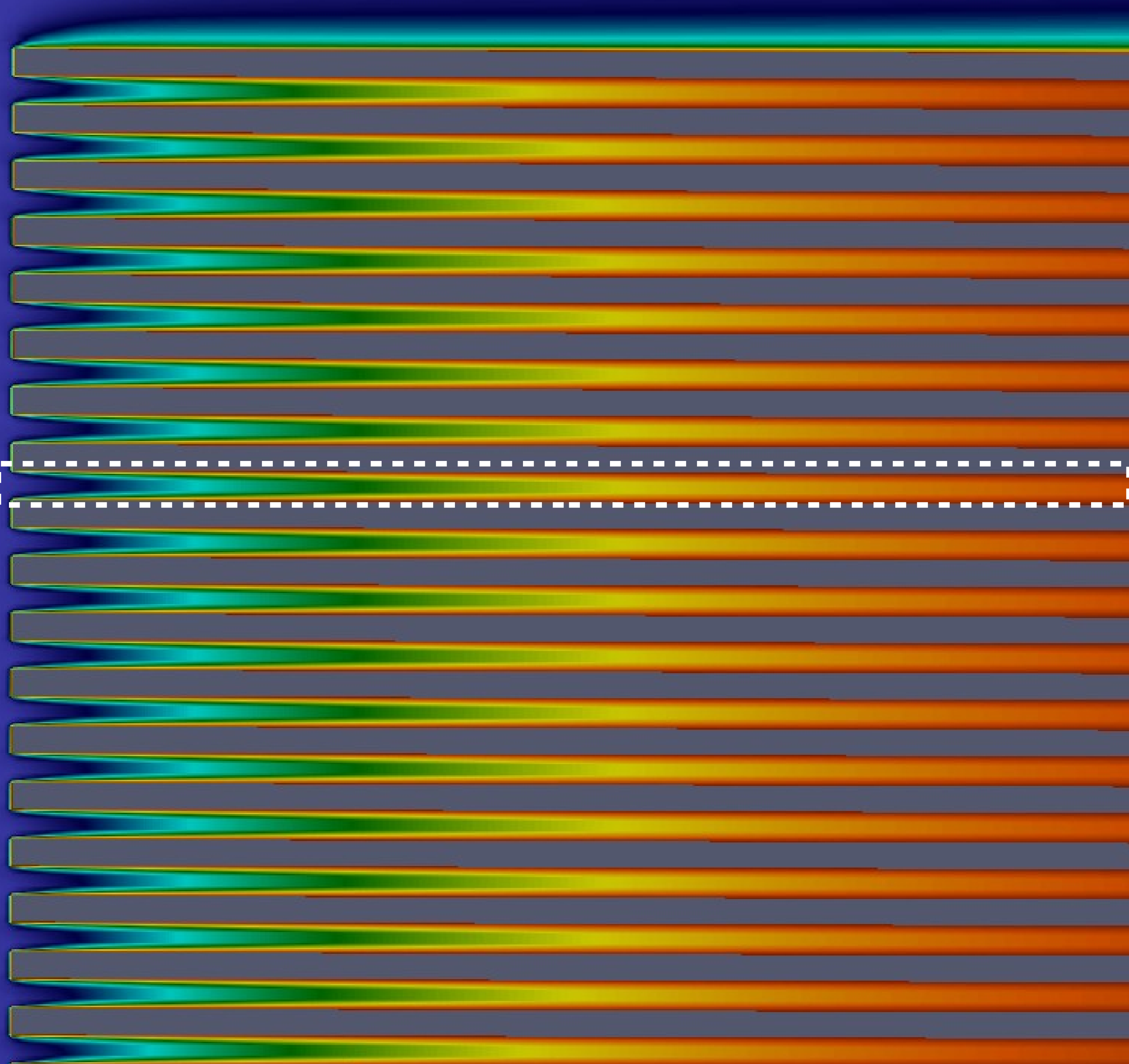


Figure: courtesy of P.Peltonen

Fluid dynamical and heat transfer conditions

Velocity scale — Reference length scale

$$\text{Reynolds number: } \mathrm{Re} = \frac{U\,L}{\nu}$$

Kinematic viscosity

$$\text{Prandtl number: } \mathrm{Pr} = \frac{\nu}{\alpha} = \frac{\text{Viscous diffusion}}{\text{Thermal diffusion}} = \frac{\mu/\rho}{k/(c_p\,\rho)}$$

Heat transfer coeff. — Reference length scale

$$\text{Nusselt number: } \mathrm{Nu} = \frac{h\,L}{k} = \frac{\text{Total heat transfer}}{\text{Conductive heat transfer}}$$

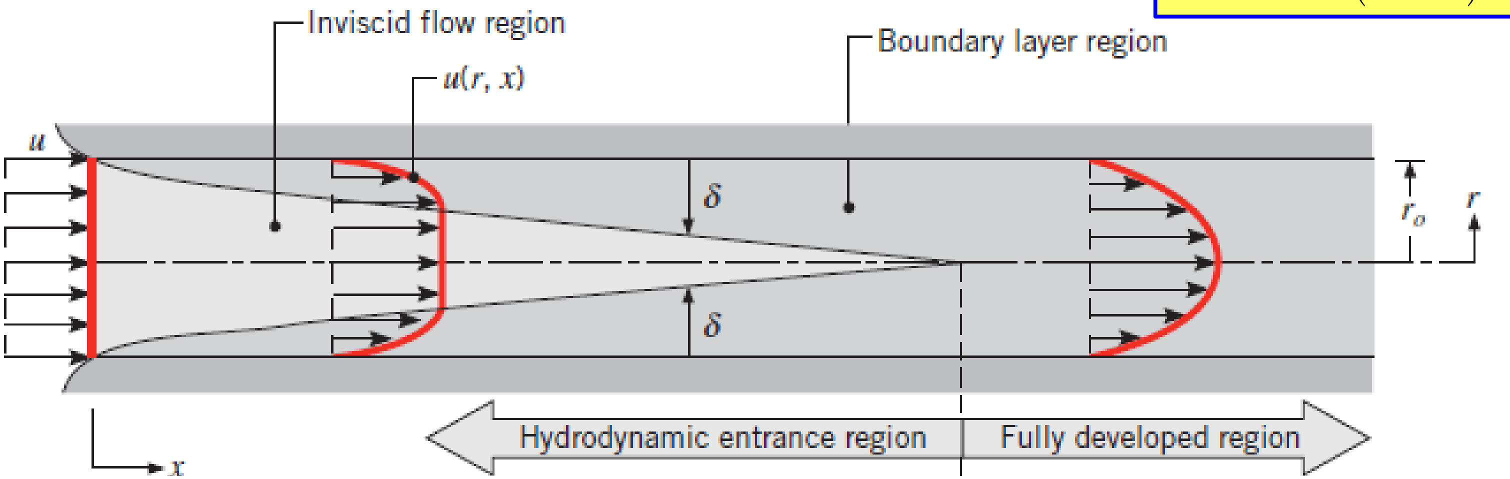
Figs. from Incropera, de Witt (Principles of Heat and Mass Transfer)

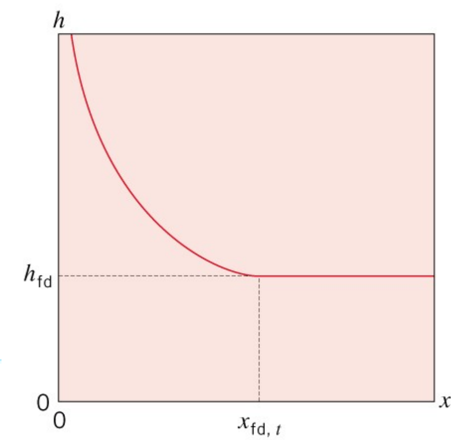
Entry region in laminar pipe/channel flow

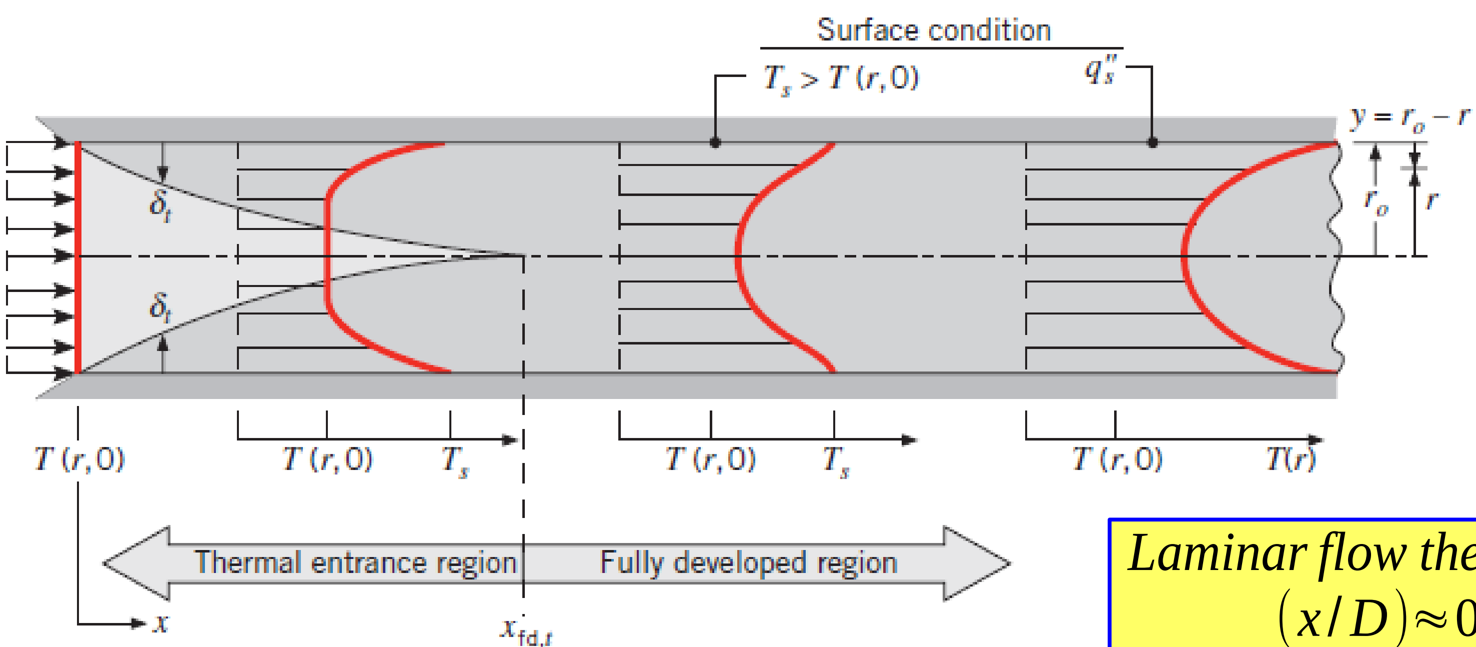
Velocity profile

Laminar flow viscous entry length:

$$(x/D) \approx 0.05\,\mathrm{Re}_D$$







Laminar flow thermal entry length:

$$(x/D) \approx 0.05\,\mathrm{Re}_D\,Pr$$

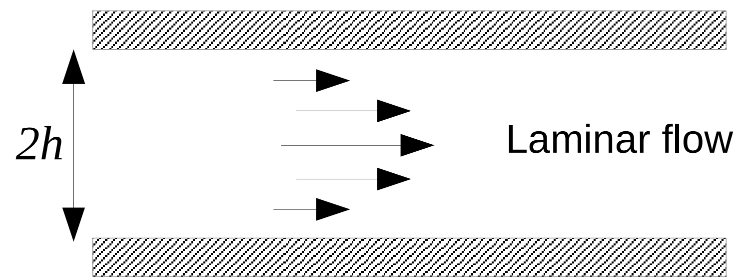
Temperature profile

Channel flow velocity between two fins can be analytically solved assuming **1)** steady state i.e. does not change in time, **2)** fully developed laminar flow (Re<2000) with constant pressure gradient, **3)** flow is only in x-direction i.e. U=U(y), V=0

$$U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}=\frac{-1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 U}{\partial x^2}+\nu\frac{\partial^2 U}{\partial y^2}$$

$$\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}=0$$

$$const.=\frac{1}{\rho}\frac{\partial p}{\partial x}=\nu\frac{\partial^2 u}{\partial y^2}$$



$$u(y)=u_{max}(1-\frac{y^2}{h^2})$$

$$u_{max}=-\frac{dp}{dx}\frac{h^2}{2\rho\nu}$$

Wall boundary conditions
Velocity: No-slip wall $u(+h) = u(-h) = 0$

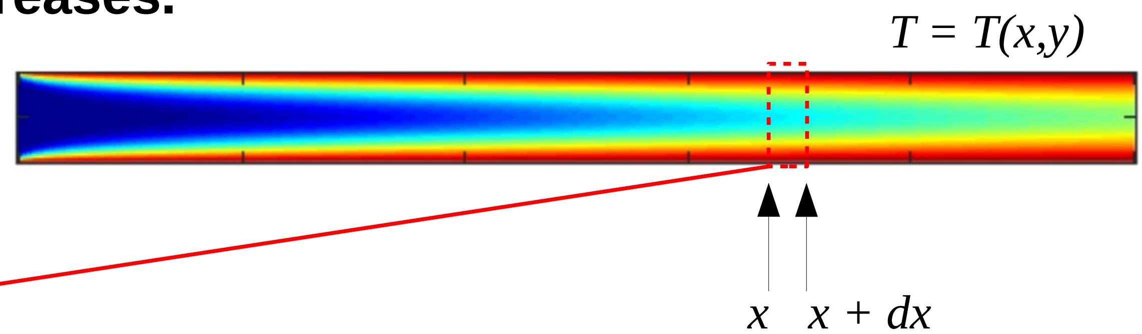
Practical question 1: two parallel plates are heated.
How long distance should the fluid travel between the plates to reach a target temperature?

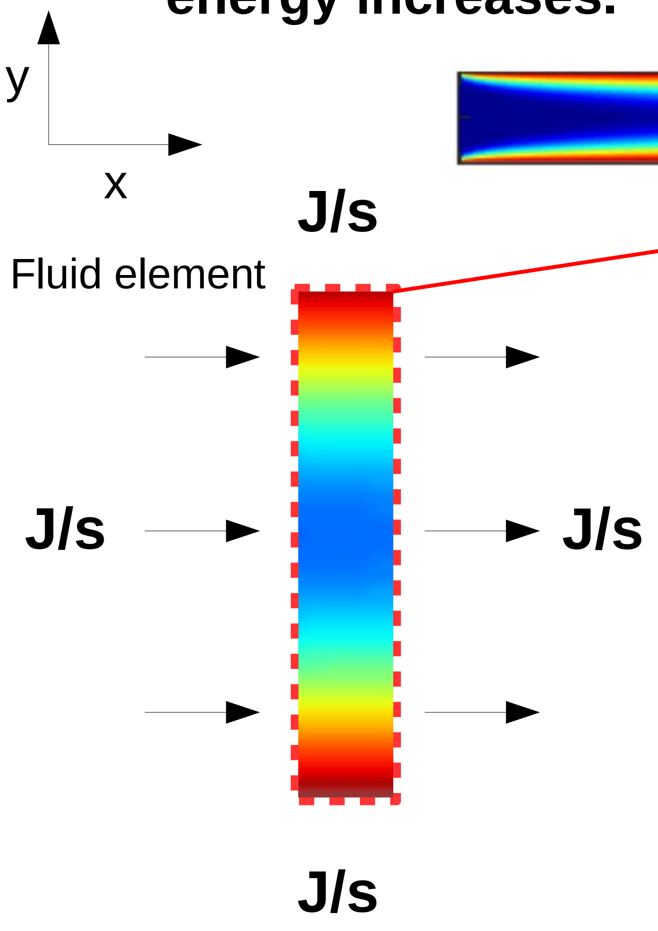
→ Need to find $T_m(x)$

Energy balance for a fluid element between heated parallel plates

(**relevance:** finding mean temperature in streamwise direction)

Wall provides a heat flux q [W/m^2] to the fluid so that a fluid element thermal **energy increases.**





L_z = channel depth out of plane

D = channel diameter

T_m = mean temp.

Fluid element energy increase during $x \ldots x+dx$

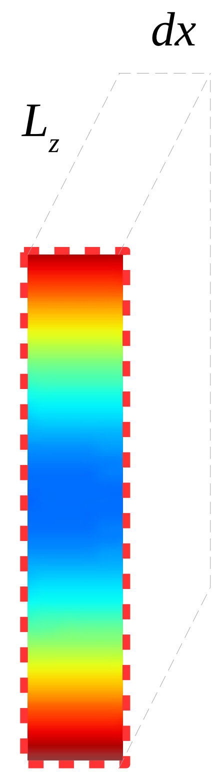
$$\rho c_p U D L_z [T_m(x+dx) - T_m(x)] = 2 q \, dx \, L_z$$

"J/s"="J/s" → units match

Mean temperature obeys an ordinary diff. eqn.

$$\frac{dT_m}{dx} = \frac{2q}{c_p \rho U D}$$

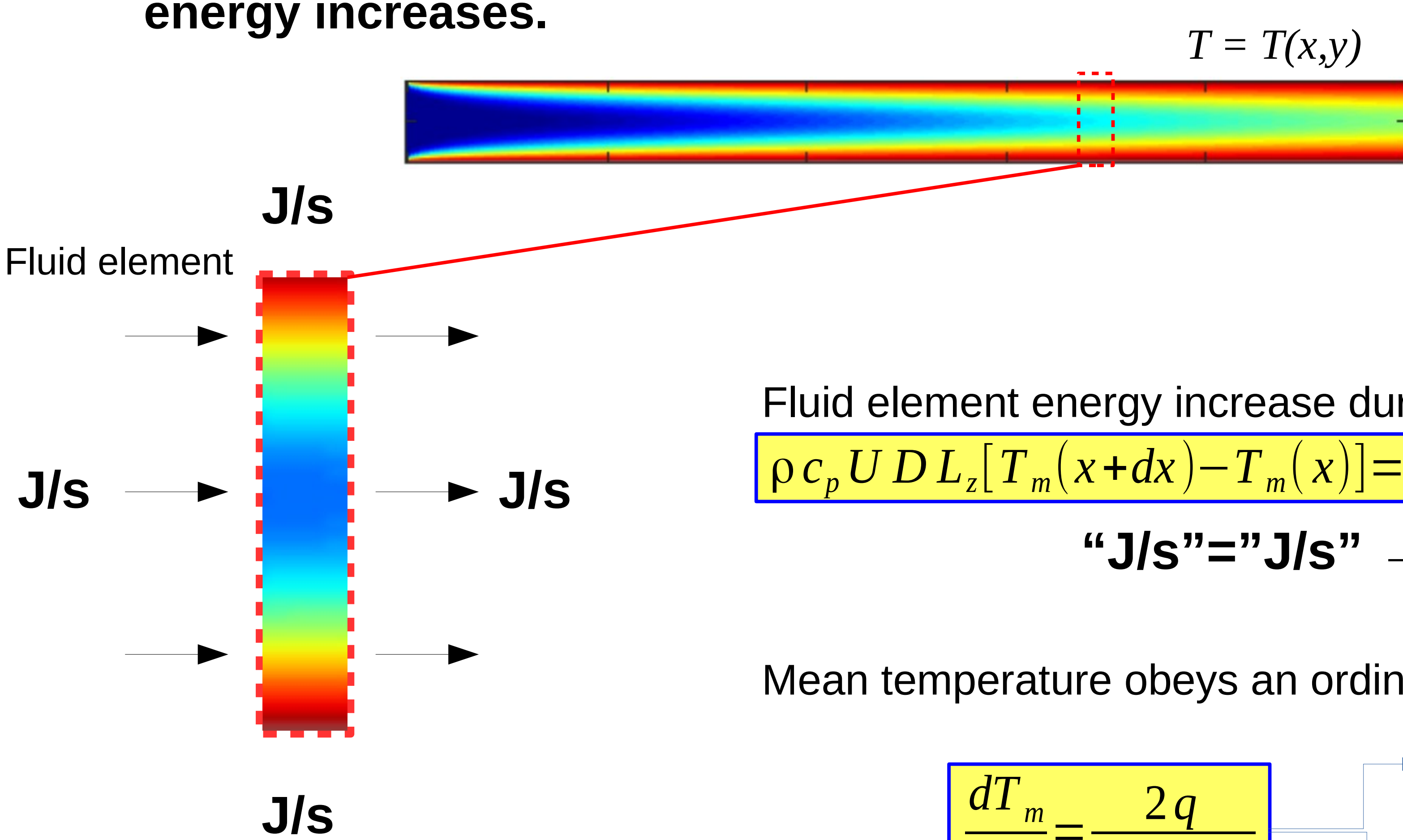
Note that the length of the surface element in z-direction cancels out



Power escaping through top and bottom plates $= 2\, q\, dx\, L_z$

Energy balance for a fluid element between heated parallel plates (**relevance:** finding mean temperature in streamwise direction)

Wall provides a heat flux q [W/m^2] to the fluid so that a fluid element thermal **energy increases.**



L_z = channel depth out of plane

D = channel diameter

T_m = mean temp.

Fluid element energy increase during $x \ldots x+dx$

$$\rho\, c_p\, U\, D\, L_z [T_m(x+dx) - T_m(x)] = 2\, q\, dx\, L_z$$

"J/s"="J/s" → units match

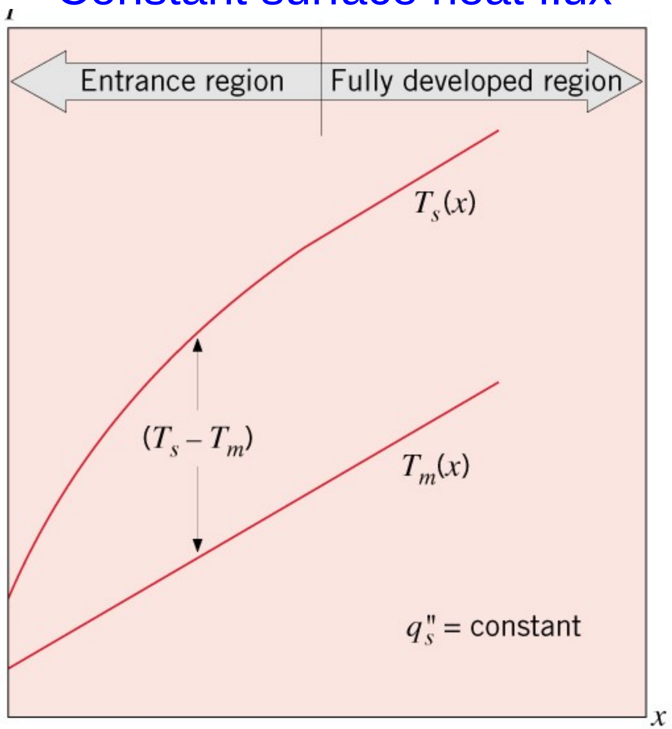
Mean temperature obeys an ordinary diff. eqn.

$$\frac{dT_m}{dx} = \frac{2\,q}{c_p \rho\, U\, D}$$

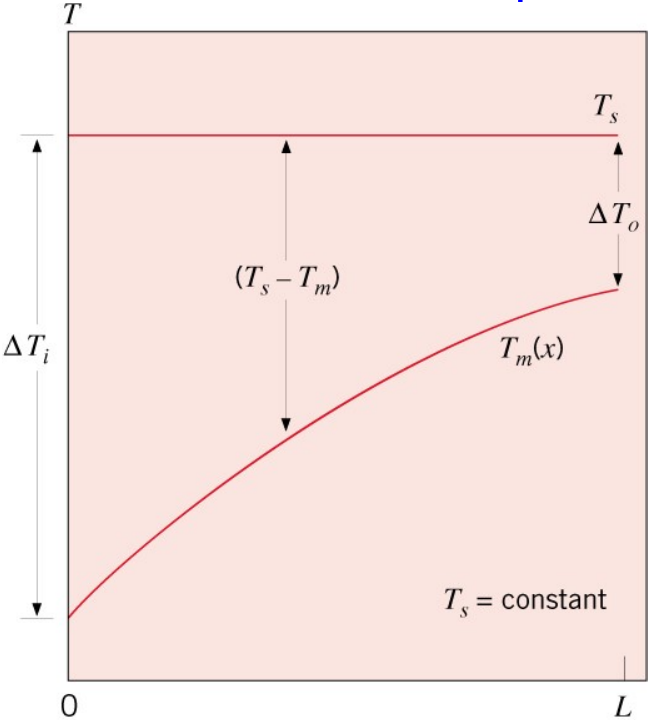
- T_s = fixed → need for Newton's cooling law to get q
- q = fixed → integrate directly

Axial mean temperature in a pipe or channel

Constant surface heat flux



Constant surface temperature



Figs. from Incropera, de Witt (Principles of Heat and Mass Transfer)

For constant surface heat flux

Notes:

1) q is const. → Surface temperature T_s follows.

2) Surface temperature $T_s = T_s(x)$. If the surface is heated then T_s must increase along the channel when $T_m(x)$ increases.

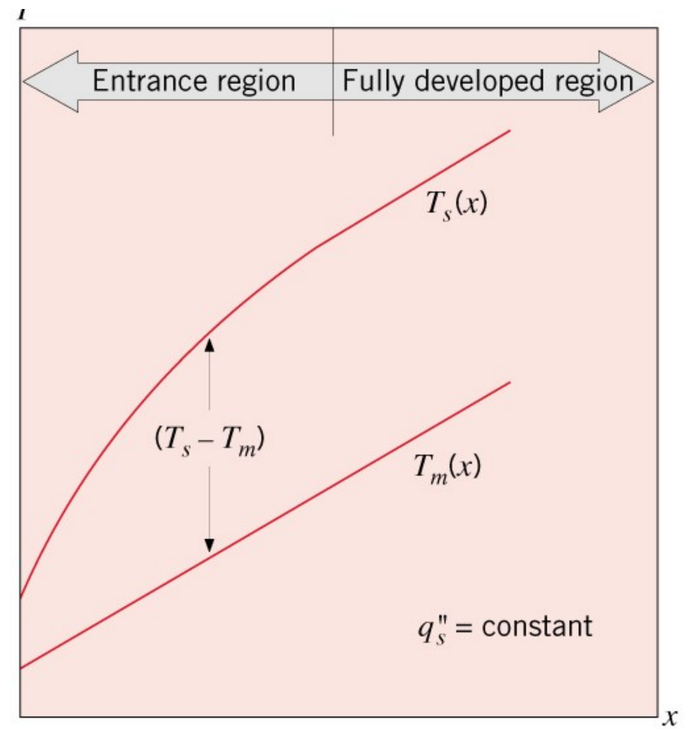
3) Newton's Cooling law states: $T_s(x) - T_m(x) = const.$

$$\frac{dT_m}{dx} = \frac{2q}{c_p \rho U D} = \text{constant}$$

$$\int_{x=0}^{x} \frac{dT_m}{dx} dx = \int_{x=0}^{x} \frac{2q}{c_p \rho U D} dx$$

$$T_m(x) = T_m^{\text{in}} + \frac{2q}{c_p \rho U D} x$$

→ Linear increase in mean temperature



For constant surface temperature:
at fully developed conditions when *h=const.*

After thermal entry region

$$Nu = \frac{hD}{k_{fluid}} \approx 7.52$$

$$\frac{dT_m}{dx} = \frac{2q(x)}{c_p \rho U D} = \frac{2h(T_s - T_m)}{c_p \rho U D}$$

$$\int_{T_m = T_{in}}^{T_m(x)} \frac{dT_m}{T_s - T_m} = \int_{x=0}^{x} \frac{2h}{c_p \rho U D} dx$$

$$\log \frac{T_m(x) - T_s}{T_{in} - T_s} = \frac{-2h}{c_p \rho U D} x$$

$$\frac{T_m(x) - T_s}{T_{in} - T_s} = \exp\left(\frac{-2h}{c_p \rho U D} x\right)$$

→ Mean temperature increases according to exp function

→ Total heat flux can be calculated based on log mean temperature

$$q_{tot} = h A \Delta T_{lm}$$

The main points:

0) We do not know q_{tot} because when T_s fixed then heat flux follows.

1) T_s-$T_m(x)$ is not constant i.e. $q=q(x)$.

2) Thus, one can not use the average of inlet and outlet temperature in Newton's law directly because mean temp. increases non-linearly.

3) Need for log-mean temperature concept.

See: Incropera Ch. 8 Eqn. (8.43)

Practical question 2: two parallel plates are heated.
The plate thickness is d and the temperature outside the plates is known. How long distance should the fluid travel to reach a target temperature?

→ Need to find $T_m(x)$

Let's think that the fluid in the channel is warm and outside cooler. Thus the flowing fluid cools in the channel. ($T_m(x)$ = ?)

We can express the lost heat (J/s):

(1) Warm fluid to surface (W):

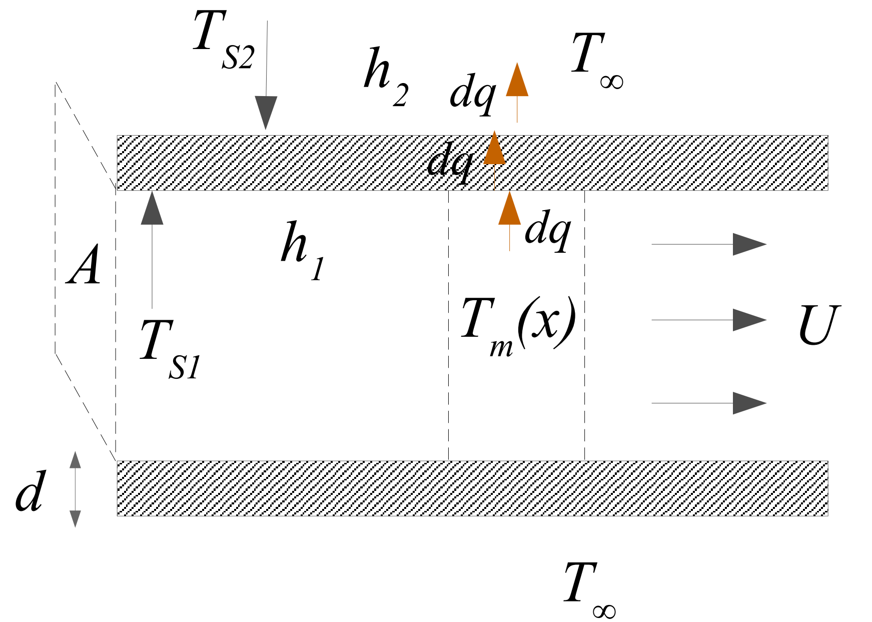
$$dq = h_1 (T_m - T_{S1}) dA$$

(2) Through the solid (W):

$$dq = k \frac{T_{S1} - T_{S2}}{d} dA$$

(3) From solid outer surface to ambient (W):

$$dq = h_2 (T_{S2} - T_\infty) dA$$



Total heat transfer coefficient h_{tot} (W/m²K) can be easily solved from (1)-(3):

$$h_{tot} = \frac{1}{\frac{1}{h_1} + \frac{1}{h_2} + \frac{d}{k}}$$

Energy balance for the cooling fluid element:

$$\rho U A c_p dT_m = \rho U L_z D c_p dT_m = -2 h_{tot} (T_m(x) - T_\infty) L_z dx$$

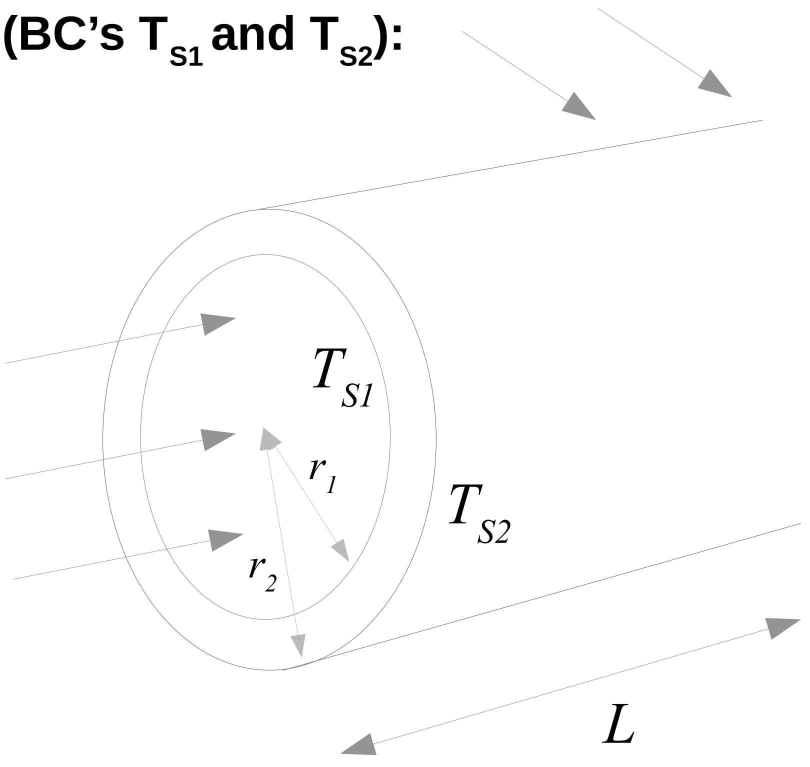
$$\frac{dT_m}{dx} = \frac{-2 h_{tot}}{\rho U D c_p} (T_m(x) - T_\infty)$$

$$\frac{T_m(x) - T_s}{T_{in} - T_s} = \exp\left(\frac{-2 h_{tot}}{c_p \rho U D} x\right)$$

In HW you need to use cylindrical coordinates to understand how the total heat transfer coefficient is then formed. (Sec. 3.3/Incropera)

Steady state heat eqn in cylindrical coordinates (BC's T_{S1} and T_{S2}):

$$\frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right)=0$$



Heat rate across a cylindrical surface

$$q_r=-k(2\pi r L)\frac{dT}{dr}$$

We can integrate heat eqn twice to obtain:

$$T(r)=\frac{T_{S1}-T_{S2}}{\ln(r_1/r_2)}\ln\left(\frac{r}{r_2}\right)+T_{S2}$$

→ **Heat transfer rate (W):**

$$q_r=(2\pi L k)\frac{(T_{S1}-T_{S2})}{\ln(r_2/r_1)}$$

Thermal resistance:

$$R_{cond}=\frac{\ln(r_2/r_1)}{2\pi L k}$$

Please see Sec. 3.3 for more help wrt HW.

The table below illustrates Nusselt numbers (non-dim.heat trans.coefficient) for different channel types with different boundary conditions. D_h = hydraulic diameter.

Cross Section	$\frac{b}{a}$	$Nu_D \equiv \frac{hD_h}{k}$ (Uniform q''_s)	$Nu_D \equiv \frac{hD_h}{k}$ (Uniform T_s)	$f\,Re_{D_h}$
	—	4.36	3.66	64
a, b	1.0	3.61	2.98	57
a, b	1.43	3.73	3.08	59
a, b	2.0	4.12	3.39	62
a, b	3.0	4.79	3.96	69
a, b	4.0	5.33	4.44	73
b	8.0	6.49	5.60	82
	∞	8.23	7.54	96
Heated / Insulated	∞	5.39	4.86	96
	—	3.11	2.49	53

In HW3 we want to check if
we can get the value
Nu = 7.54 from
numerical simulation.

Table 8.1 from Incropera, de Witt (Principles of Heat and Mass Transfer)

Strong relevance to HW3 - Heat flux balance at the surface:
Fourier's law (physics) equals to Newton's law (engineering)

Diffusive heat flux (Fourier) immediately at the wall on the fluid side =
Heat flux from Newton's law of cooling

$$-k_f \left(\frac{\partial T}{\partial y} \right)_{y=wall} = h\left(T_s - T_{mean}\right)$$

If temperature gradient in wall-normal direction would be known at each x location → we could calculate h (W/m^{2}K) every single surface point

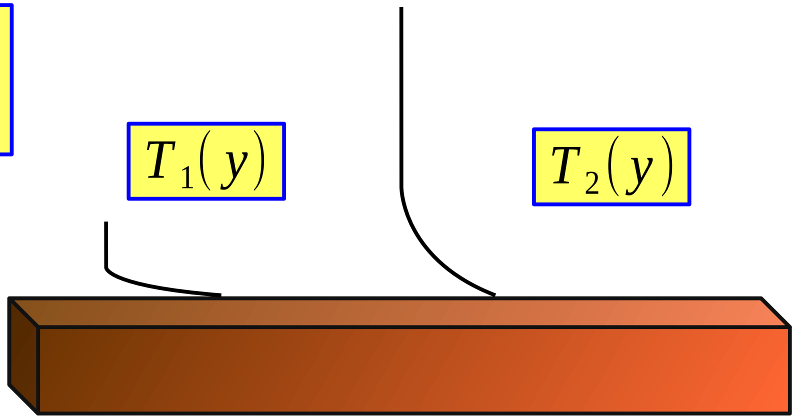


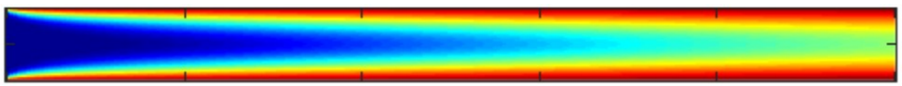
Figure: temperature profiles on bottom wall

Note:
even in convective heat transfer
the heat first diffuses i.e. conducts near the wall because
$u,v \rightarrow 0$ next to the wall

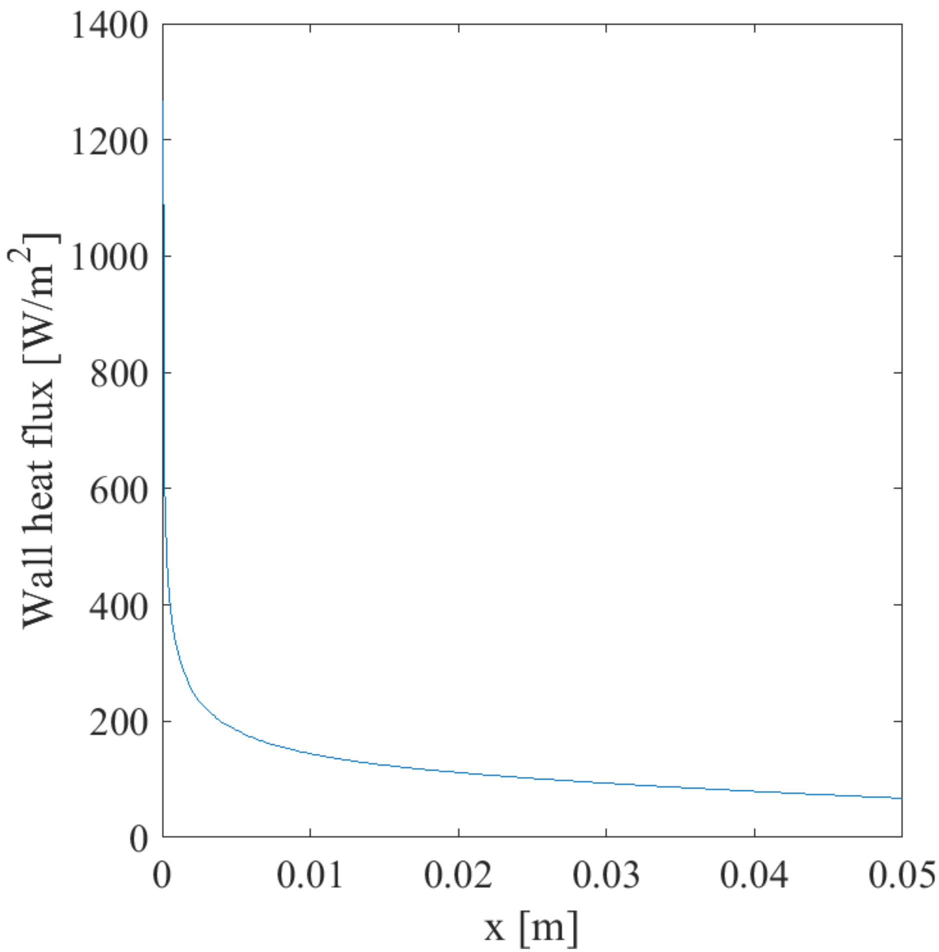
$$h = \frac{-k_f \left(\frac{\partial T}{\partial y} \right)_{y=wall}}{T_s - T_{mean}}$$

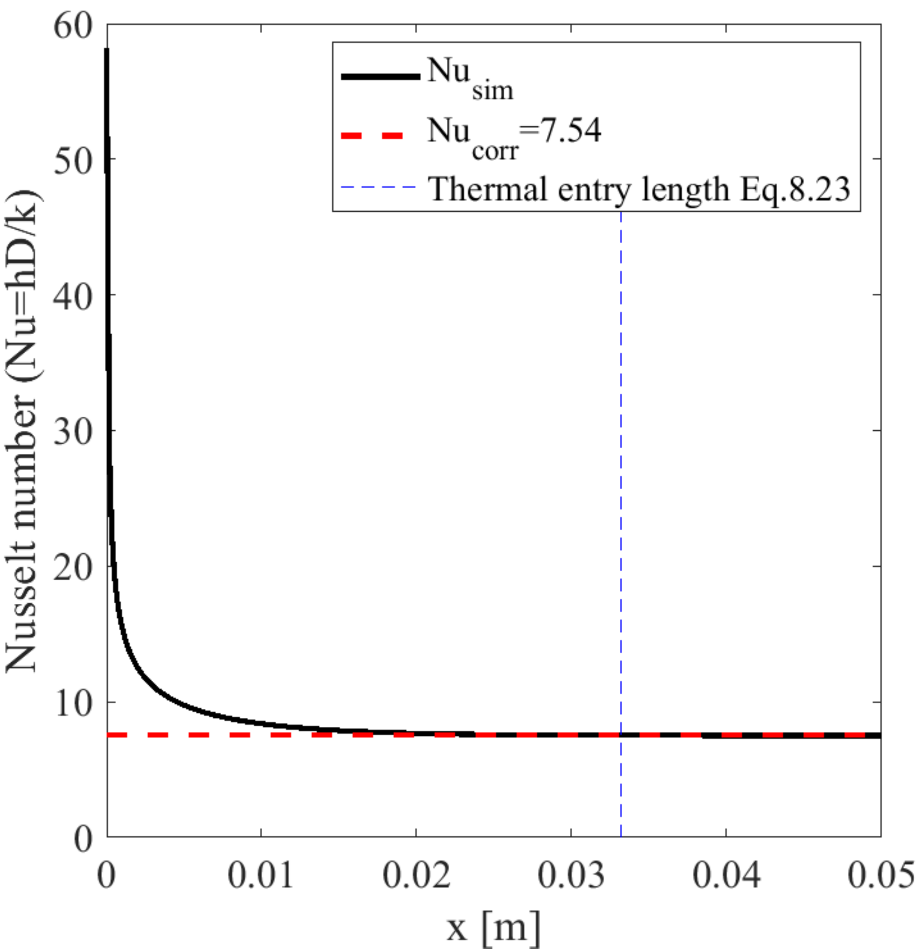
$$[h] = W/m^2K$$

Think:
How can we maximize h ?
How do h and heat flux vary in the flow direction ?



For constant wall temperature BC some example results using code `heat2d.m`





Lecture 3.2 Numerical approach: a Matlab solver for the 2d convection-diffusion equation to describe temperature transport

ILO 3: Student can write the governing equations of fluid/heat flow in a channel, estimate the energy balance and estimate temperature rise for different heating conditions. The student can confirm the channel heat transfer using generated/provided simulation data.