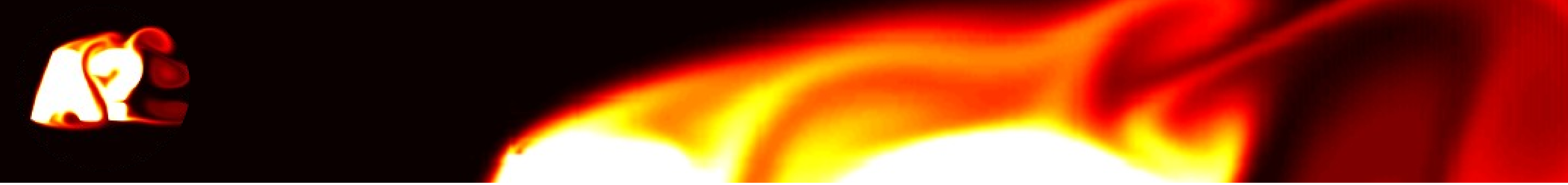


EEN-1020 Heat transfer

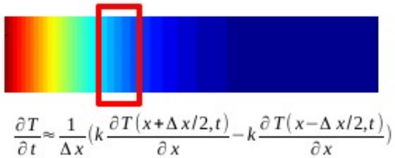
Week 4: Convective Heat Transfer External Flow

Prof. Ville Vuorinen

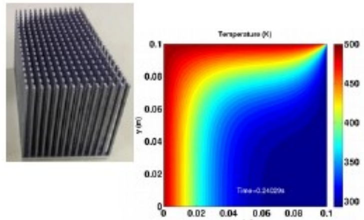
November 14th-15th 2023

Aalto University, School of Engineering

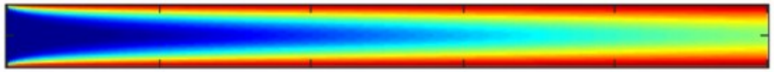
Week 1: Energy conservation, heat equation, conduction Fourier/Newton



Week 2: Fin theory, conduction, intro to convection

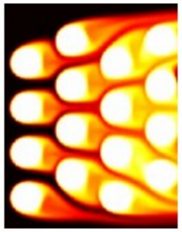


Week 3: convective heat transfer – internal flow (channel)



Week 4: convective heat transfer – external flow (fin systems)

Week 5: natural convection, boiling, correlations



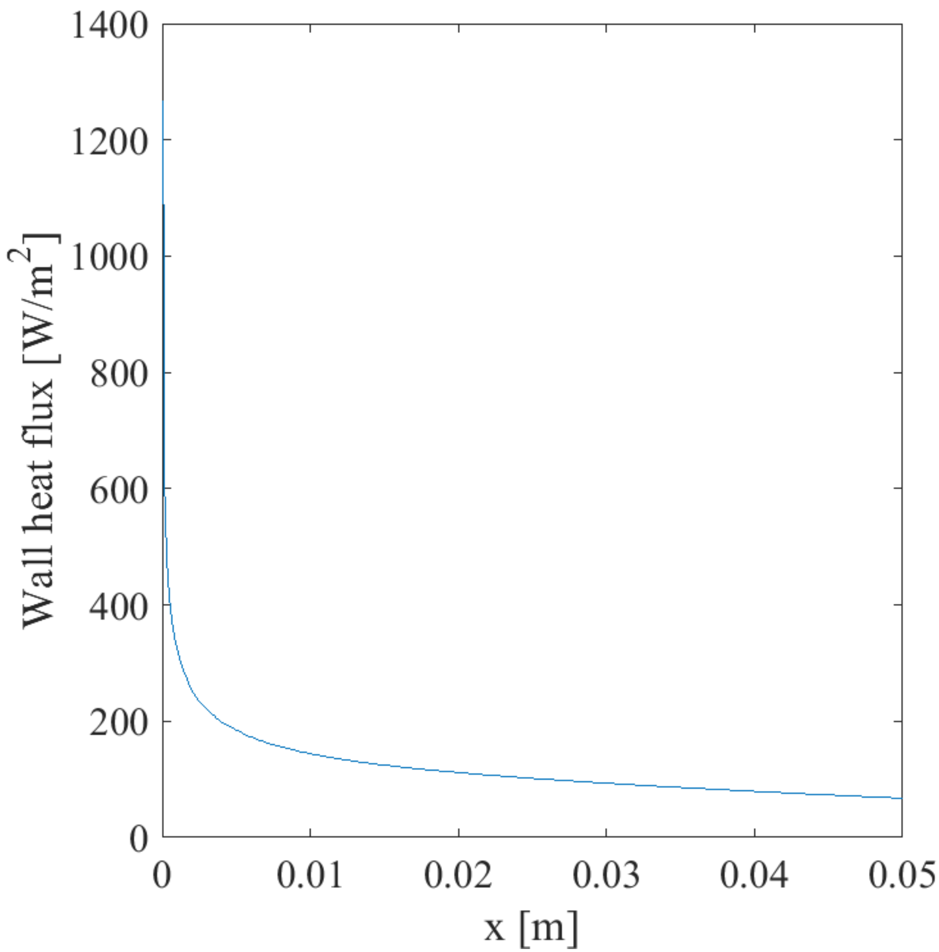
On the heat transfer course, we have “5 friends” i.e. 5 main principles that are used to explain heat transfer phenomena

1) Energy conservation: “J/s thinking”
2) Fourier’s law
3) Newton’s cooling law
4) Energy transport equation – convection/diffusion equation
5) Momentum transport equation – Navier-Stokes equation

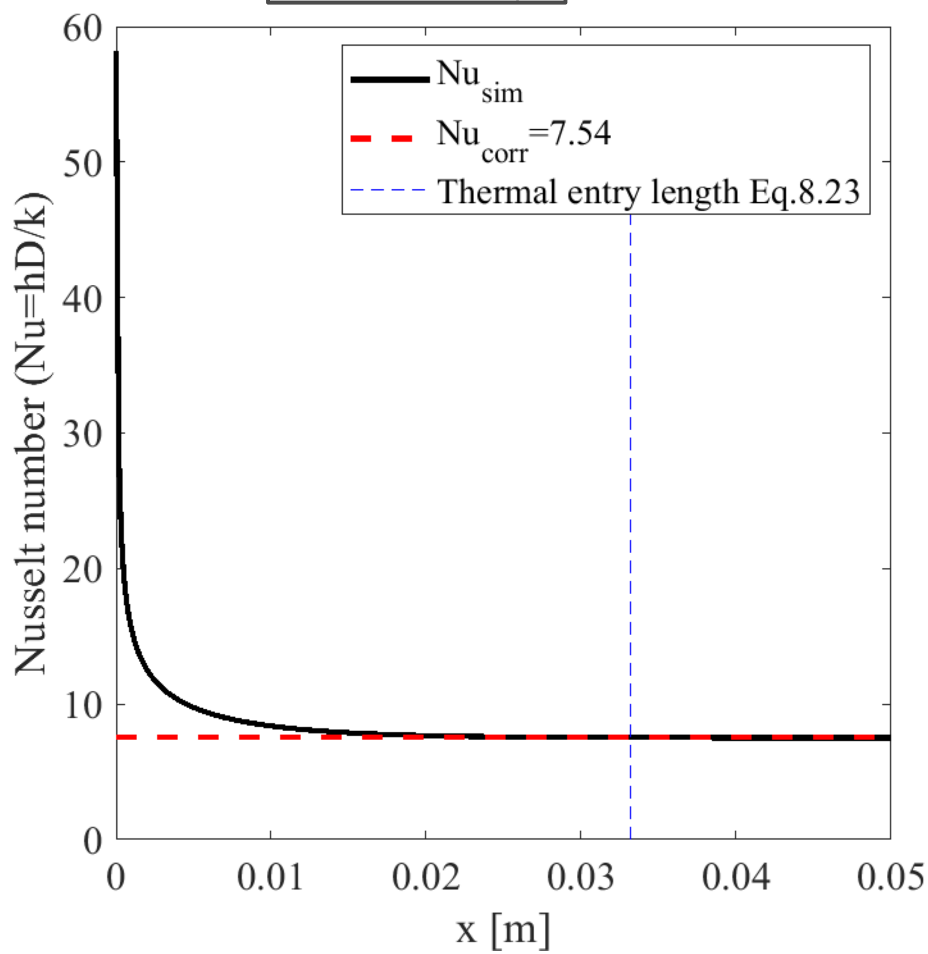
Small recap from last week on wall heat flux $q_{wall}(x)$ along channel for laminar fluid flow

For const. wall temperature laminar channel flow, the local wall heat flux is needed to get energy balance to find the local Nusselt number

$$q_{wall}(x) = k_f \left(\frac{\partial T}{\partial y}\right)_{wall}$$



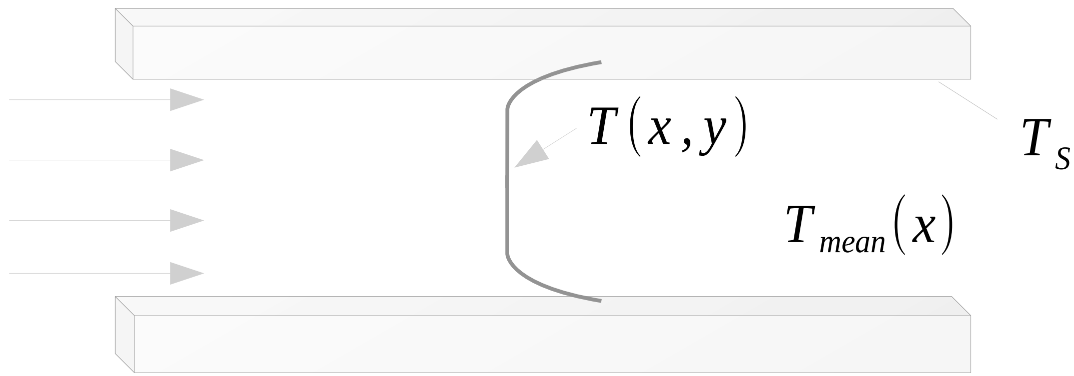
$$Nu(x) = \frac{hD}{k_f}$$



Relevance to HW3 - Heat flux balance at the surface

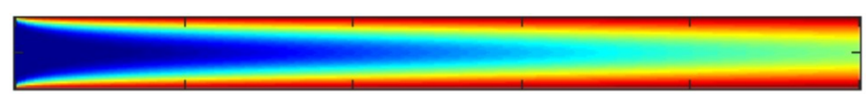
Diffusive heat flux (Fourier) immediately at the wall on the fluid side = Heat flux from Newton's law of cooling

$$k_f \left(\frac{\partial T}{\partial y} \right)_{y=wall} = h \left(T_s - T_{mean} \right)$$



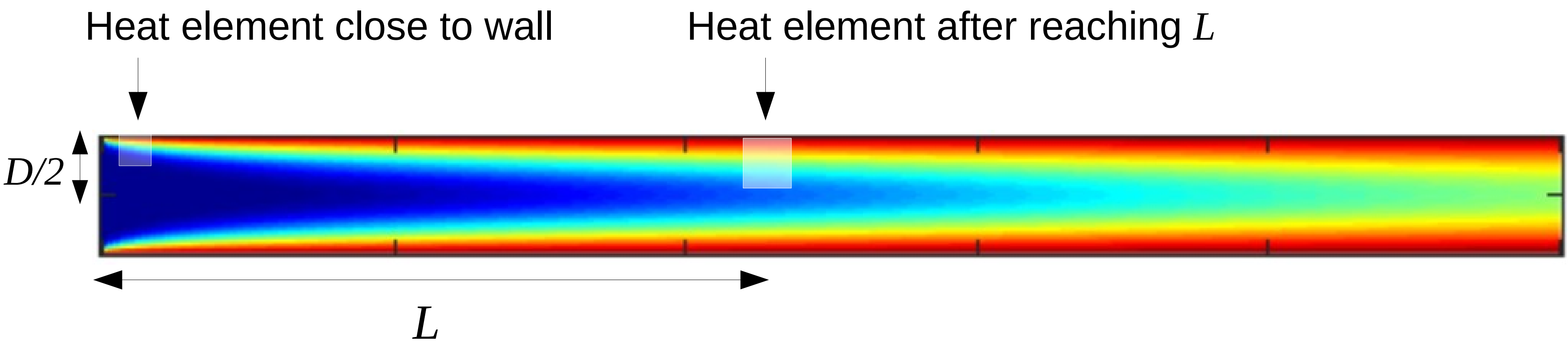
We can now calculate $h=h(x)$ (W/m^{2}K) at every point along the surface.

$$h = \frac{k_f \left(\frac{\partial T}{\partial y} \right)_{y=wall}}{T_s - T_{mean}}$$



Note: even in convective heat transfer the heat first purely diffuses i.e. conducts near the wall because $u,v \rightarrow 0$ next to the wall.

Thermal entry length derivation for laminar channel flow



- **Heat transport in *y*-direction:** Since V=0 it follows that heat diffuses in y-direction.
- Diffusion time to center line can be estimated as: $\tau_{diff} = (D/2)^2/\alpha$
- **Heat transport in *x*-direction:** During diffusion time heat is convected in x-direction distance L
- Approximate convection time: $\tau_{conv} = L/U$
- Equate: $\tau_{conv} = \tau_{diff}$

Our estimate:
$$(x/D) \approx 0.25\,\mathrm{Re}_D\,Pr$$

Text book formula for thermal entry length:
$$(x/D) \approx 0.05\,\mathrm{Re}_D\,Pr$$

Slide summary: physics and way of thinking ok but
the prefactor (0.25) is wrong (not 0.05) but explainable and refinable.

Internal flow: When Reynolds number increases large enough (below >5300), pipe and channel flows become **turbulent**. Velocity field below indicates that also wall-normal velocity emerges along with chaotic swirling due to turbulence → much higher heat transfer via enhanced convection and mixing

https://link.springer.com/article/10.1007/s10494-013-9482-8

$$\mathrm{Re} = \frac{U\,D}{\nu}$$

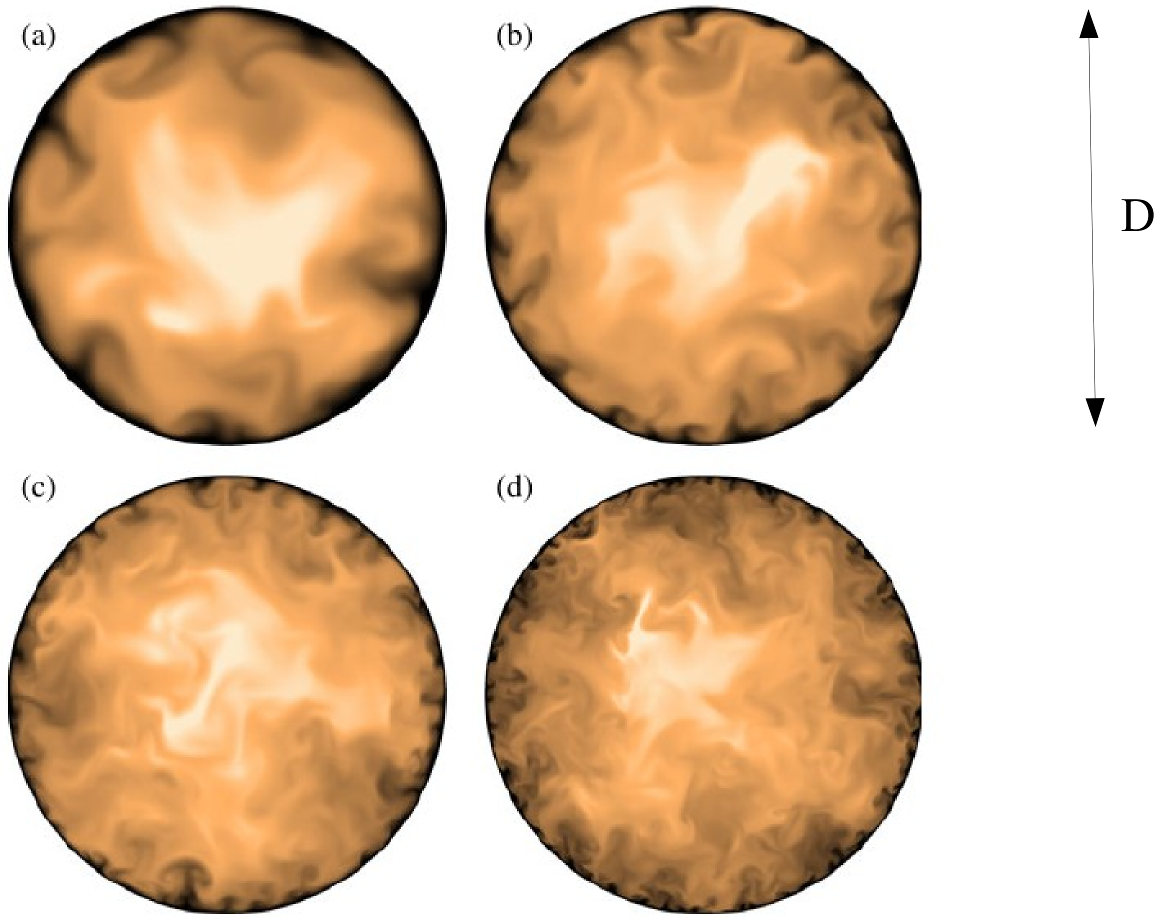


Fig. 4 Pseudo-colour visualisation of the instantaneous axial velocity u_z normalised by the bulk velocity U_b. **a** $Re_b = 5{,}300$; **b** $Re_b = 11{,}700$; **c** $Re_b = 19{,}000$; **d** $Re_b = 37{,}700$. Here, the colours vary from 0 (*black*) to 1.3 (*white*)

El-Khoury et al. (2013)

Lecture 4.1 Theory and experiment: Heat transfer of flow over a cylinder

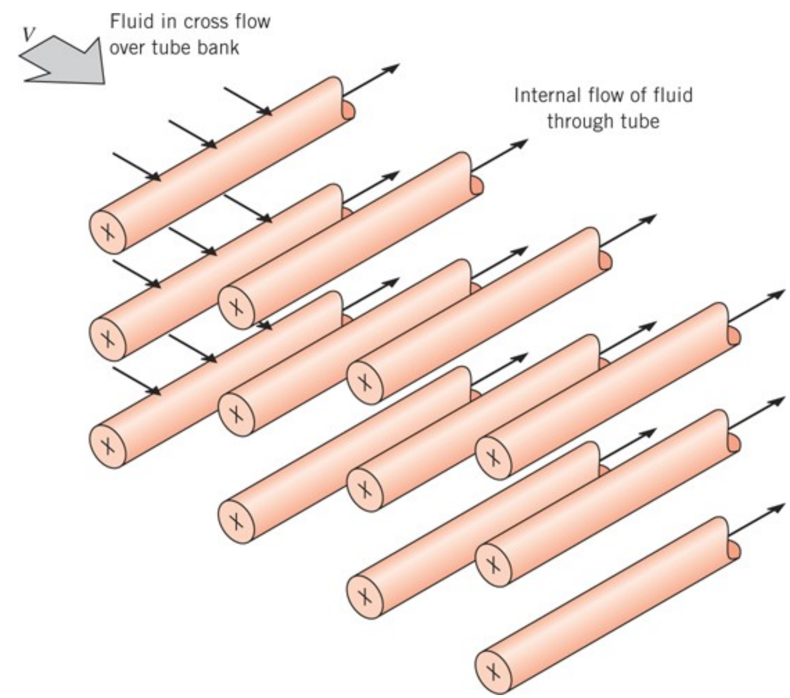
ILO 4: Student can formulate energy balance for external flow heat transfer systems and use basic correlations. The student can confirm the analysis using generated/provided simulation data.

Example: Heat transfer increases when fluid velocity relative to the object increases → higher Reynolds number → higher heat transfer coefficient

Source: https://upload.wikimedia.org/wikipedia/commons/1/17/Professional_swimmer_%2835609151296%29.jpg

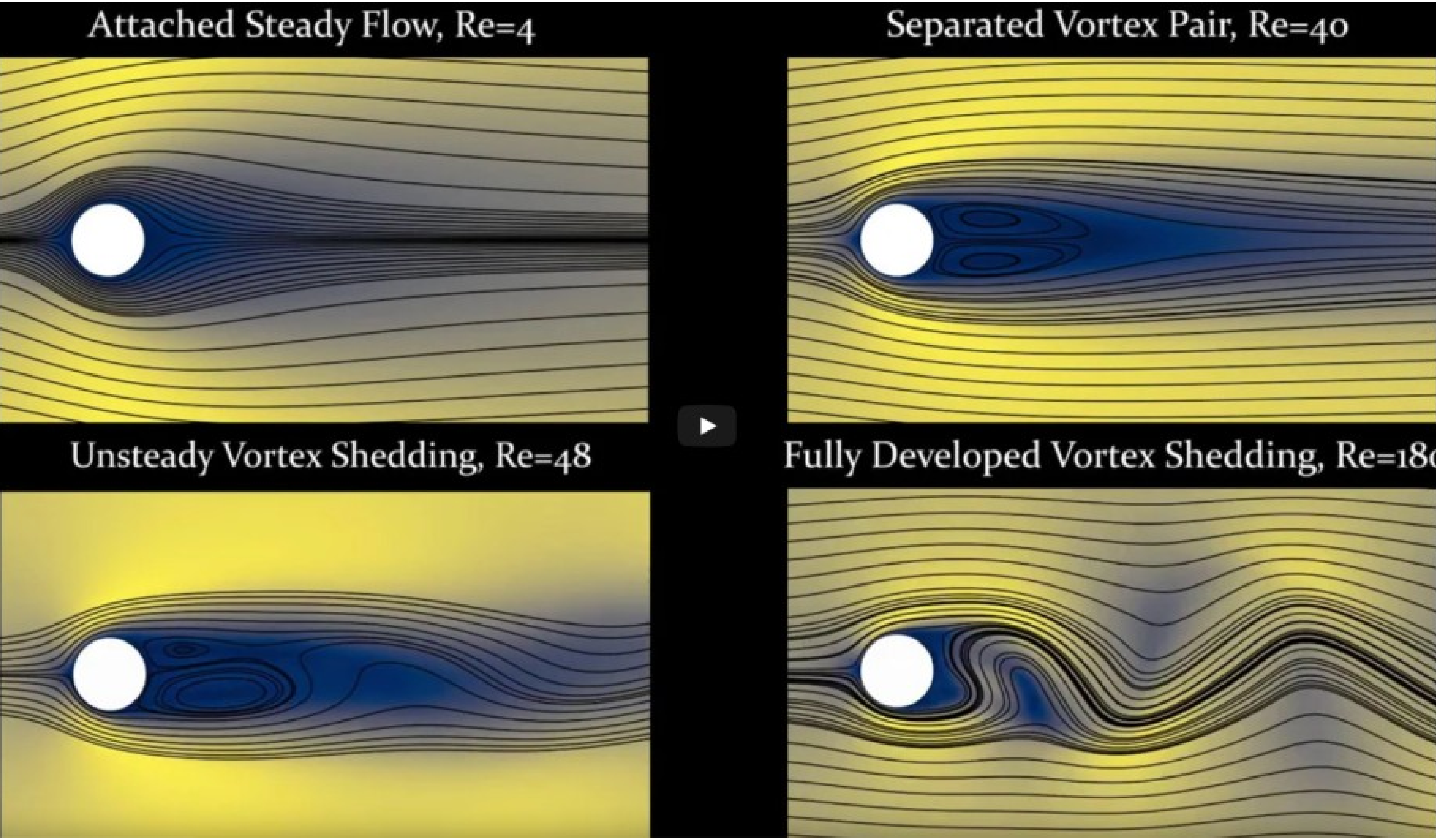
Cylindrical fins are common in energy applications

- Cylinder or pin fin beds very commonly used in heating and cooling applications
- E.g. cooling system in a ship where cool sea water is pumped through pipes and air is blown by fan over the array for AC cooling of cabins
- Understanding single cylinder heat transfer is really essential.



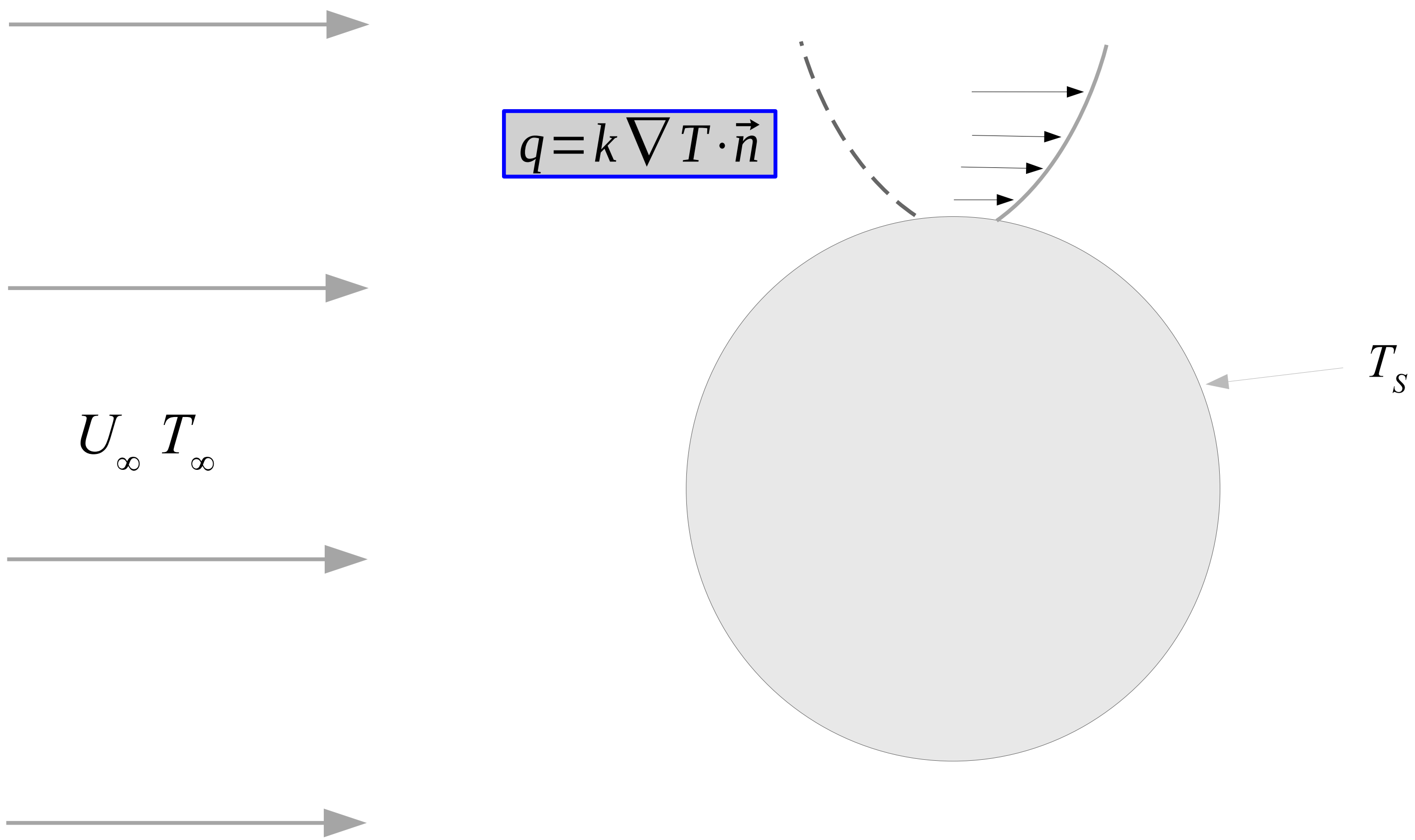
External flows: The fluid dynamical flow pattern over a cylinder depends on the Reynolds number ($Re=UD/\nu$).

Video below courtesy of: J.Ramsay, M.Sellier, W.Ho



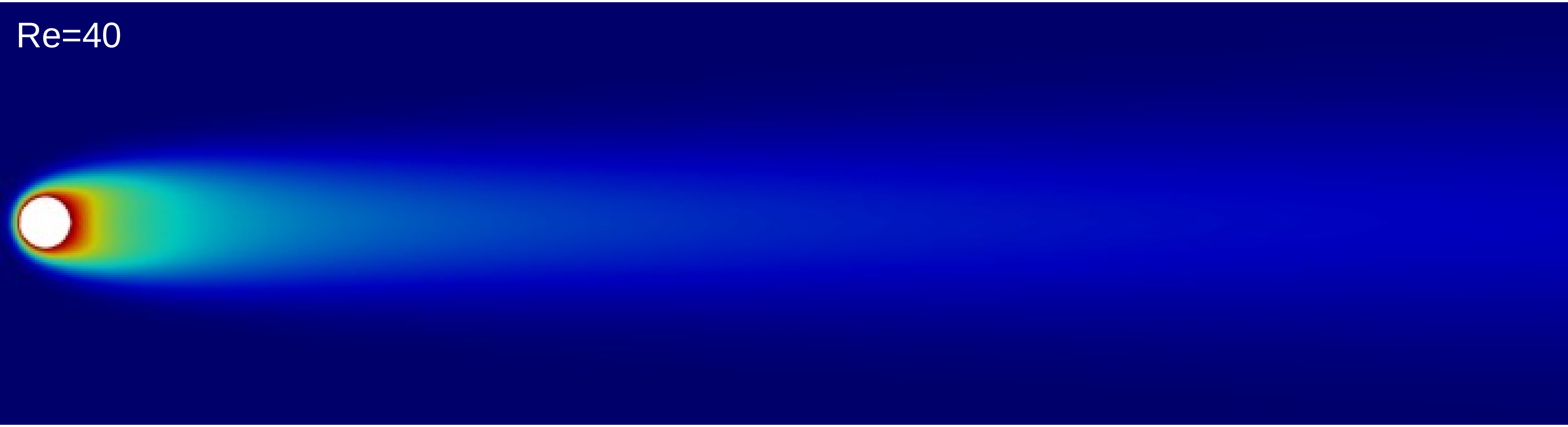
https://www.youtube.com/watch?v=pW0JfEBE9h8

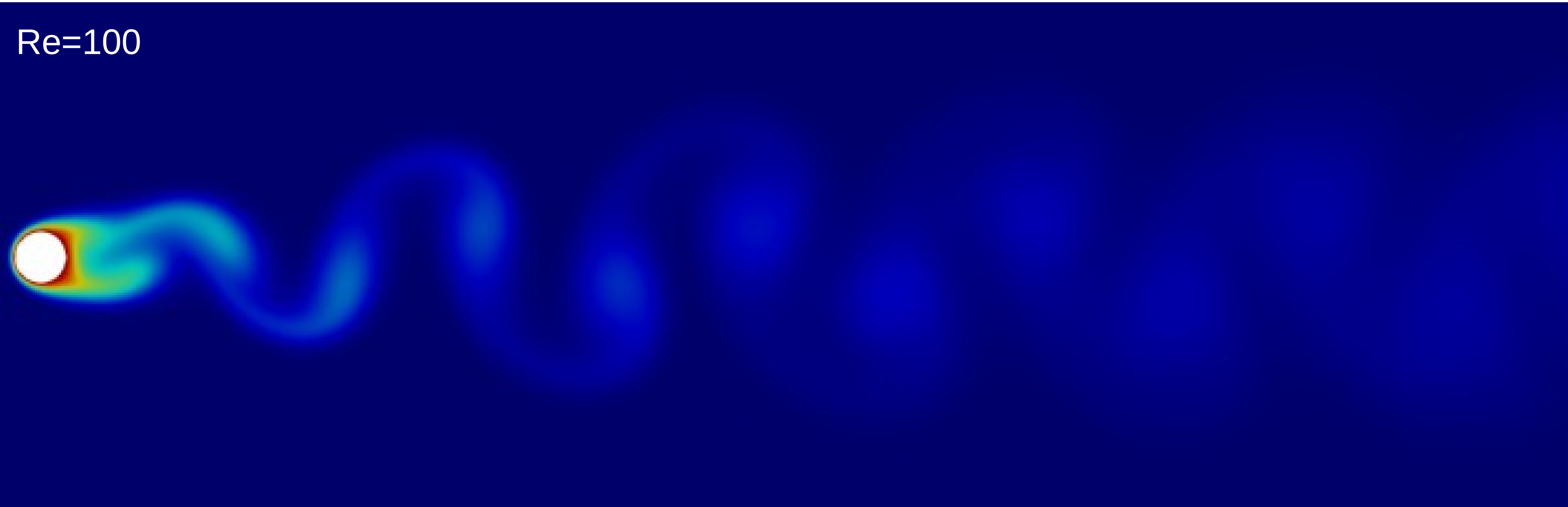
Heat transfer takes place on the surface of the cylinder



Two examples of instantaneous temperature fields from CFD simulation. **Note:** low Re → thicker thermal boundary layer.

Courtesy of: Gizem Ersavas Isitman





Energy balance (J/s thinking) for gas flow when the gas is heated at power P (W). "Control volume" thinking.

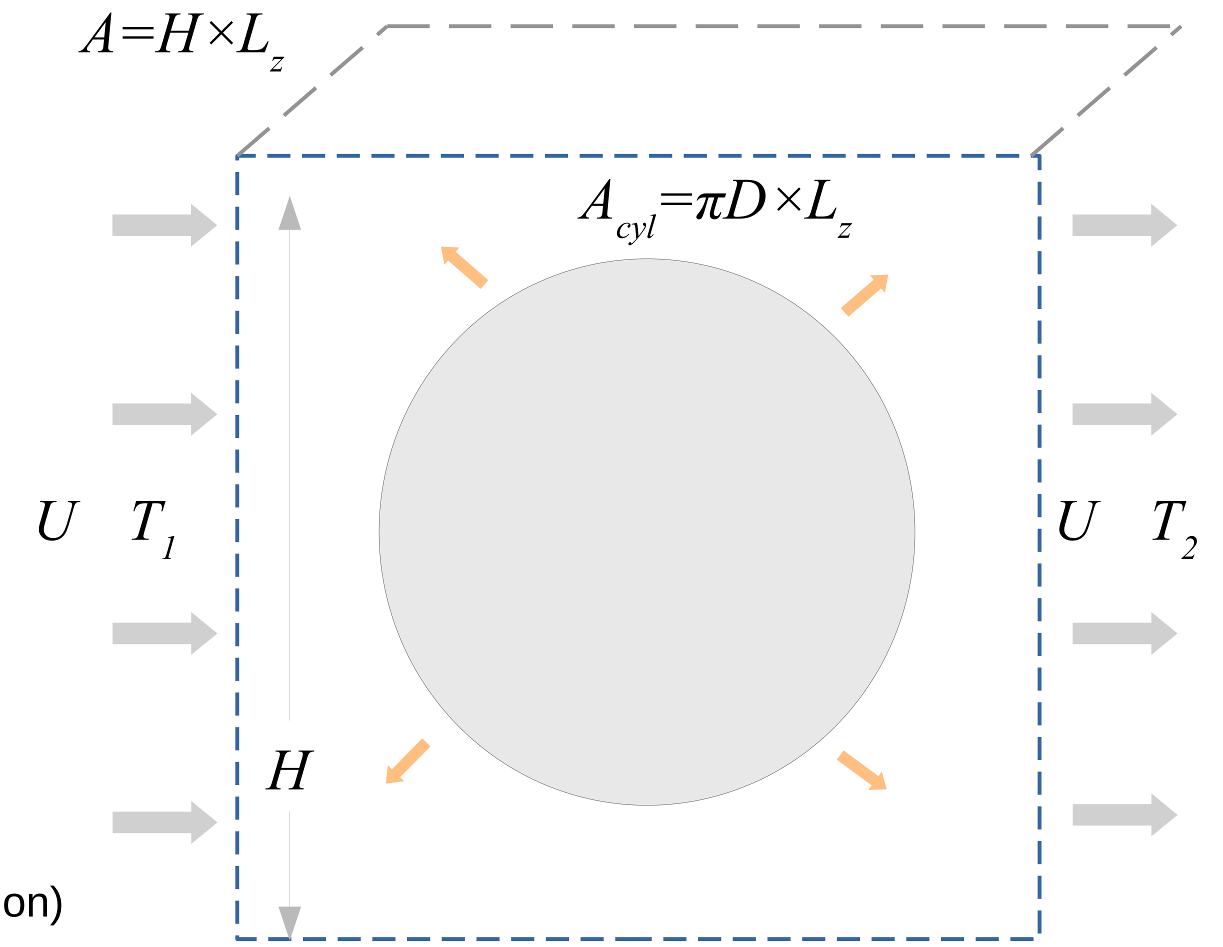
$$Q_{\mathrm{OUT}} - Q_{\mathrm{IN}} = c_p \dot{m} \Delta T_{ave} = P$$

Mass flow rate of the gas (kg/s):

$$\dot{m} = \rho U A$$

Average temperature change:

$$\Delta T_{ave} = T_2 - T_1$$

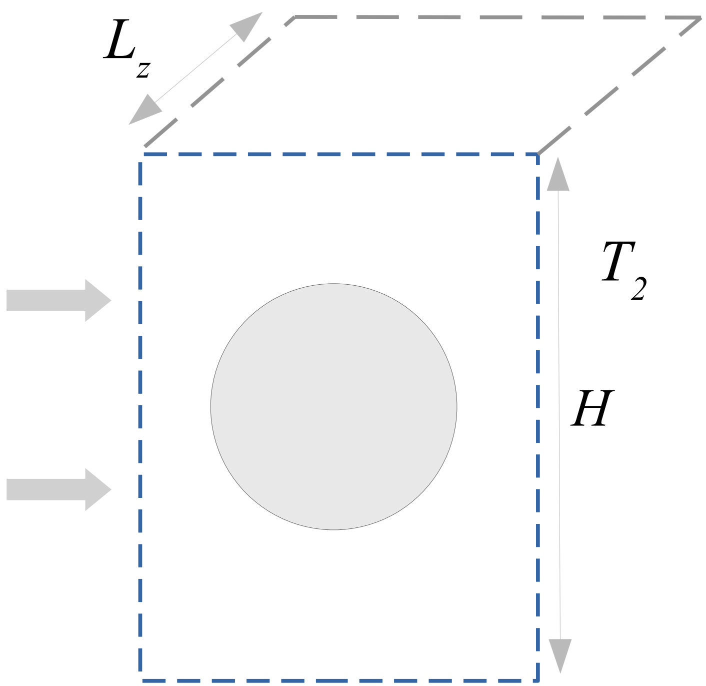


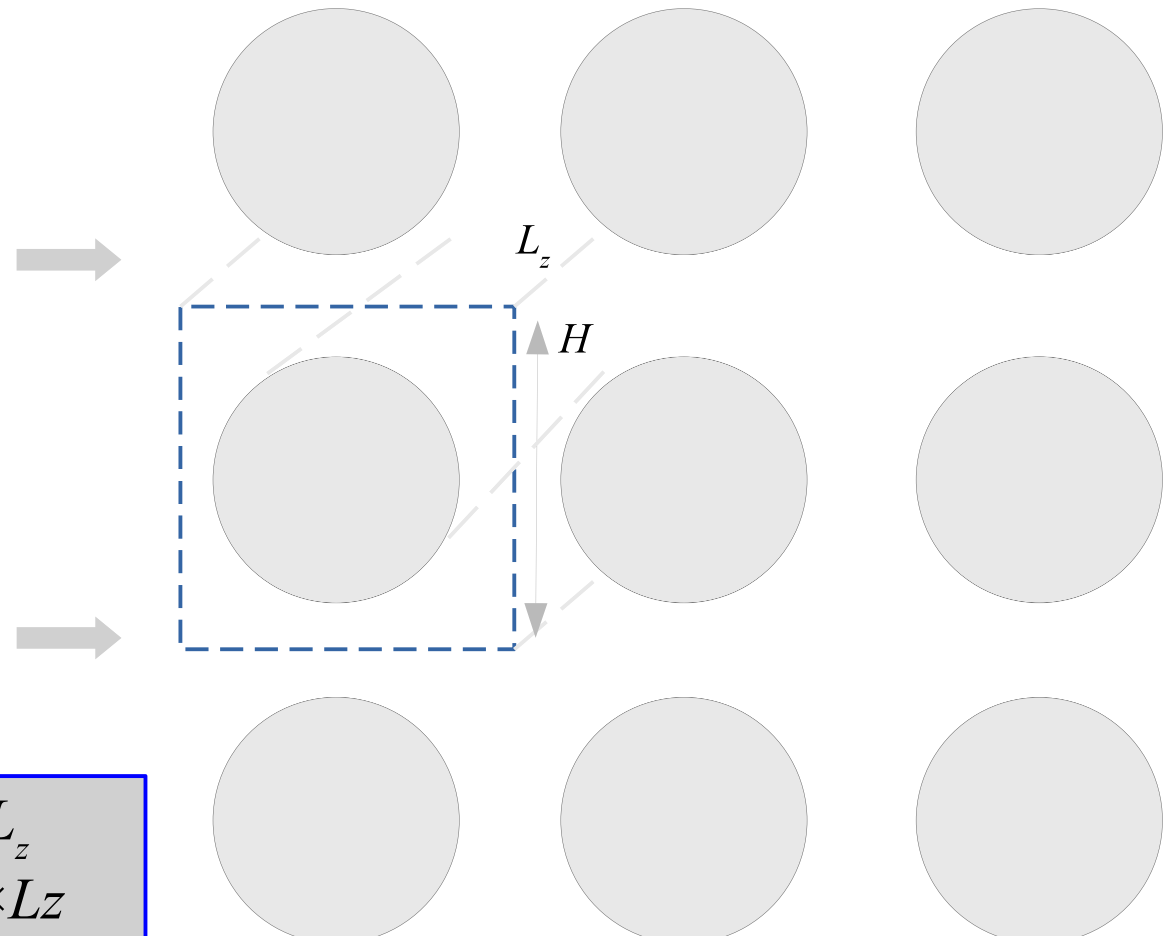
A=cross-sectional area (normal to flow direction) of control volume
U=flow average velocity

Note: Choosing H has been a common source of confusion.

Free cylinder:
T_2 depends obviously on the chosen height of the control volume volume (same energy goes to smaller/larger volume). L_z cancels out (think 1m).

Cylinder bed:
The natural height of control volume relates to the symmetry of the problem which enables estimation of temperatures inside and after pipe heat exchangers.

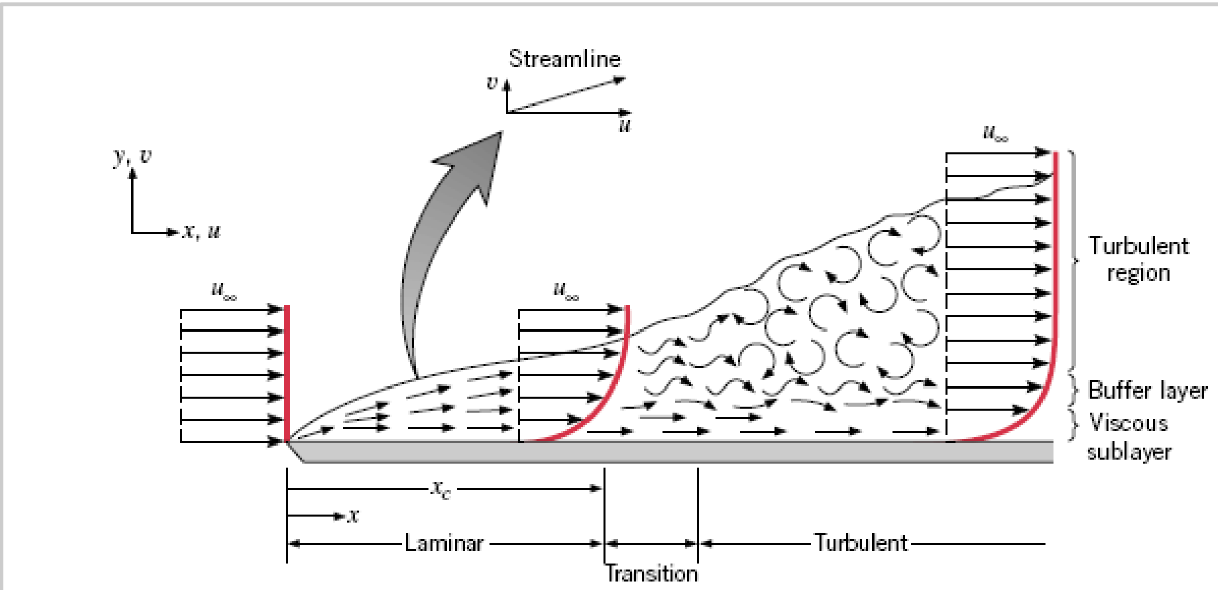


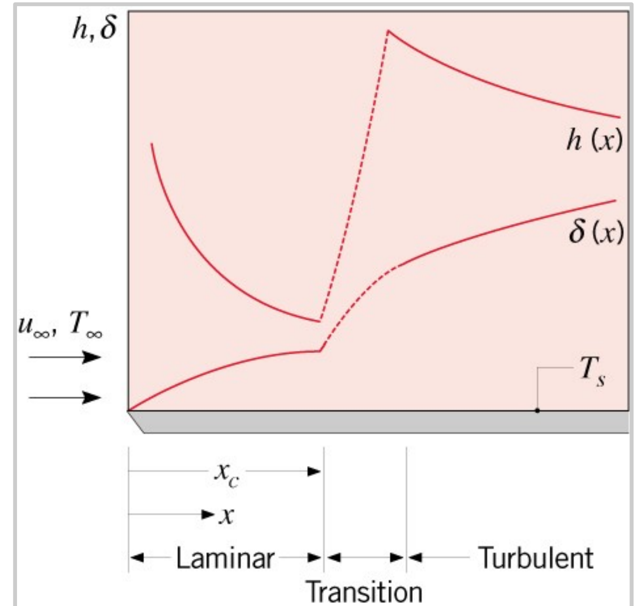


$$A = H \times L_z$$
$$A_{cyl} = \pi D \times Lz$$

External flows: Many turbulent flow systems pose laminar to turbulence transition region before turbulence has developed. Heat transfer differs between different regions → needs to be taken into account. Example: free boundary layer.

Here: “Free” refers to “free of the opposite wall”





$$Re_c = \frac{\rho U_\infty x_c}{\mu}$$

Critical Reynolds number at the onset of laminar to turbulence transition.

$$10^5 < Re_c < 3 \cdot 10^6$$

$$U / U_\infty = 0.99$$

$$\delta = \frac{5.0}{\left(\frac{U_\infty}{\nu x}\right)^{1/2}} = \frac{5x}{Re_x^{1/2}}$$

Laminar boundary layer thickness grows along the plate.

$$\delta / \delta_T = Pr^{1/3}$$

Thickness ratio between viscous/thermal BL’s depends on *Pr*.

$$Nu_x = h_x x / k = 0.332 Re_x^{1/2} Pr^{1/3}$$

Local Nusselt number scaling depends on Re_x and *Pr*.

https://www.youtube.com/watch?v=e1TbkLIDWys&t=82s

Purpose of empirical heat transfer correlations:

For a broad range of Reynolds numbers and Prandtl numbers, express average heat transfer coefficient h in a non-dimensional form called Nusselt number: $Nu=Nu(Re,Pr)$

Different average Nusselt number correlations for heated cylinders have been developed

$$\overline{Nu}_D = \frac{\bar{h} D}{k}$$

Hilpert correlation (see Table 7.2)

$$\overline{Nu}_D = C \, \mathrm{Re}_D^m \, Pr^{1/3}$$

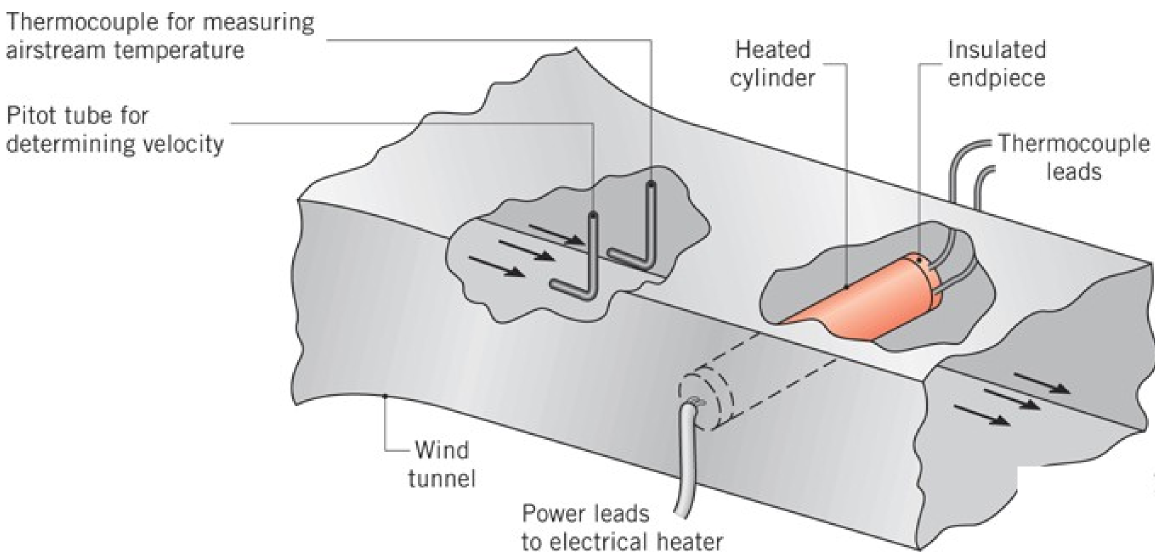
Churchill & Bernstein correlation (broad applicability)

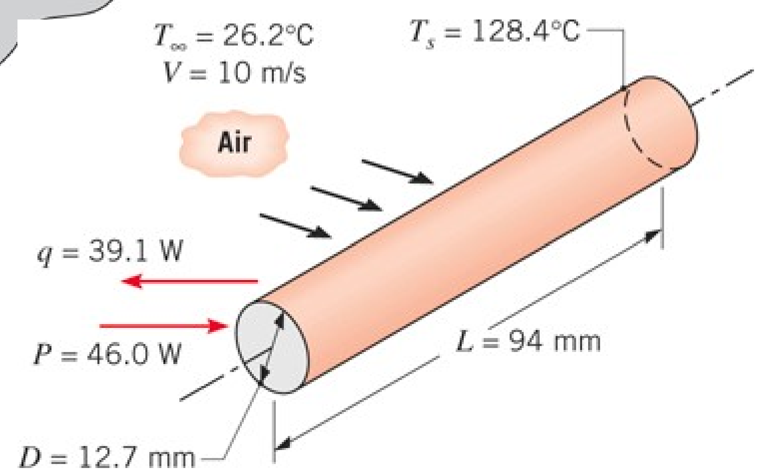
$$\overline{Nu}_D = 0.3 + \frac{0.62 \, \mathrm{Re}_D^{1/2} \, Pr^{1/3}}{\left[1 + \left(0.4/Pr\right)^{2/3}\right]^{1/4}} \left[1 + \left(\mathrm{Re}_D/282000\right)^{5/8}\right]^{4/5}$$

Zukauskas correlation (broad applicability, see Table 7.4)

$$\overline{Nu}_D = C \, \mathrm{Re}_D^m \, Pr^{1/3} \left(\frac{Pr}{Pr_s}\right)^{1/4}$$

Example 7.4: Heated cylinder in cross-flow





$$\bar{h} = ?$$

+ compare the result to correlation equation

First, we get the convection coefficient directly.

$$\bar{h} = \frac{q}{A\left(T_s - T_\infty\right)} = 102\, W/m^2 \cdot K$$

Zukauskas relation:

$$\overline{Nu}_D = C\, \mathrm{Re}_D^m Pr^n \left(Pr/Pr_s\right)^{1/4}$$

$$\mathrm{Re}_D = 7992 \rightarrow C = 0.26\,, m = 0.6\,, Pr = 0.7 < 10 \rightarrow n = 0.37$$

Table 7.2

Utilization of Zukauskas relation gives:

$$\overline{Nu}_D = 50.5$$

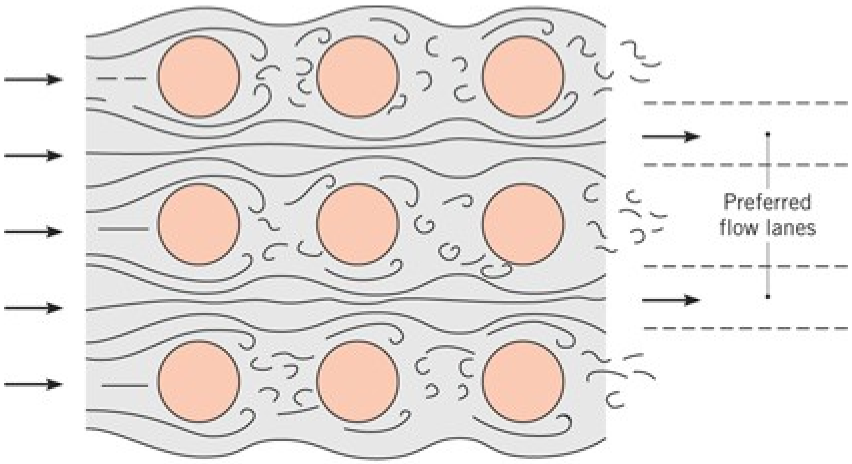
$$\bar{h} = \overline{Nu}_D k/D = 105\, W/m^2 \cdot K$$

$\bar{h} = 105\, W/m^2 \cdot K$ → close to first observed value $\bar{h} = 102\, W/m^2 \cdot K$

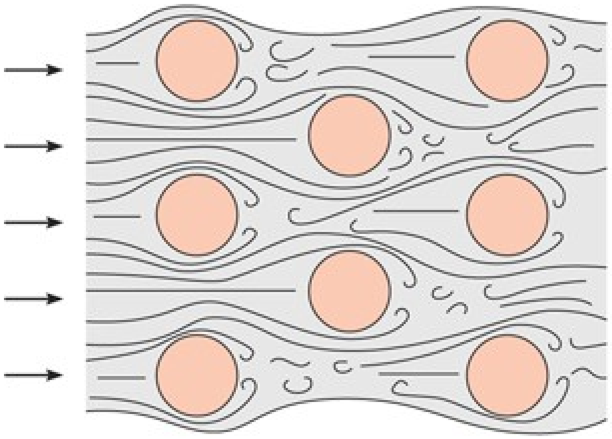
Lecture 4.2 Numerical approach: 2d heat transfer over a fin bed using Matlab

ILO 4: Student can formulate energy balance for external flow heat transfer systems and use basic correlations. The student can confirm the analysis using generated/provided simulation data.

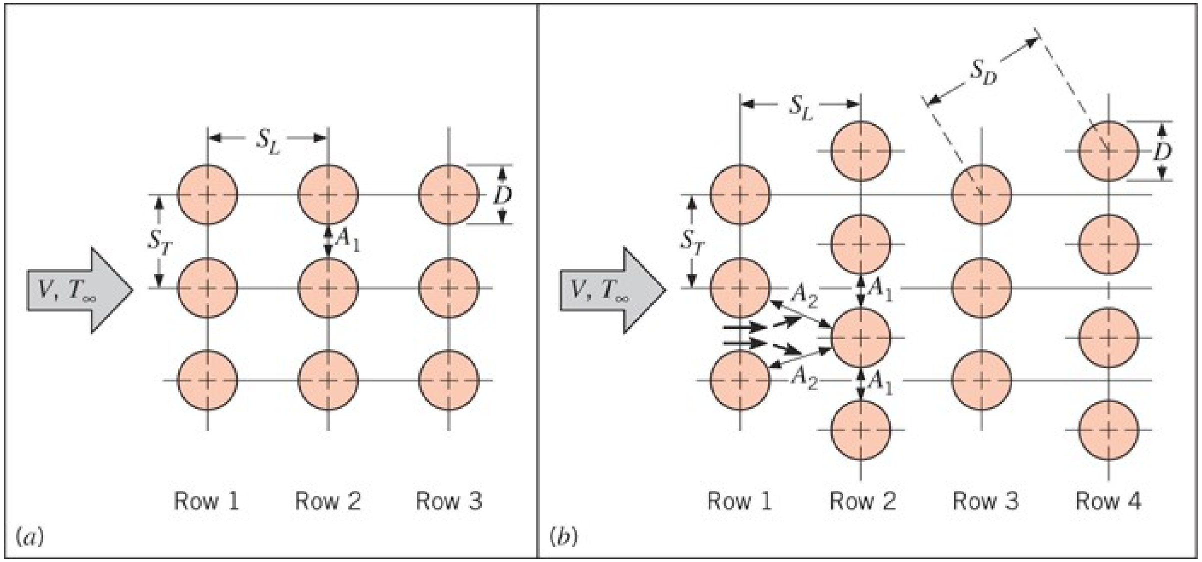
Fin arrays



(a)



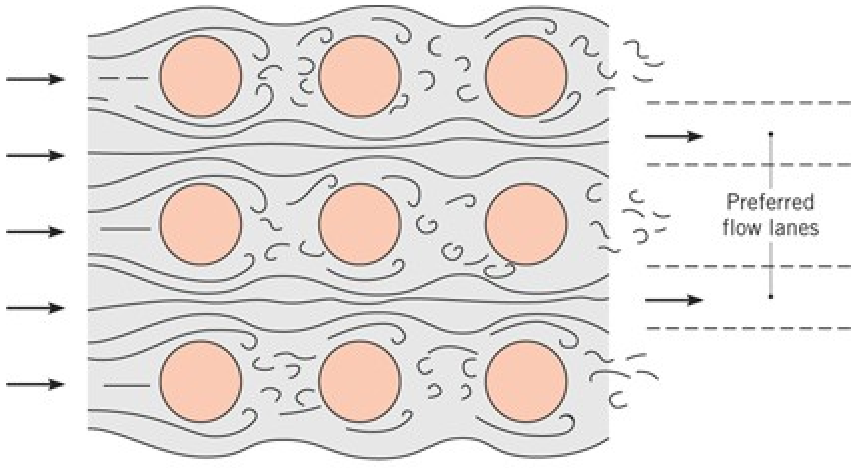
(b)



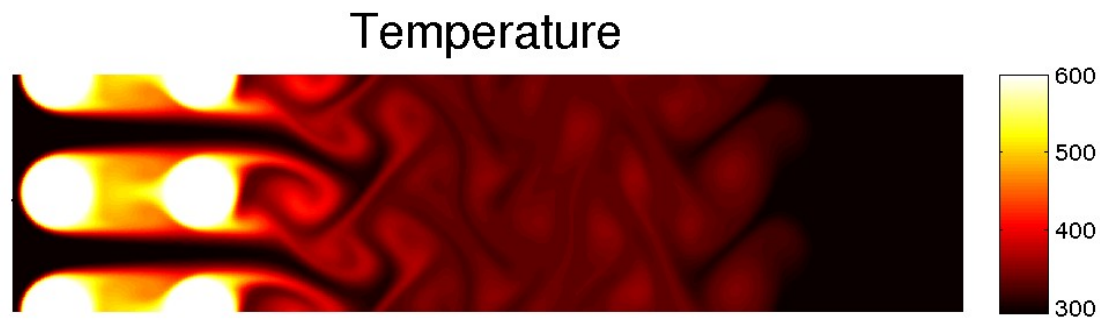
S_T = transverse pitch

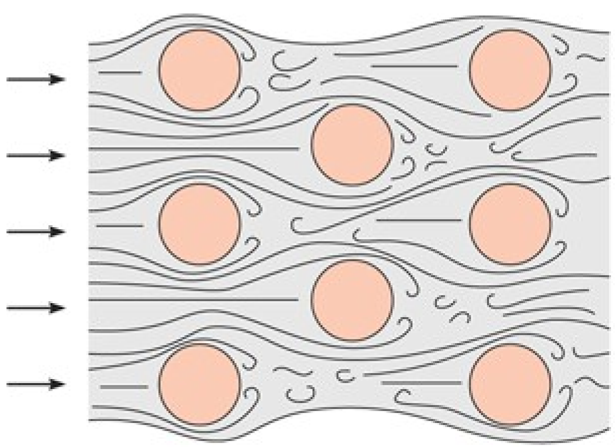
S_L = longitudinal pitch

Aligned vs staggered configurations

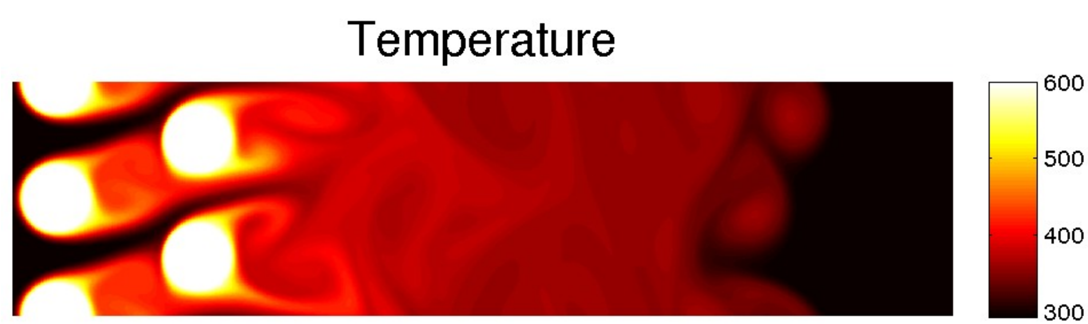


(a)



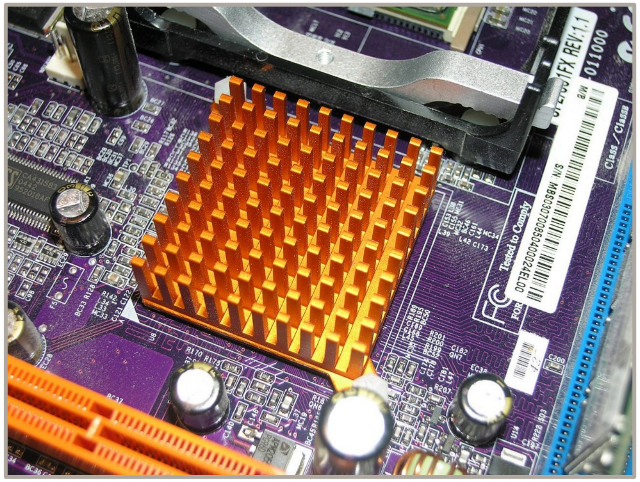


(b)



What do you think which one would pose better heat transfer ?

HW4: Pin fin configuration



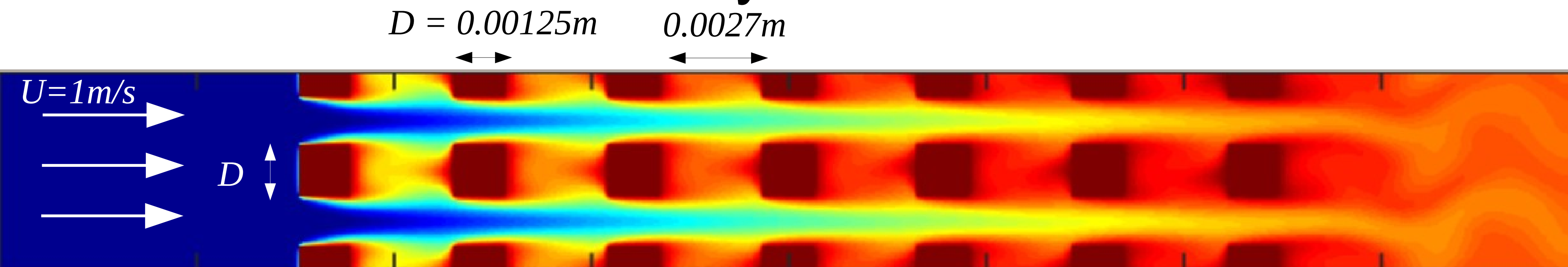
In this session we will look mostly into heat transfer in a pin fin bed. Could occur in electronics cooling or air conditioning system.

Other assumptions during the session:

1) pin fin walls at T_{wall}=+29.6 deg C

2) inflow temperature is T_{left} = +22.6 deg C

3) velocity field is fully developed and laminar and mean inflow velocity is close to 1m/s

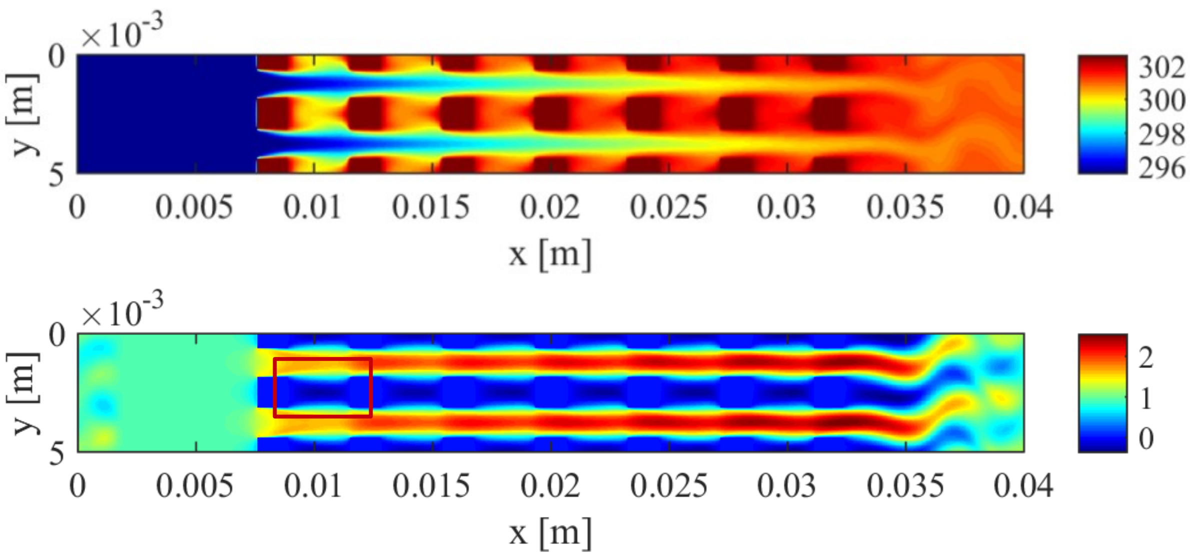


HW4: Convection-diffusion equation for temperature to estimate Nu

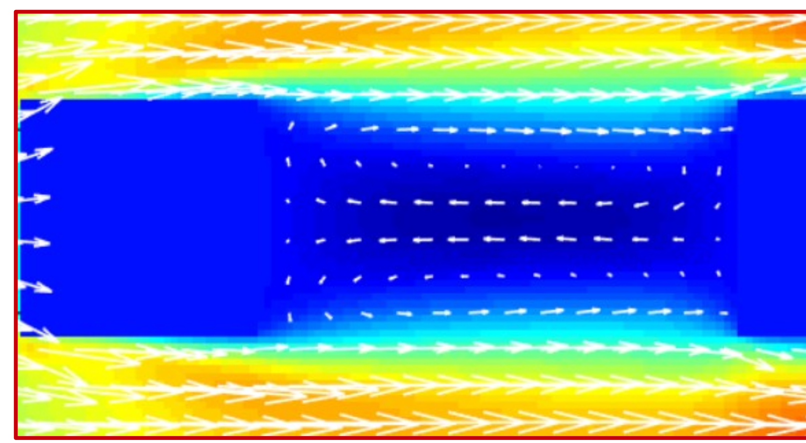
The Matlab session will focus on investigating heat transfer in a 2d fin bed using a provided velocity field which is assumed fixed. Constant wall T is assumed.

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial x^2} + \alpha\frac{\partial^2 T}{\partial y^2}$$

$T=T(x,y)$ in steady state 2d flow
with constant wall temperature BC



Zoom to recirculation zone showing reverse flow (every fifth vector shown)



$u=u(x,y)$ and $v=v(x,y)$ are provided from separate CFD simulation. Assume “frozen flow” and zero velocity on the walls.

STEPS 1&2: Running code and saving the result.

- download the `heat2dfins` code from MyCourses and extract – by right clicking mouse - to new folder `Week4`. The code solves a convection-diffusion eqn for temperature using velocity obtained from another previous CFD simulation.

- run the code by clicking play. You can study either single fin case or multifin case using parameter. Here, let's first practice with a single fin so set the parameter `Single=1` `(Single=2 in the homework).`

STEP 3: postprocessing.

- create a new file called `PostProcess.m` Let us visualize the velocity field U(x,y) which comes indeed from the previous simulation (is given and assumed fixed herein).

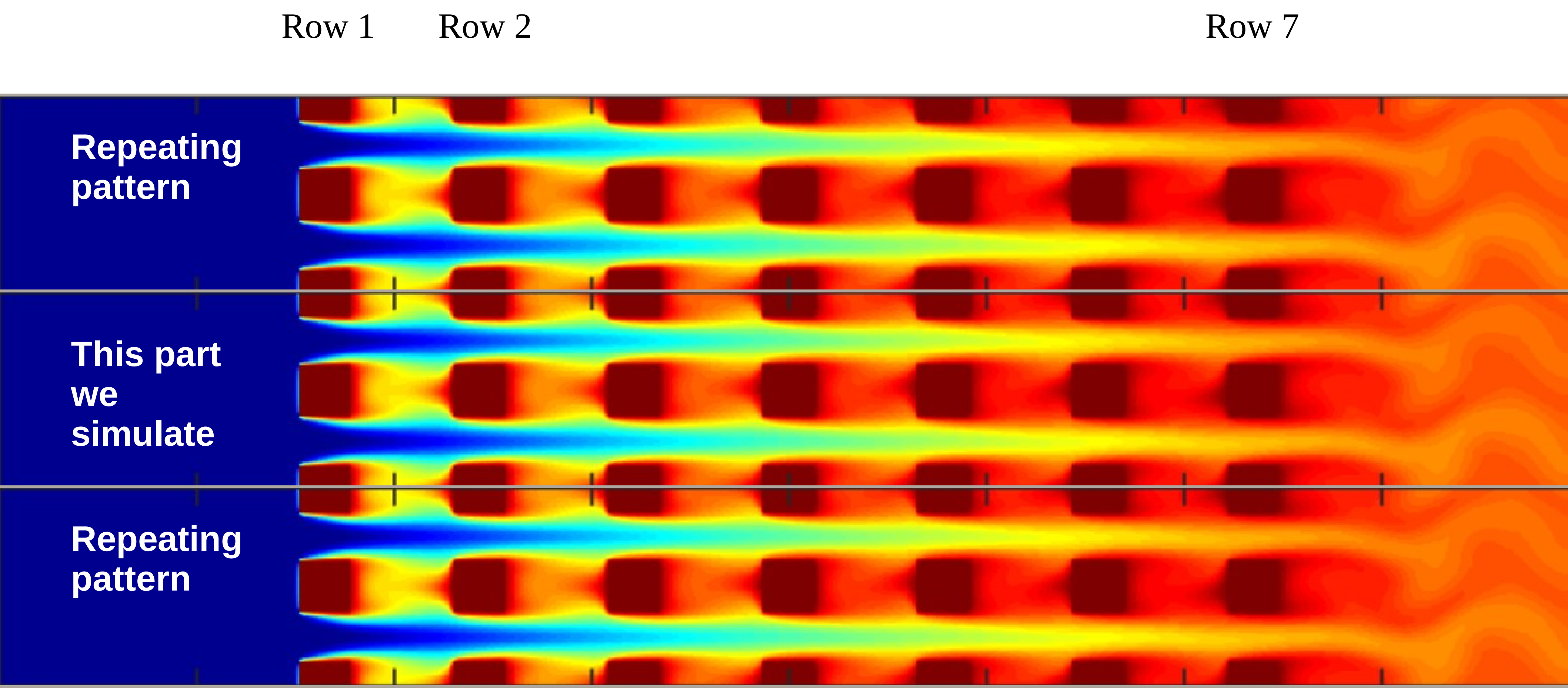
```
a=load('FinsSingle.mat');

figure(2), clf, box

% first make image of the x-velocity field U(x,y)
imagesc([min(min(a.X)) max(max(a.X))],[min(min(a.Y)) max(max(a.Y))], a.U)

axis equal, axis tight, colormap jet, hold on, colorbar

print -dpng Velocity2dfins
```

STEP 4: use the Hilpert correlation (for cylindrical fins) to estimate Nusselt number under these conditions for this single rectangular fin. You need also the Reynolds and Prandtl numbers.

STEP 5: can you get similar order of magnitude from the **simulation result** for Nusselt number from **the single fin** case ?

$$\mathrm{Nu}_{ave} = \mathrm{Nu} = \frac{h\,D}{k} = C\,\mathrm{Re}_D^m\,Pr^{1/3}$$

We can think we are in fact modeling an "infinite", repeating pattern of fins ($T_s = const.$)

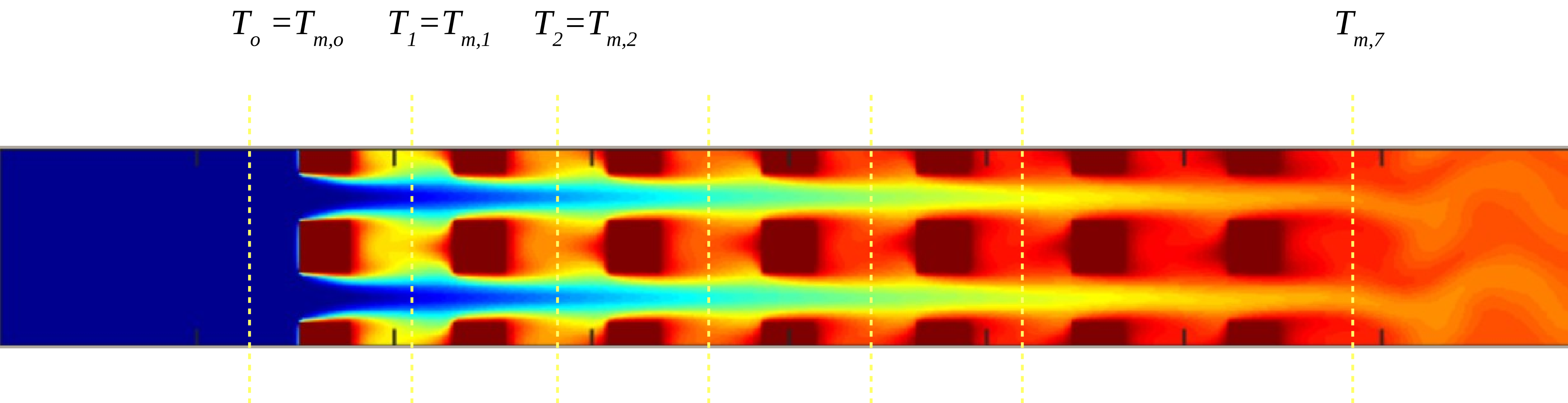


Question: How to use Newton's cooling law correctly and consistently in this situation ?
Is h constant ? What is T_m ?

$$q = hA(T_s - T_m)$$

Things we are interested in:
ΔT, h, $T_m(x)$

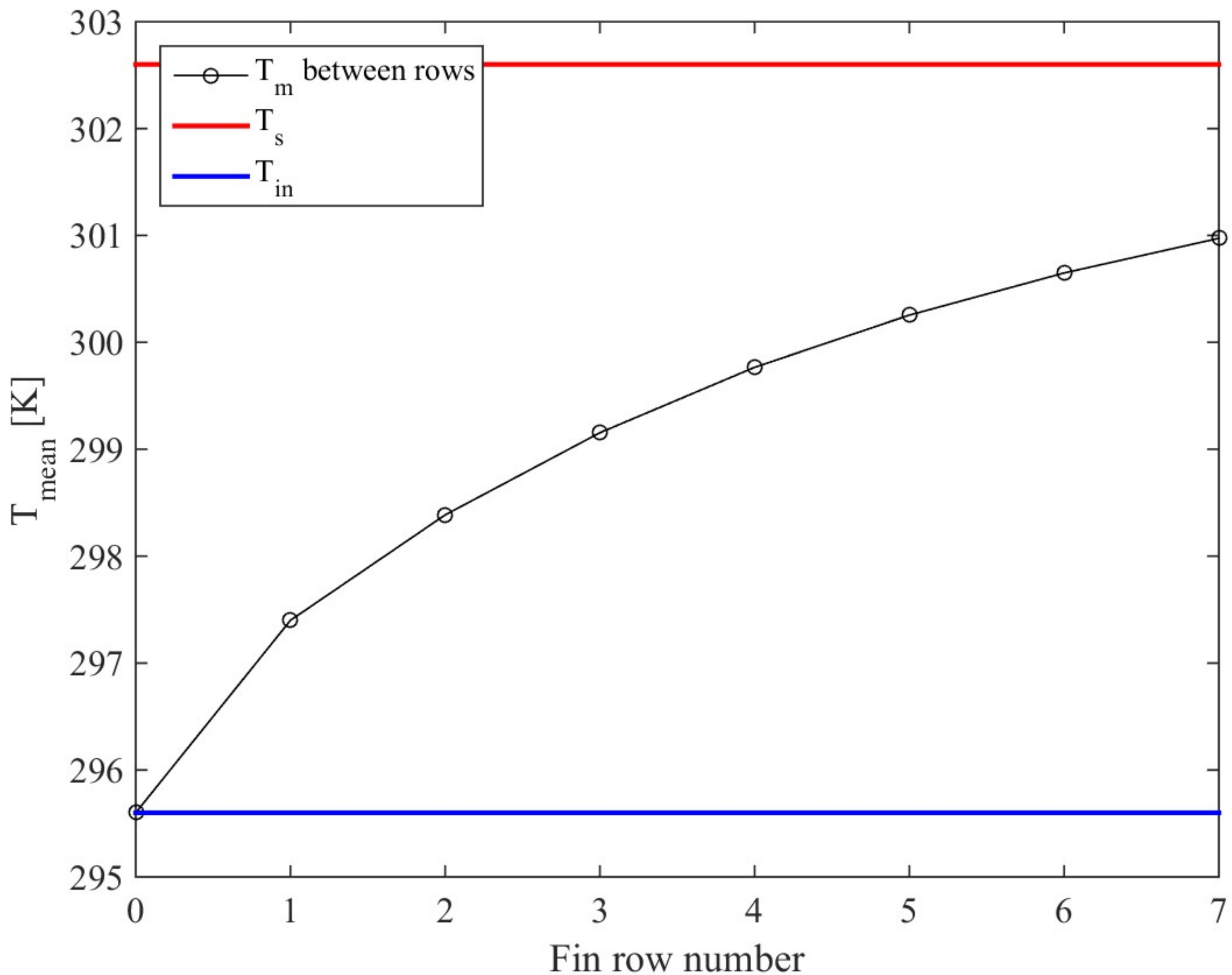
$$q = h_n A_{fin}\left(T_s - T_n\right) = c_p \rho U A_{in}\left(T_{n+1} - T_n\right)$$



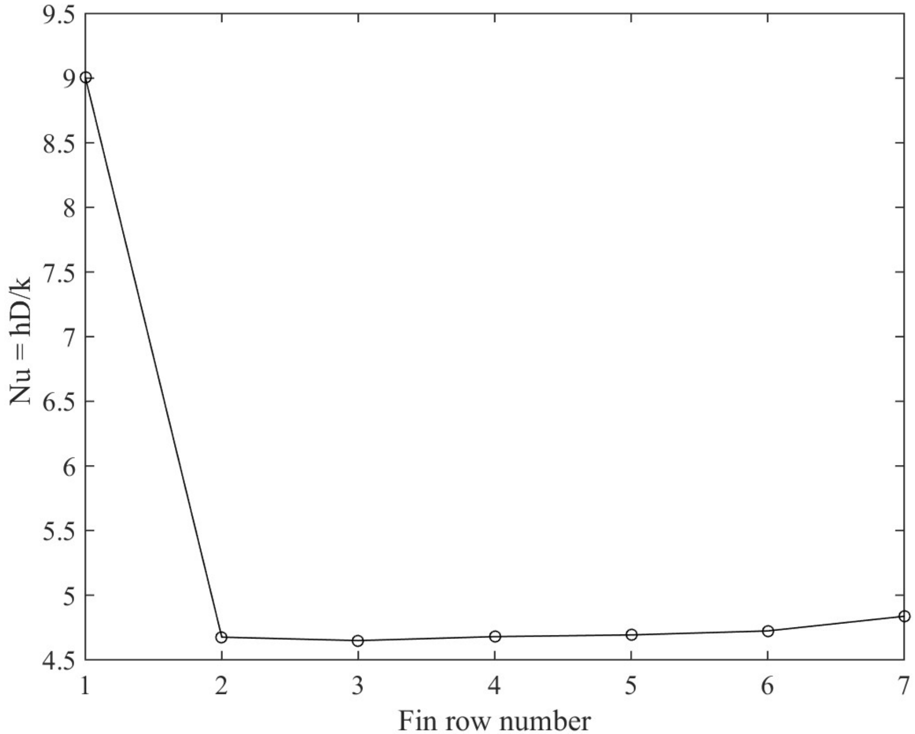
Note:
- we do not know if h is constant row-wise.
- we do not know if temperature jump per row is constant

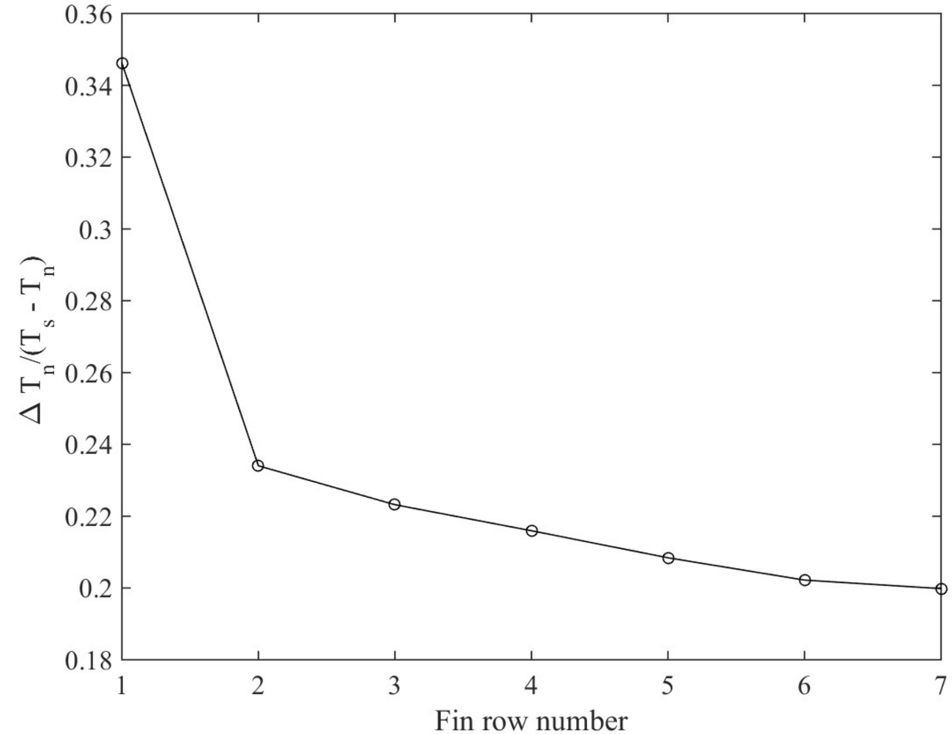
```
%Calculation of mean temperature and velocity
Tm = sum(T(iny,inx).*U)./(sum(U)); Um = mean(U);
```

Mean temperature after *n* fin rows



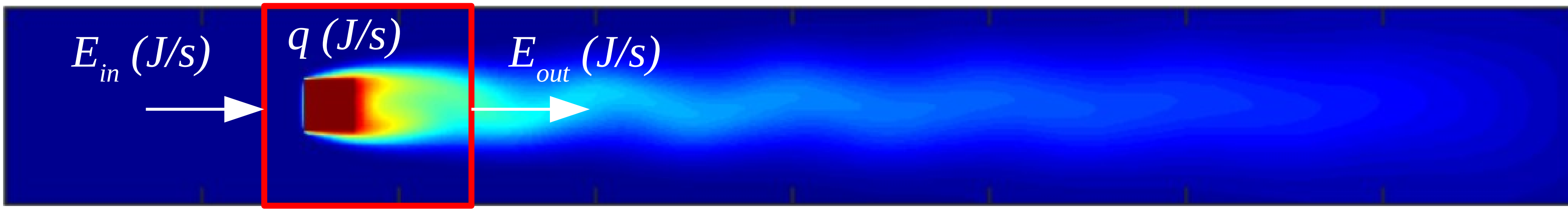
Relative temperature jump for fin array and Nusselt number





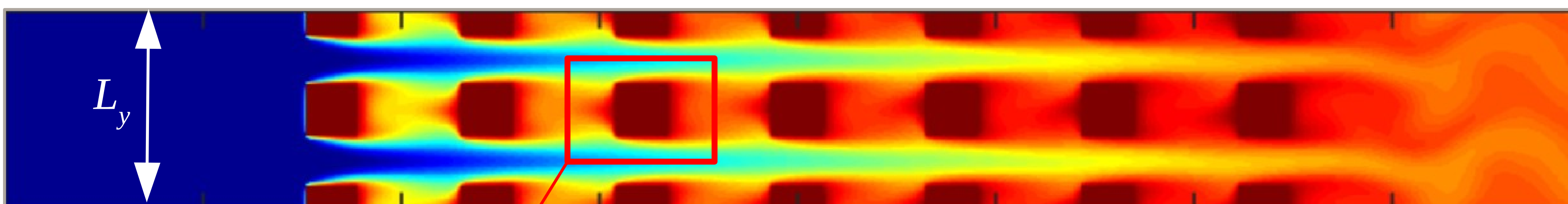
Control volume for energy conservation analysis:

In HW4 one needs to evaluate Nusselt numbers for single fin and multiple fin cases

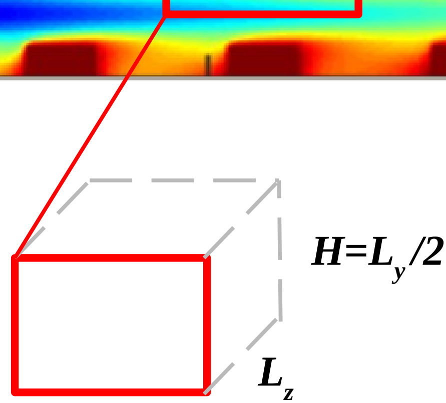


$$A_{in} = L_y \cdot L_z \text{ and } A_{fin} = 4\,D L_z$$

$$q = h_n A_{fin}(T_s - T_n) = c_p \rho U A_{in}(T_{n+1} - T_n)$$



$$A_{in} = \frac{L_y}{2} \cdot L_z \text{ and } A_{fin} = 4\,D L_z$$



Chosen control volume here