Aalto University

Basic oscillating systems

ELEC-E5610 Acoustics and the Physics of Sound

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1

Basic concepts

1 Definition of degrees of freedom

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A finite number of DOFs a simplification of real-world phenomena.

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- antiresonance notches between resonance peaks
- example: the double pendulum
  `http://www.youtube.com/watch?v=d2E5oojoXjk`
  - (two or more DOFs & nonlinearity $\Rightarrow$ chaotic behavior!)

1 Continuums

In reality, physical objects are not point-like, but distributed systems (at least in classical mechanics), i. e. their coordinate systems are continuous functions.

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- finite systems (e. g. a finite string) have countably infinite degrees of freedom
- infinite systems have non-countably infinite degrees of freedom

1 Practical simplifications

Typically, some practical simplifications are made:

Number of DOFs : 1DOF systems are sufficiently accurate approximations of real systems, if

- the bandwidth is limited
- the eigenfrequencies are well separated in frequency

Linearity : it is assumed that the frequency and waveform of the vibration do not depend on amplitude

1 Practical simplifications II

Deterministicity : it is possible to determine the future state of a system using a “snapshot” of a previous state

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- displacement, velocity, and acceleration typically are in different phases

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After a transient excitation, a system will vibrate at its eigenfrequencies.

1 Eigenmodes

Each eigenfrequency corresponds to a certain shape of vibration, called **eigenmode**.

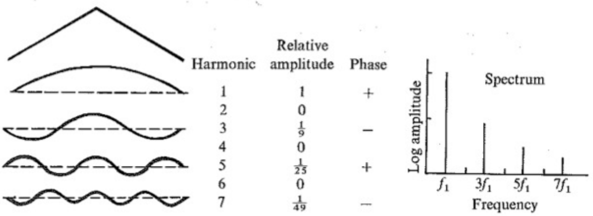

Fig. 2.5. Frequency analysis of a string plucked at its center. Odd-numbered modes of vibration add up in appropriate amplitude and phase to give the shape of the string.

Figure: Vibrational modes of a plucked string, R&F p. 39

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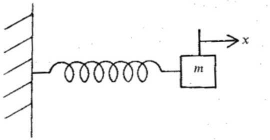

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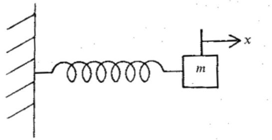

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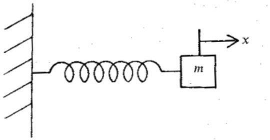

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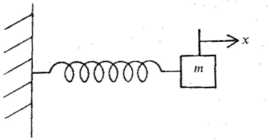

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A general solution to the motion equation (1) is
$x = A\cos(\omega_0 t + \phi)$, where ω_0 is the eigenfrequency (rad/s).

2

Complex representation

2 Complex exponential representation

FF appendix 1

Harmonic vibration has the general form:

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and interpret Eq. (2) as the real part of an exponential function:

$$f(t) = \mathrm{Re}\{Ae^{i(\omega t+\phi)}\} = \mathrm{Re}\{Ae^{i\phi}e^{i\omega t}\}. \tag{3}$$

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Graphic representation is a rotating phasor:

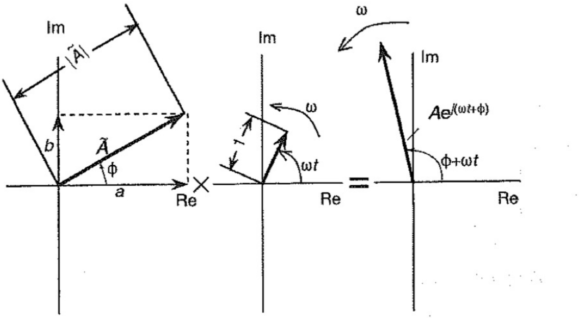

Fig. A1.1 Complex exponential representation of harmonic signals by phasors.

Interpretation:

- the projection of this rotating vector on the real axis denotes the state at each time instant
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- superposition of harmonic signals by summing the phasors (`http://tinyurl.com/2bo7bcj`):

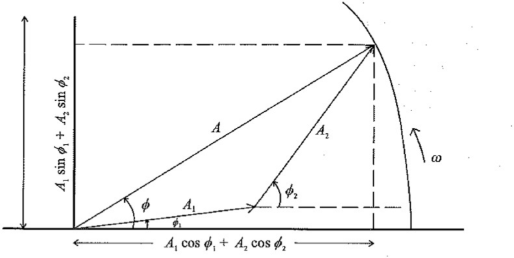

Fig. 1.4. Phasor representation of two simple harmonic motions having the same frequency.

2 Complex representation of a 1DOF oscillator

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and acceleration

$$\tilde{a} = \dot{\tilde{v}} = \ddot{\tilde{x}} = -\omega_0^2 \tilde{A}e^{i\omega_0 t}$$

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- the $\mathrm{Re}\{\}$-operator is often left unwritten in the literature

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Lossy 1DOF systems

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- $\Rightarrow$ the motion equation of a damped 1DOF oscillator

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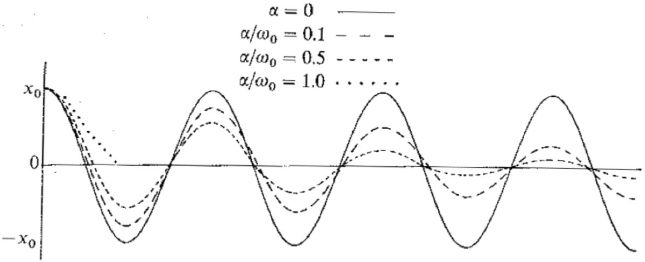

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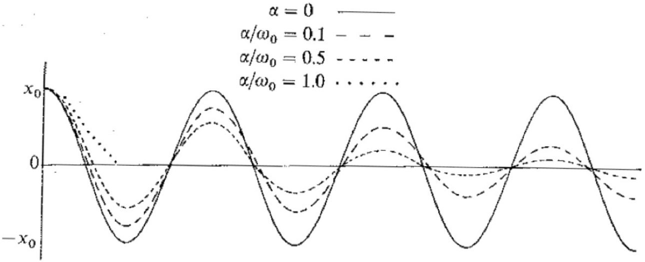

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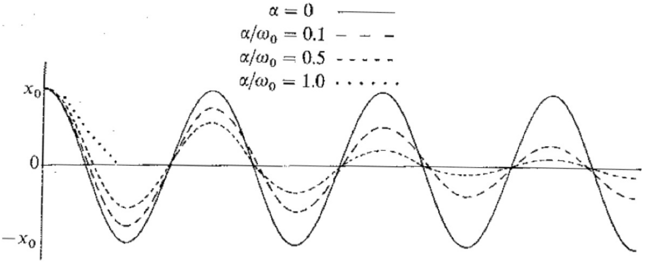

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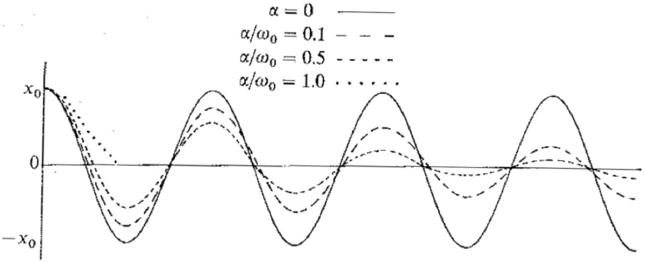

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3 Time constant and quality factor

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quality factor $Q = \frac{K}{R_\mathrm{m}\omega_0} = \frac{\omega_0}{2\alpha}$ denotes the sharpness of the resonance peak. The higher the Q, the sharper the resonance.

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if the excitation is sinusoidal $f(t) = \mathrm{Re}\{\tilde{F}\} = \mathrm{Re}\{Fe^{i\omega t}\}$:

$$\tilde{x} = \frac{\tilde{F}/m}{\omega_0^2 - \omega^2 + i\omega 2\alpha} \qquad \text{(R\&F:(1.60))}$$

$$\tilde{v} = \frac{\tilde{F}\omega/m}{2\omega\alpha + i(\omega^2 - \omega_0^2)} \qquad \text{(R\&F:(1.61))}$$

By dividing Eqs. (R&F:(1.60) and (1.61)) with $\tilde{F}$ and taking the absolute value, one obtains the response of displacement and velocity for a constant force excitation as a function of frequency:

$$\frac{|\tilde{x}|}{|\tilde{F}|} = \frac{1}{\sqrt{(K-\omega^2 m)^2 + (\omega R)^2}}$$

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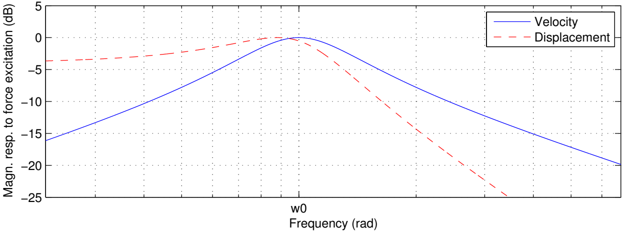
...these may be interpreted as **the transfer functions between input force and resulting displacement (or velocity)!**

3 Graphic interpretation

Transfer functions for force input as a function of frequency:



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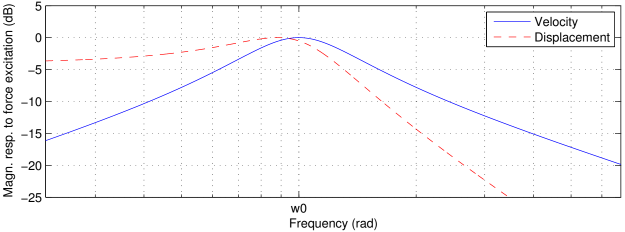

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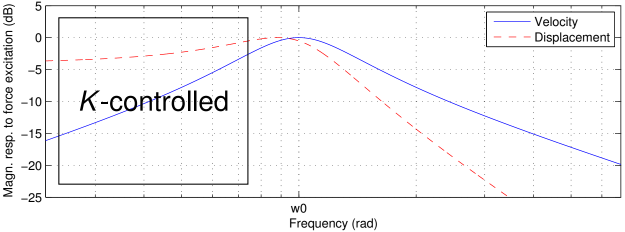

$$\Rightarrow \frac{|\tilde{x}|}{|\tilde{F}|} = \frac{1}{\sqrt{(\omega_0^2 m - \omega^2 m)^2 + (\omega R)^2}}, \quad \frac{|\tilde{x}|}{|\tilde{F}|} \approx \begin{cases} \frac{1}{K} & \text{if } \omega << \omega_0, \\ \frac{1}{\omega R} & \text{if } \omega \approx \omega_0, \\ \frac{1}{\omega^2 m} & \text{if } \omega >> \omega_0 \end{cases}$$

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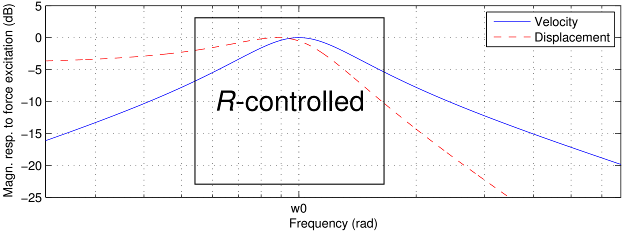

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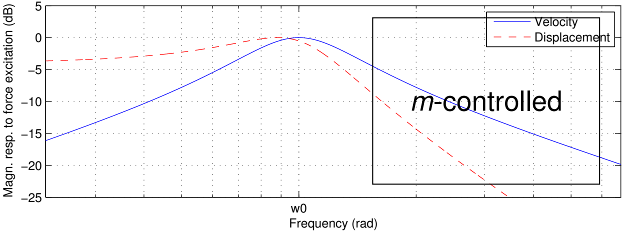
3 Graphic interpretation

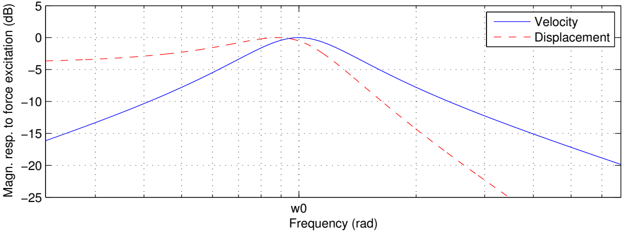
Transfer functions for force input as a function of frequency:



$$\Rightarrow \frac{|\tilde{x}|}{|\tilde{F}|} = \frac{1}{\sqrt{(\omega_0^2 m - \omega^2 m)^2 + (\omega R)^2}}, \quad \frac{|\tilde{x}|}{|\tilde{F}|} \approx \begin{cases} \frac{1}{K} & \text{if } \omega << \omega_0, \\ \frac{1}{\omega R} & \text{if } \omega \approx \omega_0, \\ \frac{1}{\omega^2 m} & \text{if } \omega >> \omega_0 \end{cases}$$

3 Graphic interpretation

Transfer functions for force input as a function of frequency:



$$\Rightarrow \frac{|\tilde{x}|}{|\tilde{F}|} = \frac{1}{\sqrt{(\omega_0^2 m - \omega^2 m)^2 + (\omega R)^2}}, \quad \frac{|\tilde{x}|}{|\tilde{F}|} \approx \begin{cases} \frac{1}{K} & \text{if } \omega << \omega_0, \\ \frac{1}{\omega R} & \text{if } \omega \approx \omega_0, \\ \frac{1}{\omega^2 m} & \text{if } \omega >> \omega_0 \end{cases}$$

3 Combinations of springs and masses