Aalto University
School of Electrical Engineering

Plane and Spherical Waves, Intensity

ELEC-E5610 Acoustics and the Physics of Sound, Lecture 4

Ville Pulkki

Department of Signal Processing and Acoustics
Aalto University School of Electrical Engineering

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Plane Wave

1 Plane Wave

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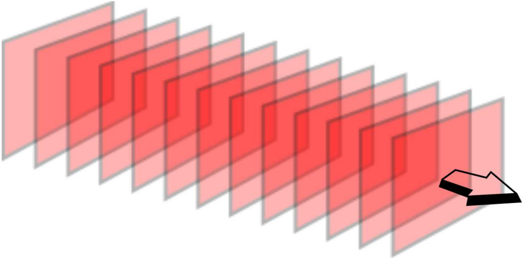

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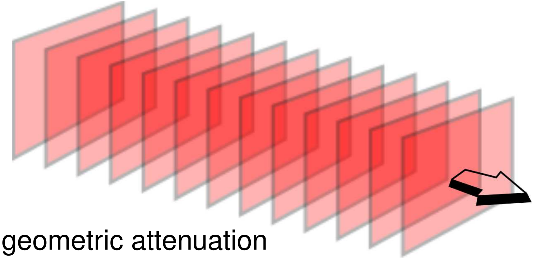

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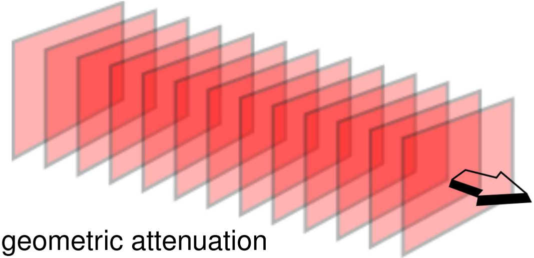

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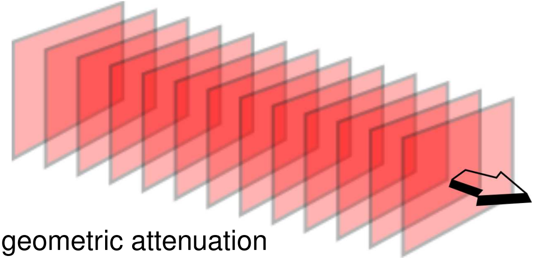

- no geometric attenuation
- pure plane waves do not exist in reality
- sound fields act as plane waves far from the source

1 Plane Wave II

Practical plane- and cylindrical wave approximations can be obtained with line array constructions

- typically used at large outdoor concerts
- lower attenuation of sound as a function of distance (compared to spherical waves)

1 Complex pressure wave

A plane wave propagating in the positive x-direction can be given using the complex representation

$$p = \tilde{A}e^{-ikx}e^{i\omega t} \qquad \text{(FF:(7.9),R\&F:(6.16))}$$

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Complex pressure wave
Characteristic Impedance
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Note: exponents are purely imaginary $\Rightarrow$ no losses!

1 Characteristic Impedance of a Plane Wave

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For other wave types (spherical, cylindrical, etc.) the [often complex] characteristic impedance depends on the type of the sound field and spatial coordinates.

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- Eq. (1): $\frac{p}{u} = \rho c \Rightarrow u = \frac{p}{\rho c}$
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- $u = \frac{20\,\mathrm{Pa}}{1.2\,\mathrm{kg/m^3}\;343\,\mathrm{m/s}} \approx 49$ mm/s

2 Intensity

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- Energy density $D = ||\mathbf{u}||^2(\rho_0/2) + |p|^2/(2\rho_0 c^2)$ (acoustic) energy in unit volume
- consists of kinetic (D_k) and potential (D_p) energy densities
- for plane waves $D = 2D_\mathrm{k} = 2D_\mathrm{p} = \rho\mathbf{u}\cdot\mathbf{u} = \frac{p^2}{K}$
- assume bulk modulus independent of pressure $\Rightarrow K = \rho c^2$

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- $|I| = Dc = \frac{p^2}{\rho c^2} \cdot c = \frac{p^2}{\rho c} = \frac{p^2}{z_c} = |pu|$
- Generally for 3D vectors: $\mathbf{I} = p\mathbf{u}$

2 Intensity of a Plane Wave II

As a conclusion, one can estimate the intensity of a plane wave if two of the following quantities are known:

- pressure p
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Acoustic domain	Electric domain
$p = z_c u$	$U = ZI$
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Remember: **intensity is a power quantity**!

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(just keep this small distinction in mind, we'll be talking about RMS quantities for the remainder of this lecture, unless otherwise stated)

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  - $\Rightarrow$ intensity is a vector quantity!
- generally, intensity is complex
- can be divided into real and imaginary parts
  - real part called **active intensity** (or average intensity)
  - imaginary part called **reactive intensity**

2 Acoustic Intensity in General II

For harmonic fields, the following holds:

For plane waves:

$$\mathbf{I} = \frac{|p|^2}{\rho_0 c}\mathbf{e_u}$$

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For general fields:

$$\mathbf{I} = p\mathbf{u}^* = \frac{z_\mathrm{c}|p|^2}{|z_\mathrm{c}|^2}\mathbf{e}_{\mathrm{u}^*}$$

$$\mathrm{Re}(\mathbf{I}) = \frac{\mathrm{Re}(z_\mathrm{c})|p|^2}{|z_\mathrm{c}|^2}\mathbf{e}_{\mathrm{u}^*}$$

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2 Practical Importance

Acoustic intensity can be used e. g. in

- identifying noise sources
  - note that the pressure fields may often be very complicated!
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  - Note: shape of the enclosing surface is arbitrary

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Problem: hard to measure particle velocity!

- classic solution: measure the pressure difference between two microphones $\propto$ particle velocity
  - average pressure of microphones $\approx$ pressure between the microphones
- nowadays also devices which directly measure the particle velocity

2 Intensity Measurements II

Figure: A Brüel&Kjær intensity probe

2 Intensity Measurements III

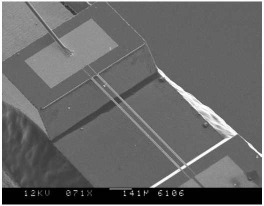


Figure: Velocity sensor based on heated platinum wires (Microflown Inc) Scale: micrometers.

Figure: Measurement of 3D intensity with 3 velocity sensors directed at XYZ axes, and a coincident pressure sensor. Scale: centimeters.

2 Case Study: Loudspeaker Intensity (FF:p.83-84)

Plane Wave
RMS vs. Peak Values
General
Practical Importance
Measurements
Case Study I

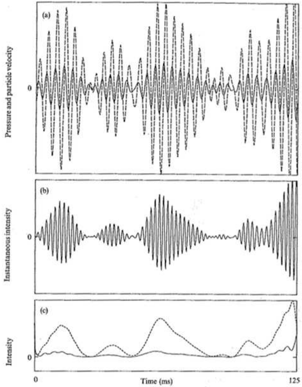


Fig. 5.5 Measurements in the near field of a loudspeaker: (a) sound pressure ——; normalized particle velocity ------; (b) instantaneous intensity; (c) complex instantaneous intensity, Re ——, Im ------. One-third octave band centred on 250 Hz. Reproduced with permission from Jacobsen, F. (1991) 'A note on instantaneous and time-averaged active and reactive intensity', *Journal of Sound and Vibration* **147**: 489–496.

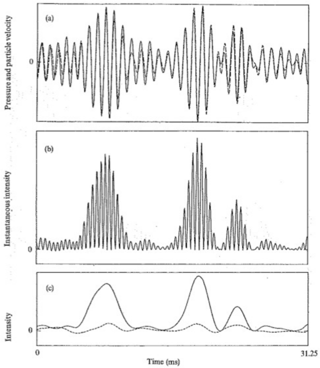


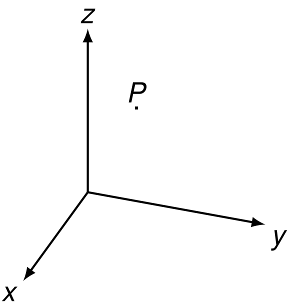
Fig. 5.6 Measurements in the near field of a loudspeaker: (a) sound pressure ——; normalized particle velocity ------; (b) instantaneous intensity; (c) complex instantaneous intensity, Re ——, Im ------. One-third octave band centred on 1 kHz. Reproduced with permission from Jacobsen, F. (1991) 'A note on instantaneous and time-averaged active and reactive intensity', *Journal of Sound and Vibration* **147**: 489–496.

3

Spherical waves

3 Spherical Coordinates

A point *P* can be given in spherical coordinates using



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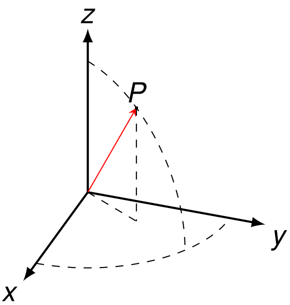

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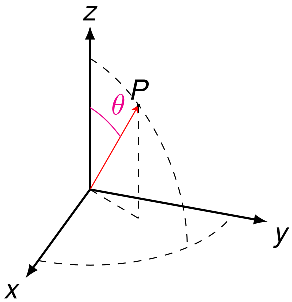
- angle θ (from z-axis)



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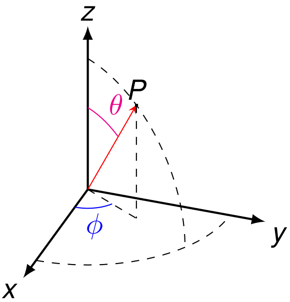

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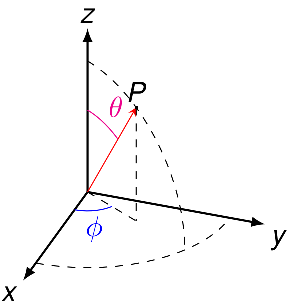
Relation to cartesian coordinates:

$$x = r \sin\theta \cos\phi \qquad r = \sqrt{x^2 + y^2 + z^2}$$

$$y = r \sin\theta \sin\phi \qquad \theta = \arccos(z/r)$$

$$z = rcos\theta \qquad \phi = \arctan(y/x)$$



3 Spherical Coordinates II

Calculations for point sources become simpler when switching from cartesian to spherical coordinates

- set the origin at the source
- the sound field is symmetric w.r.t. origin, angles θ and ϕ become irrelevant

3 Definitions

Some definitions related to spherical sound fields:

acoustic center is the center of spherical sound waves radiating outward a sound source

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$1/r^2$-law illustrates the behavior of the power w.r.t. distance from the source.

- in effect, intensity decreases 6 dB when the distance doubles. Why?

3 Characteristic Impedance of a Spherical Wave

The characteristic impedance of a spherical wave can be derived from wave equation:

$$z_{\mathrm{s}} = \frac{p}{u} = \rho c \left(\frac{ikr}{1 + ikr} \right) \qquad \text{(R\&F:(6.29), FF:(3.45))}$$

$$= \rho c \left(\frac{k^2 r^2}{1 + k^2 r^2} + i \frac{kr}{1 + k^2 r^2} \right)$$

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What is the limit value far away from the source?

- when $kr \to \infty \Rightarrow z_\mathrm{s} \to \rho c$
- in other words, the wavefront starts to resemble a plane wave!

3 Characteristic Impedance of a Spherical Wave II

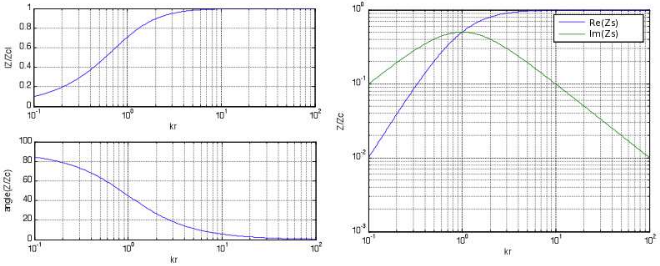

Figure: The characteristic impedance of a spherical wave in real and imaginary parts (right) and its relation to the characteristic impedance of a plane wave (left).

3 Near- and Far Fields

Near field: the relation between p and u differs from the plane-wave case

- close to the sound source when considering the
  - wavelength
  - dimensions of the source
- some wave components attenuate quickly when moving away from the source

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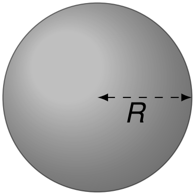
Near field: the relation between *p* and *u* differs from the plane-wave case

- close to the sound source when considering the
  - wavelength
  - dimensions of the source
- some wave components attenuate quickly when moving away from the source

Far field: the wave field acts locally as a plane wave

3 Elementary Monopole

Consider a sphere with radius R.



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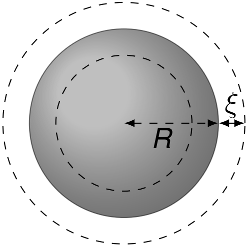
- the radial displacement is ξ



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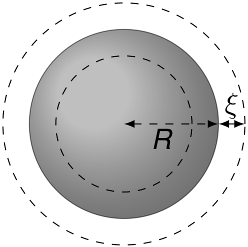

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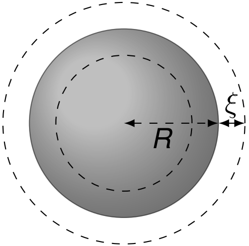

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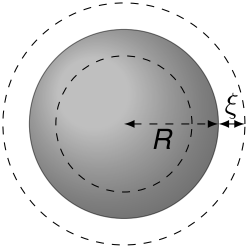
If $R \to 0$, the source becomes an **elementary monopole**.



3 Pressure Field Created by an Elementary Monopole

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What is the pressure field created by a point source?

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For a point source $R = 0 \Rightarrow$

$$\tilde{p} = \frac{i\omega\rho q_0}{4\pi r}e^{-ikr} \qquad \text{(FF:6.20,R\&F:7.4)}$$

3 Intensity Created by an Elementary Monopole

Active intensity in the radial direction ($\tilde{p}$ is peak, not RMS):

$$\mathrm{Re}[\mathbf{I}] = \frac{1}{2}|\tilde{p}|^2 \mathrm{Re}\left[1/z_{\mathrm{r}}\right]\mathbf{e}_{\mathrm{r}} = \frac{1}{2\rho c}|\tilde{p}|^2 \mathbf{e}_{\mathrm{r}} \qquad \text{(FF:6.27)}$$

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Acoustic power can be computed by integrating Eq. (FF:6.27) over a spherical surface:

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What can you say about the point source as a bass source?

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Poor radiator at low frequencies. Also, power is mainly a property of the source.

3 Mechanical Radiation Impedance

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The radiation impedance of a pulsating sphere is given as

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Remember $k = \omega/c$. For small ω, $\mathrm{Im}[z_{\mathrm{mrad}}] \gg \mathrm{Re}[z_{\mathrm{mrad}}]$.

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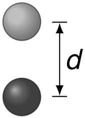
is called **attached mass**. This attached mass represents the inertial effect of the surrounding fluid, and can be approximated for low frequencies as $3\times$ the mass of the fluid replaced by the sphere.

3 Elementary Dipole

Placing two elementary monopoles with opposite phases at a distance d between them creates an **elementary dipole**.



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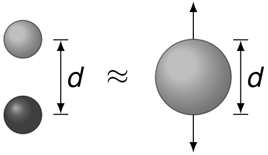
A sphere with diameter d, vibrating back and forth, may be considered as an elementary dipole, if $kd \ll 1$ (diameter must be small w.r.t. wavelength).



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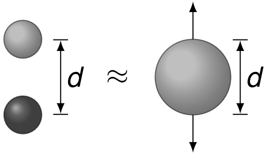
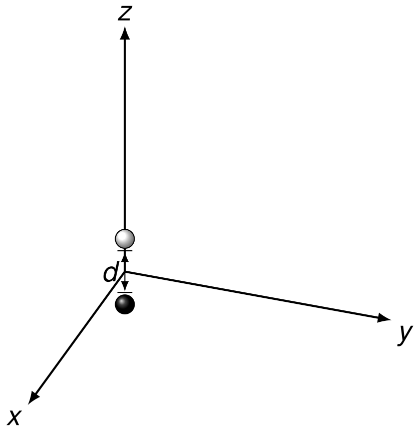
We will study the sound field created by an elementary dipole in what follows.



3 Elementary Dipole II

Consider a case where each monopole is placed on the z-axis.



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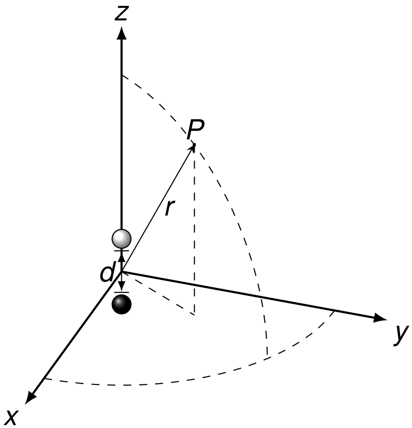

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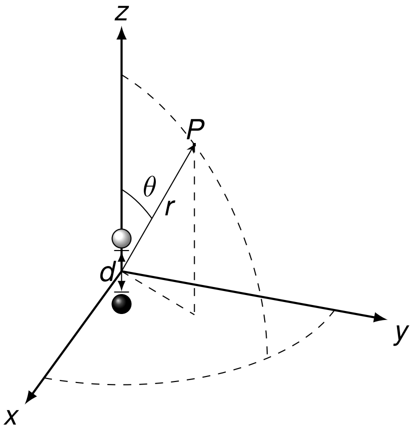

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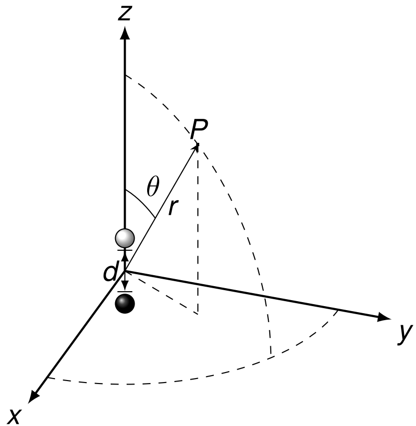

Note the rotational symmetry around *z*-axis!

3 Pressure Field

The expression for the sound pressure at P is

$$\tilde{p}(r,\theta) = \frac{\omega^2 \rho}{4\pi c r}\left(1 + \frac{1}{ikr}\right) e^{-ikr} \mu \cos\theta \qquad \text{(R\&F:7.7)}$$

where $\mu = q_0 d$ is the **dipole moment**.

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Equations for the particle velocity, characteristic impedance, and impedance can be found in FF:p.115.

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What can you say about the bass response of the elementary dipole?
- Extremely poor radiator at low frequencies (acoustic short-circuit)!