Aalto University
School of Electrical
Engineering

Waves in fluids and rectangular enclosures

ELEC-E5610 Acoustics and the Physics of Sound, Lecture 3

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1
Fluids

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- liquids
  - certain volume in a certain temperature
  - strong interaction between particles
  - the relative deformation of volume is smaller than the deformation of pressure

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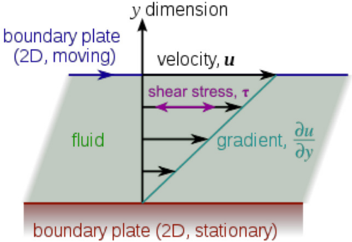

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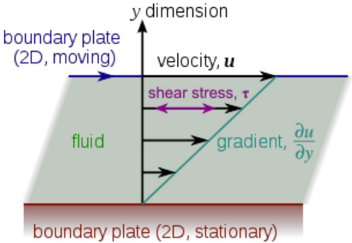
- no heat conduction



Bose-Einstein condensates (a.k.a *superfluids*, e. g. Helium below $2.2K^{\circ}$) fully act as ideal fluids (`http://www.youtube.com/watch?v=2Z6UJbwxBZI`).

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Although the ideal gas is an idealization, it is a **good approximation** to the behavior of many gases under many conditions.
Remember that pressure in gases arises from random molecular motion, it is scalar.

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- however, a more convenient model can be obtained by considering the medium as a *continuum*
  - $\Rightarrow$ forget the molecular structure of the fluid. Valid assumption, if

$$Kn = \frac{\Lambda}{L} \ll 1,$$

where Kn is Knudsen’s number, Λ is the average free space between molecular collisions, and L is the length in the dimension of observation.

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$\Rightarrow$ yes, air can be considered continuous for audio frequencies!

1 Disturbance

Sound is a disturbance of air density and pressure

- it is so rapid that no flow occurs
- no heat convection between compression and rarefaction occurs
- in a linear case we assume that the disturbance is the same regardless of the static air pressure

So, let's add a small disturbance to the static pressure P_0 and density ρ_0

$$P = P_0 + P_e \tag{1}$$

$$\rho = \rho_0 + \rho_e \tag{2}$$

so that $P_0 \gg P_e$ and $\rho_0 \gg \rho_e$

1 Features

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$\Rightarrow$ See also:
`www.feynmanlectures.caltech.edu/I_47.html`

1 Combining the features

III Spatial gradients generate gas motion

$$\rho_0 \frac{\partial^2 \chi}{\partial t^2} = -\frac{\partial P_e}{\partial x}$$

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$$\frac{\partial^2 \chi}{\partial t^2} = \kappa \frac{\partial^2 \chi}{\partial x^2}$$

where $\kappa = 1/c^2 = \rho/\gamma P$ for an adiabatic process

1 Combining features

So, our one-dimensional wave equation for a fluid looks very familiar:

$$\frac{\partial^2 \chi}{\partial t^2} = \kappa \frac{\partial^2 \chi}{\partial x^2}$$

We have the inertial force from the fluid mass, and restoring force from the pressure.
Note that we now derived the equation for displacement, the same applies for pressure.
[Check again Feynman "Lectures on physics" from previous slides for details]

1 Adiabatic process - nonlinearity

Note that in the adiabatic process the pressure-density relation is not linear

$$P = \alpha \rho^{\gamma}$$

However, the changes associated with audible sound are so small that we can approximate their variation with the tangent in the equilibrium point, like we did above.

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- normally, air behaves as adiabatic (contrary to what Newton thought)
- near solids and in porous materials air behaves as isothermal $\Rightarrow$ porosity slows down sound

1 Speed of sound II

A practical formula for the speed of sound in an ideal gas:

$$c = c_0\sqrt{T/T_0}, \qquad \text{(R\&F(6.13))}$$

where T is temperature and c_0 is sound velocity at temperature T_0.

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Typical sound speeds

- gases: 200-1000 m/s
- liquids: 1200-1600 m/s
- solids: 3000-6000 m/s

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Loss Mechanisms

2 Loss Types

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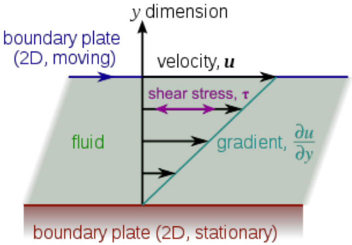
- geometric attenuation
- losses due to boundaries
- losses within the medium
  - viscosity
  - heat conduction
  - relaxation

2 Viscosity

Viscosity causes a shear stress (force) σ_{xy} in the direction of the movement.



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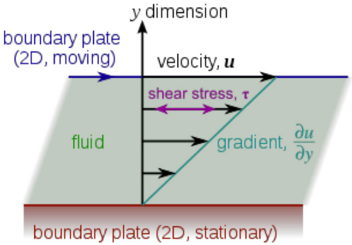

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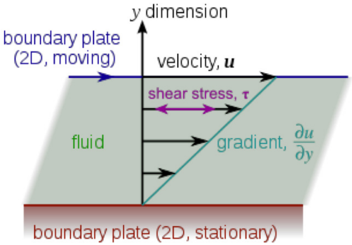

$$\sigma_{xy} = \mu \frac{\partial u_x}{\partial y}, \text{ where}$$

- μ is the *shear viscosity* (property of the fluid)
- u_x is the velocity of the fluid

2 Heat Conduction

- heat flows from compressed areas to uncompressed ones
- reduces pressure difference between wave maxima and minima
  - $\Rightarrow$ wave attenuates
- compressibility no longer adiabatic
- typical case near solids

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- it takes a certain time until the energy transfer is finished and an equilibrium is found
  - this time is called **relaxation time**

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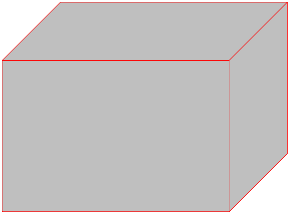

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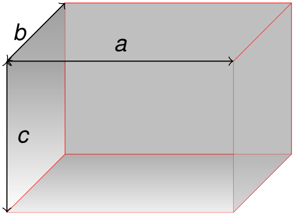
Consider a rectangular enclosure with dimensions $a \times b \times c$.



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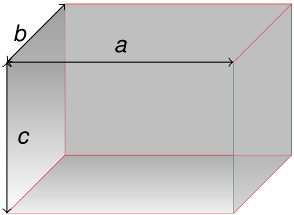
Consider a rectangular enclosure with dimensions $a \times b \times c$. Assume rigid walls

- total reflection
- $u_\mathrm{n} = 0$ at the boundary
- p maximum at the boundary



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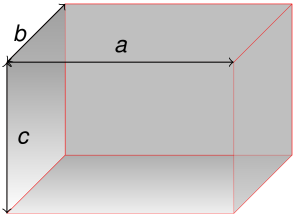

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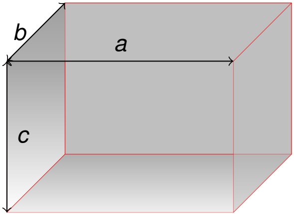

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with the boundary condition $z_\mathrm{wall} = \infty$

3 Solution of the Wave Equation

$$p(x,y,z,t) = \sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} p_{lmn}(x,y,z,t)$$

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(R&F:6.55,FF:9.1)

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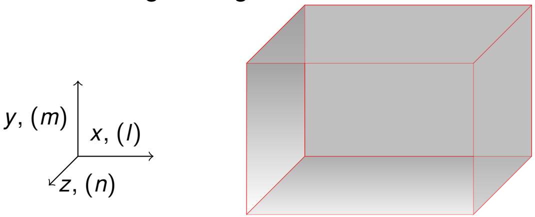
3 Room Modes

Mode numbers l, m, n represent the number of pressure zero-crossings along the coordinate axes.



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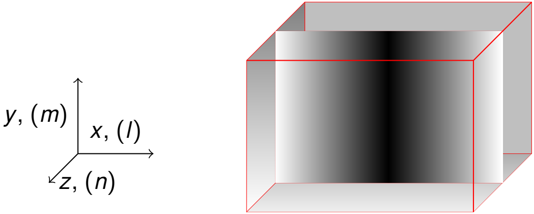

Let's visualize the pressure distribution with grayscale color (white denotes high absolute pressure value, black denotes pressure zero-crossing):

3 Room Modes

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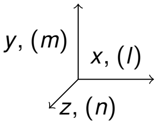

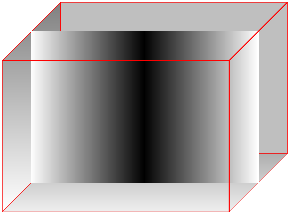

Let's visualize the pressure distribution with grayscale color (white denotes high absolute pressure value, black denotes pressure zero-crossing):
This is mode (1,0,0), since there is one zero-crossing along the *x*-axis

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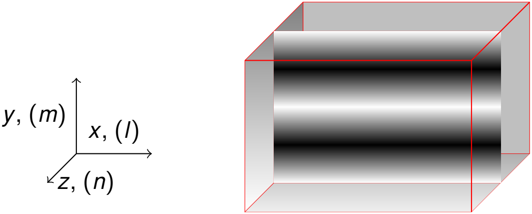

Let's visualize the pressure distribution with grayscale color (white denotes high absolute pressure value, black denotes pressure zero-crossing):
How about this?

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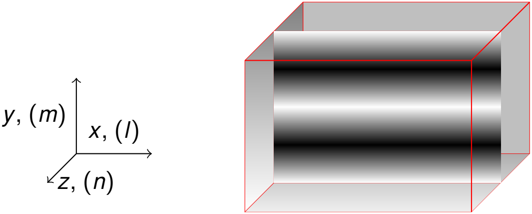

Let's visualize the pressure distribution with grayscale color (white denotes high absolute pressure value, black denotes pressure zero-crossing):
This is mode (0,2,0), since there are two zero-crossings along the *y*-axis

3 Room Modes

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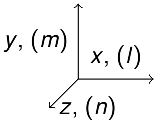

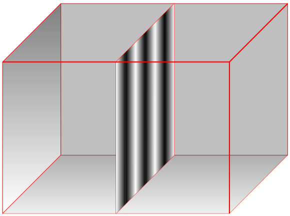

Let's visualize the pressure distribution with grayscale color (white denotes high absolute pressure value, black denotes pressure zero-crossing):
And what about this?

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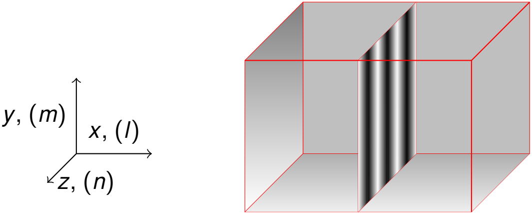

Let's visualize the pressure distribution with grayscale color (white denotes high absolute pressure value, black denotes pressure zero-crossing):
This is mode (0,0,3), since there are three zero-crossings along the *z*-axis

3 Room Modes

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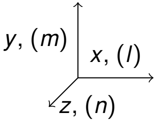
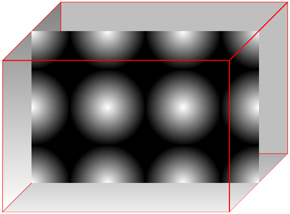
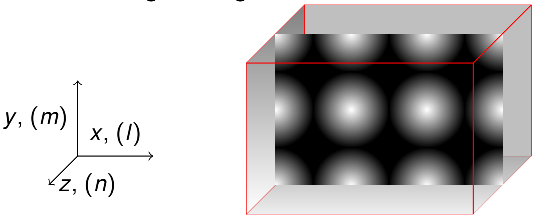




Let's visualize the pressure distribution with grayscale color (white denotes high absolute pressure value, black denotes pressure zero-crossing):
Bonus: which mode is this?

Mode numbers *l*, *m*, *n* represent the number of pressure zero-crossings along the coordinate axes.



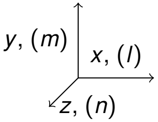
Let's visualize the pressure distribution with grayscale color (white denotes high absolute pressure value, black denotes pressure zero-crossing):

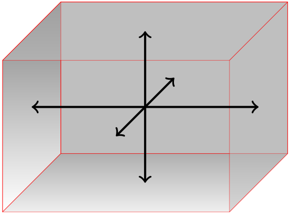
This is mode (3,2,0). See also

`http://www.falstad.com/modebox/`

3 Axial Modes

Modes for which only one number (l,m, or n) is nonzero, are called **axial modes**





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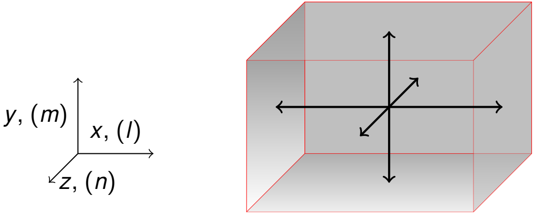
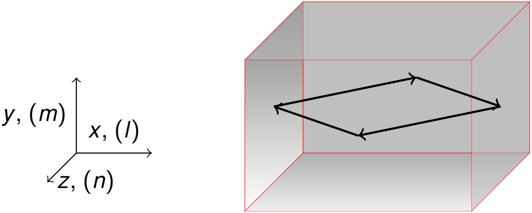


- axial modes encounter perpendicular reflection between two surfaces

3 Tangential Modes

Modes for which two numbers (l,m, or n) are nonzero, are called **tangential modes**



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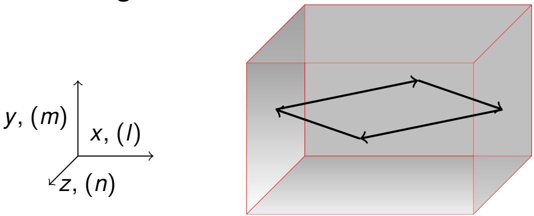
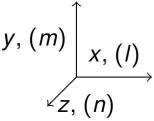
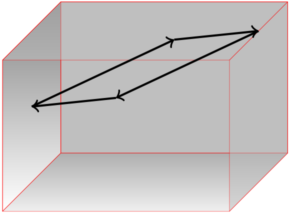


- tangential modes reflect arbitrarily between four surfaces, but within a plane normal to one of the axes

3 Oblique Modes

Modes for which all numbers (l,m, or n) are nonzero, are called **oblique modes**





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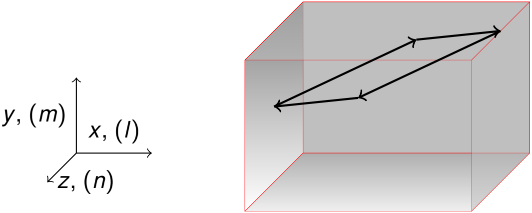

- oblique modes may reflect arbitrarily between all six surfaces

3 Eigenfrequencies

The eigenfrequencies of a rectangular enclosure are given as

$$f_{lmn} = \frac{c_0}{2}\sqrt{\frac{l^2}{a^2} + \frac{m^2}{b^2} + \frac{n^2}{c^2}}, \tag{4}$$

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- in steady state: only standing waves!

However, with a continuous source, also other frequency components exist. In other words, solution (3) is valid only in the "free vibration case".

3 Location of the Source

Similarly as with the string and membrane, the location of the source affects the resulting modes.

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Depending on the listening position, one will either measure/hear a frequency (at antinodes of standing wave) or not (at nodes of standing wave).

3 Several subwoofers

About 50% of modes are in opposite phase in two corners

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- If you put a subwoofer in each corner, and drive it with in-phase signal, what happens?

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- If you put a subwoofer in each corner, and drive it with in-phase signal, what happens?
- Antiphasic modes are cancelled

3 Sound Field Below the Lowest Eigenfrequency

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- does not depend on location
- the room acts as an acoustic capacitance

$$p = \frac{q}{i\omega C},$$

where $C = \frac{V}{K}$ is the acoustic capacitance, V is room volume, and K is the bulk modulus of air.

3 Modal Density

The average densities (eigenfrequencies per Hz) can be given for the different mode types as:

$$n_\mathrm{a} = \frac{L}{2c_0} \text{ for axial modes}$$

$$n_\mathrm{t} = \frac{\pi A f}{c_0^2} - \frac{L}{2c_0} \text{ for tangential modes}$$

$$n_\mathrm{o} = \frac{4\pi f^2 V}{c_0^3} - \frac{\pi A f}{c_0^2} \text{ for oblique modes}$$

where V is room volume, A is area of the surfaces, and L is the total length of the edges.

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where V is room volume, A is area of the surfaces, and L is the total length of the edges. An approximation for the total modal density for high frequencies is $n \approx \frac{4\pi V f^2}{c_0^3}$

3 Modal Density II

The modes should be evenly spaced in frequency to avoid distinct resonances at low frequencies

- room dimensions should not have integer relations

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  - living rooms
  - recording studios
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Table: Some widely used recommendations

Source	Dimension ratio
ASHRAE	1:1.17:1.47
Bolt	1:1.28:1.54
IAC	1:1.25:1.60
Sepmeyer	1:1.14:1.39
$1:\sqrt[3]{2}:\sqrt{2}$	1:1.26:1.41

3 Distribution of modes in two rooms

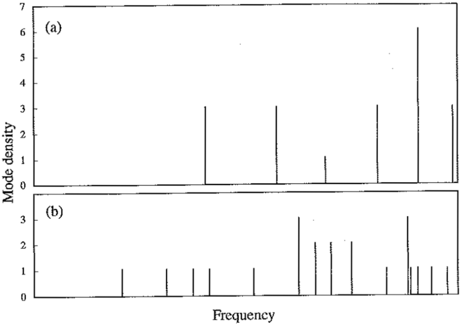

Fig. 6.3. Distribution of mode frequencies for two rectangular rooms of equal volume and dimension ratios (a) 2 : 2 : 2 and (b) 1 : 2 : 3. Where mode frequencies are coincident, the relevant line has been lengthened proportionally.

(R&F Fig 6.3)

3 Reverberation

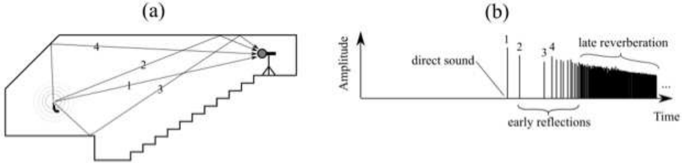

Figure 3: Sound propagation from an impulse source to a microphone inside a room (a), and the corresponding impulse response signal (b), as recorded by the microphone.

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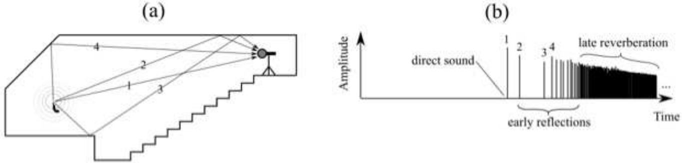

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Reverberation time, T_{60} is the time it takes for the reverberation to decay by 60 dB.

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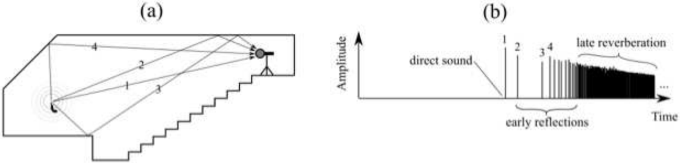

Figure 3: Sound propagation from an impulse source to a microphone inside a room (a), and the corresponding impulse response signal (b), as recorded by the microphone.

Reverberation time, T_{60} is the time it takes for the reverberation to decay by 60 dB. In diffuse sound fields with low wall absorption: approximated with *Sabine's formula*:

$$T_{60} = 0.161 \frac{V}{A_{\alpha}},$$

where A_{α} is the equivalent absorption area (calculated from material properties and absorption area).

3 Further Info

- Course ELEC-E5680 Virtual Acoustics
- H. Kuttruff, 2000, “Room Acoustics”, Taylor & Francis, ISBN: 0419245804
- http://www.falstad.com/ripple/