Aalto University
School of Electrical
Engineering

Reflection and Refraction in Fluid Boundaries

ELEC-E5610 Acoustics and the Physics of Sound, Lecture 5

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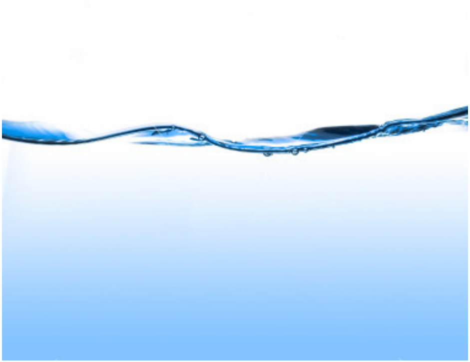
7 November 2023

1

Fluid Boundary

1 Continuity at the Boundary

Let us consider the boundary between two fluids.



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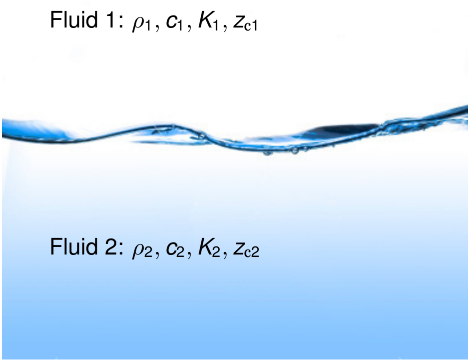

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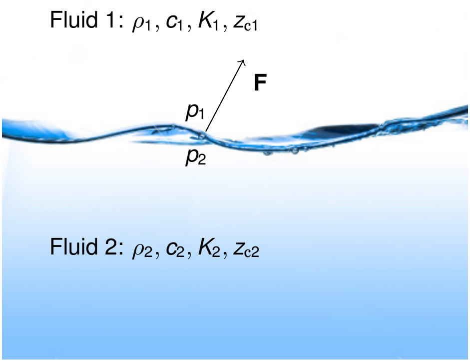

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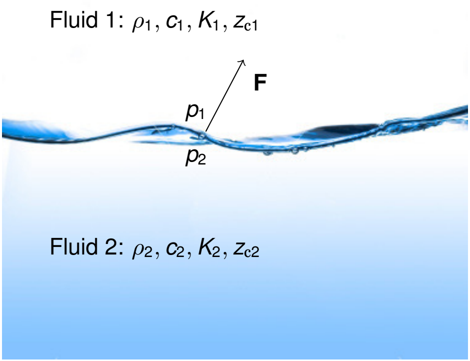

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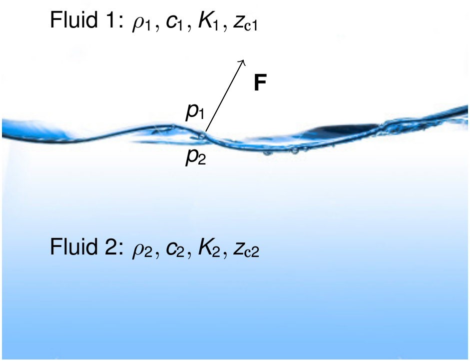

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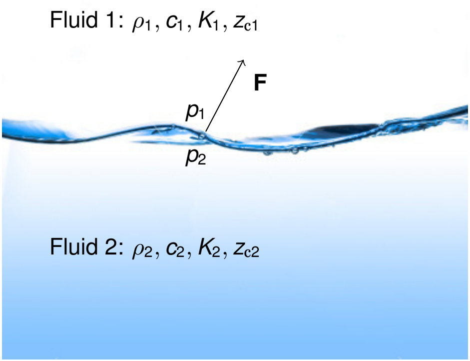

Because of infinitely small mass. Conclusion: **pressure must be continuous at the boundary**!

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- if it were not, there would either become a vacuum at the boundary, or the fluids would overlap each other
- ⇒ the normal component of the particle velocity must be zero at the boundary between an infinitely **rigid** wall and a fluid

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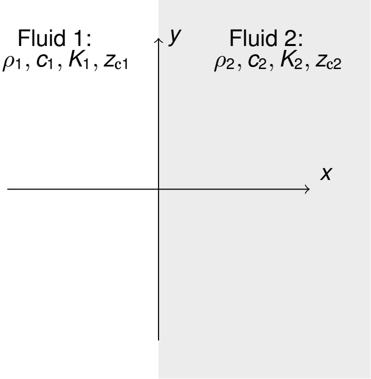
Perpendicular Reflection

2 Perpendicular Reflection: Pressure

If the incident angle is 90° w.r.t boundary, a *perpendicular reflection* occurs



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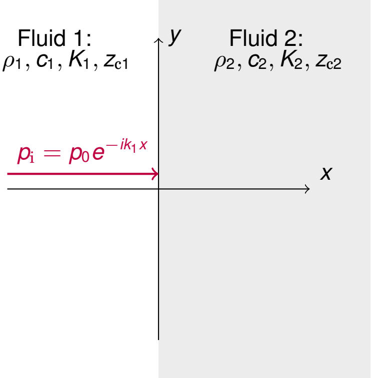

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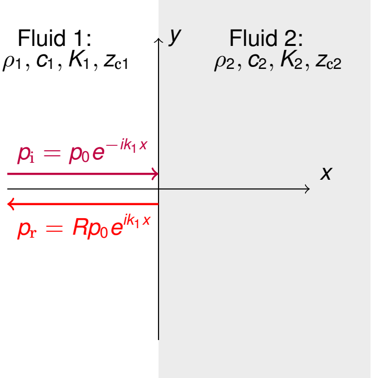

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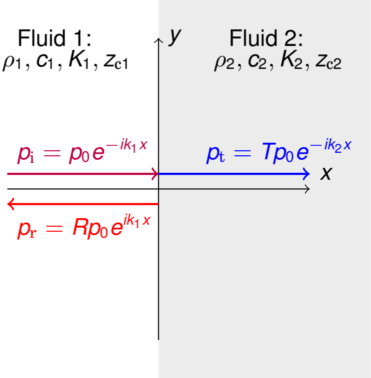

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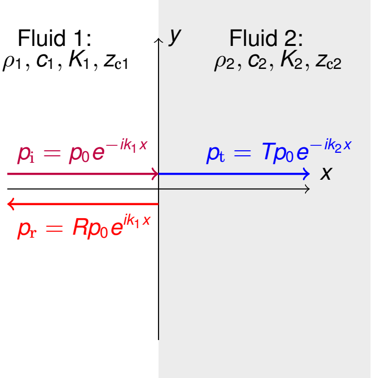

Note: $k_1 = \frac{\omega}{c_1} \neq \frac{\omega}{c_2} = k_2$ (different fluids!)

2 Perpendicular Reflection: Particle Velocity

Particle velocities behave in a similar manner

- We recall the relation between sound pressure and particle velocity: $\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$
- fluid 1:
  - incident particle velocity $u_\mathrm{i} = \frac{p_0}{z_{\mathrm{c}1}} e^{-ik_1 x}$
  - reflecting particle velocity $u_\mathrm{r} = -\frac{Rp_0}{z_{\mathrm{c}1}} e^{ik_1 x}$
  - note: the signs of the exponential and the term itself change (why?)

    (animations: `https://www.acs.psu.edu/drussell/Demos/phase-p-u/phase-p-u.html`)
- fluid 2:
  - transmitted particle velocity $u_\mathrm{t} = \frac{Tp_0}{z_{\mathrm{c}2}} e^{-ik_2 x}$

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Thus,

$$R = \frac{z_{\mathrm{c}2} - z_{\mathrm{c}1}}{z_{\mathrm{c}2} + z_{\mathrm{c}1}} \tag{1}$$

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"How can this be? Energy can't increase!"

- Remember: the characteristic impedances are different: if the particle velocity decreases, sound pressure increases
- of course, intensity can't increase

In fact, also the characteristic impedance is continuous at the boundary (see this by comparing the pressures and particle velocities on both sides)

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- $I_\mathrm{i} - I_\mathrm{r} = I_\mathrm{t}$ (check the math to verify)
- Transmitted intensity can also be given as

$$I_\mathrm{t} = I_\mathrm{i} - I_\mathrm{r} = (1 - |R|^2) I_\mathrm{i} \tag{3}$$

  - in other words, obtained using the incoming intensity and characteristic impedances!

2 Perpendicular Reflection: Intensity II

The result in Eq. (3) holds for the active intensity also when z_{c2} is complex

- denote $z_{c2} = r_2 + ix_2$, $z_{c1} = r_1$

Remember Eq. (1): $R = \frac{z_{c2} - z_{c1}}{z_{c2} + z_{c1}}$

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- the energy transfer coefficient becomes

$$1-|R|^2 = \frac{|r_2+ix_2+r_1|^2 - |r_2+ix_2-r_1|^2}{|r_2+ix_2+r_1|^2} = \frac{4r_1r_2}{(r_1+r_2)^2+x_2^2}$$

- if also z_{c2} is real, $x_2 = 0$ and we get

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$\frac{I_\mathrm{t}}{I_\mathrm{i}} = 1 - |R|^2 \approx 0.0011$
$\Rightarrow$ large impedance mismatch, poor power transfer.

2 Table: Sound from Air to Different Materials

Transmission loss in dB:

$$\mathrm{TL} = -10\log_{10}\frac{I_\mathrm{t}}{I_\mathrm{i}}$$

Table: Perpendicular reflection from air to different materials

Material	z_c [Pas/m]	$I_\mathrm{t}/I_\mathrm{i}$	TL [dB]
steel	$3.9 \cdot 10^7$	$4 \cdot 10^{-5}$	44
concrete	$9 \cdot 10^6$	$18 \cdot 10^{-5}$	38
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Note: TL gives here only the effect of a single boundary, not dissipative losses (inside the material), etc.

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The sound field in front of the boundary consists of the incoming wave and **the reflecting wave**

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$$p = p_\mathrm{i} + p_\mathrm{r} = p_0(e^{-ik_1x} + Re^{ik_1x})$$

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The sound field in front of the boundary consists of the incoming wave and the reflecting wave. Another interpretation is **a standing wave** plus

$$p = p_\mathrm{i} + p_\mathrm{r} = p_0(e^{-ik_1x} + Re^{ik_1x})$$

$$= \begin{cases} p_0(2R\cos(k_1x) + (1-R)e^{-ik_1x}), \text{ when } R \geq 0 \\ p_0(2iR\sin(k_1x) + (1+R)e^{-ik_1x}), \text{ when } R \leq 0 \end{cases}$$

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Remember: a standing wave transmits no energy (active intensity is zero). Similar equations can be derived for the particle velocity.

2 Impedance in front of a Boundary

Specific acoustic impedance at the distance $d = -x$ from the boundary can be given as

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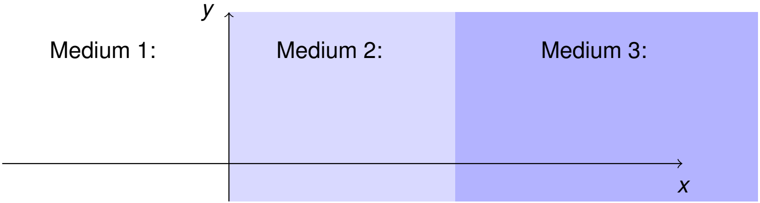
2 Reflection from Multilayer Medium

Consider a structure consisting of a multilayer medium.



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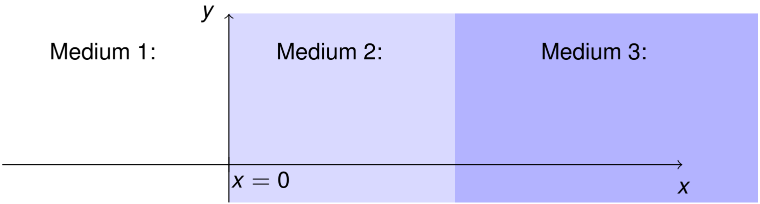

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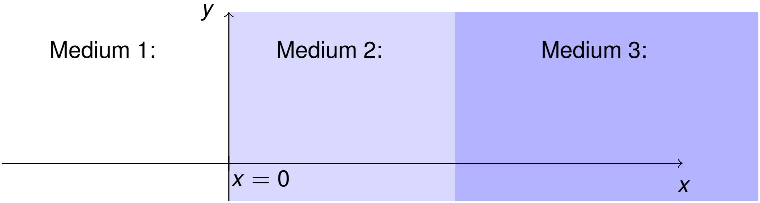

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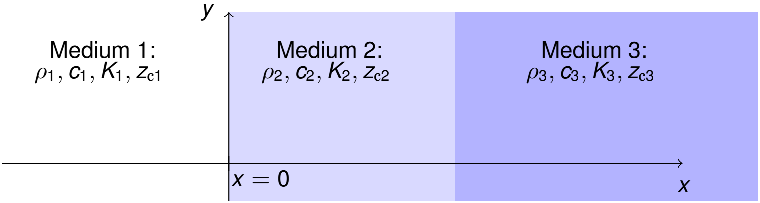

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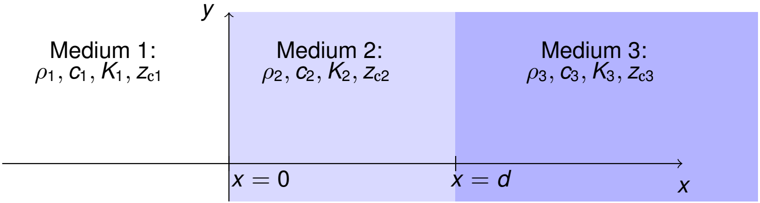

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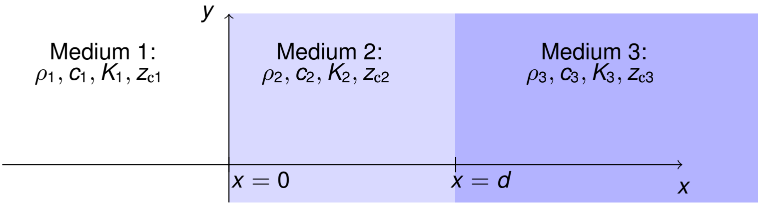

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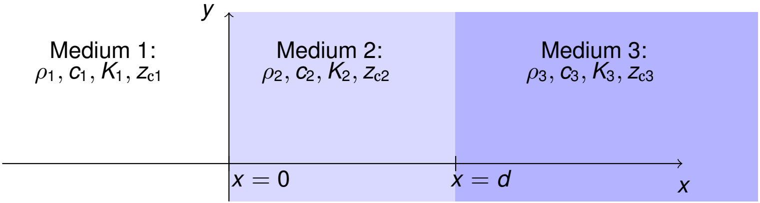
With multiple layers, one starts evaluating the boundary impedances from right to left using the above equation



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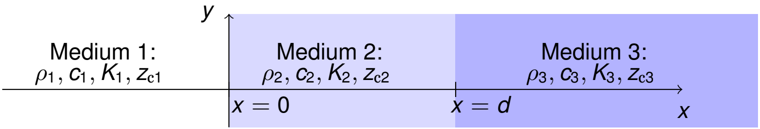

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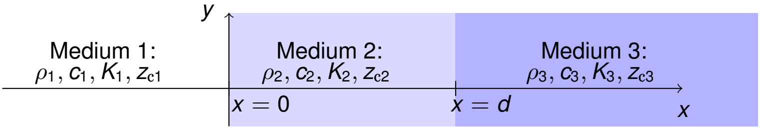

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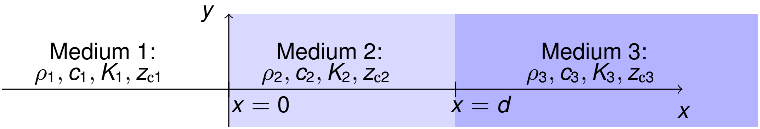
Answer: Yes, there are three such cases (check the math):



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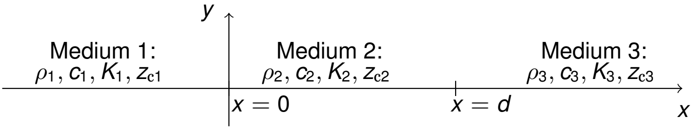

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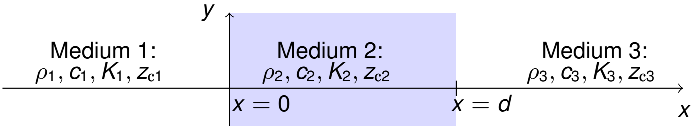

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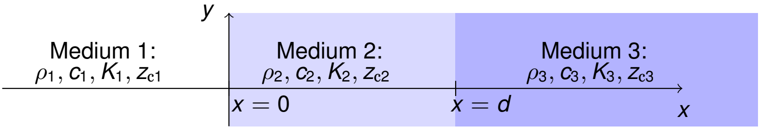
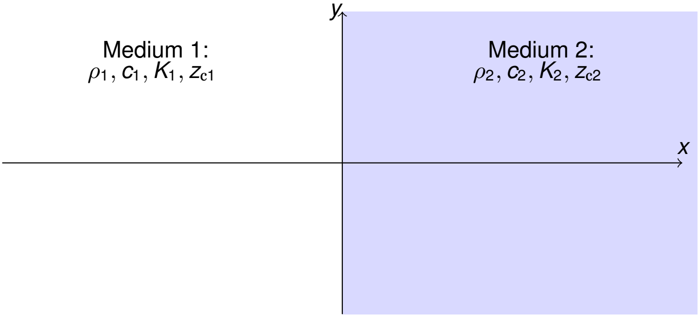
- Quarter-wave transformer: $z_{c2} = \sqrt{z_{c1} z_{c3}}$, $d = \frac{(2n+1)\lambda_2}{4}$, $n = 1, 2, 3...$



3

Reflection with Arbitrary Angle

3 Snell's Law



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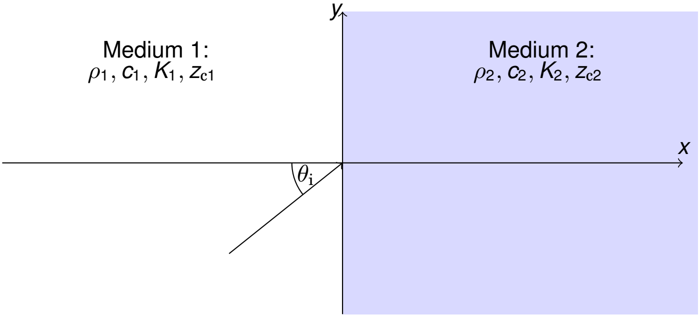
Medium 1:
ρ_1, c_1, K_1, z_{c1}
Medium 2:
ρ_2, c_2, K_2, z_{c2}
y
x
θ_i

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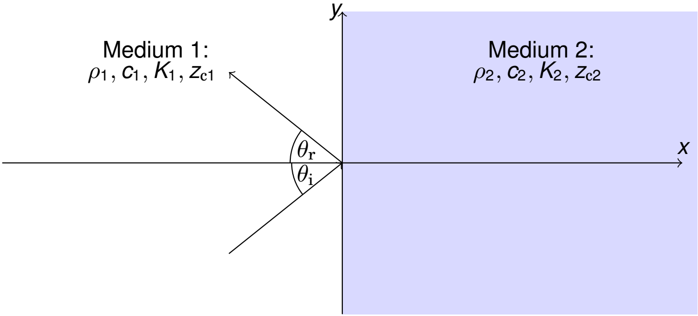
Medium 1:
ρ_1, c_1, K_1, z_{c1}
Medium 2:
ρ_2, c_2, K_2, z_{c2}
y
x
θ_r
θ_i

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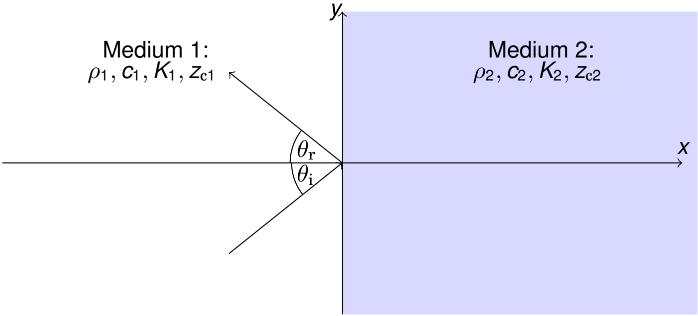

Generally,

- $\theta_i = \theta_r$

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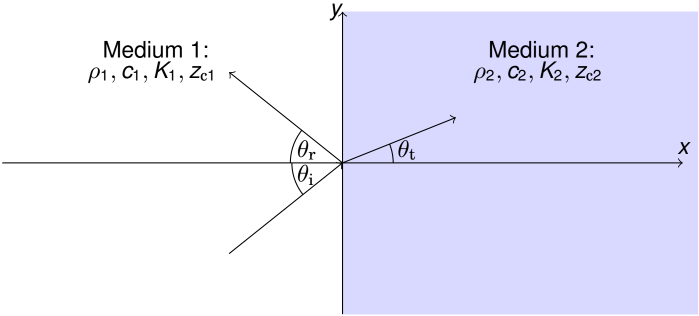

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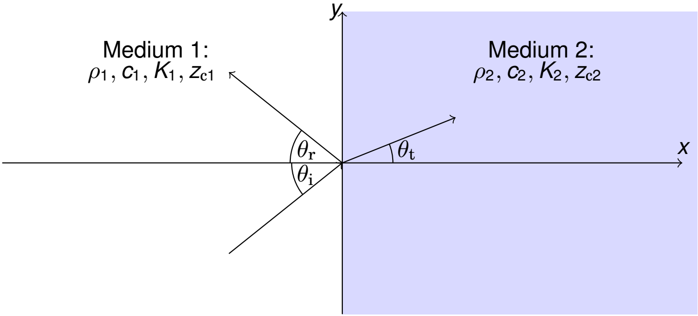
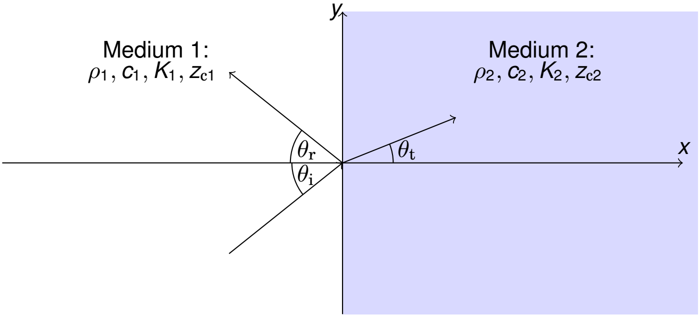


Generally,

- $\theta_i = \theta_r$
- $\theta_i \neq \theta_t$

3 Snell's Law



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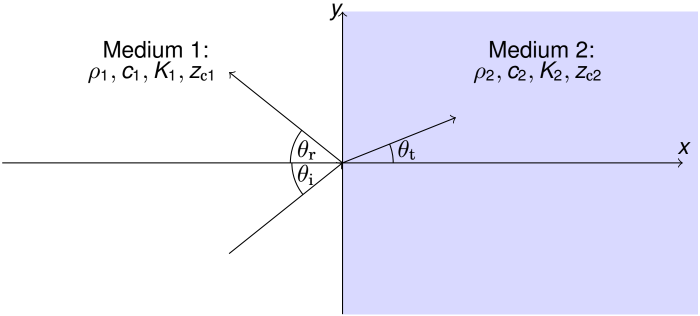

Generally,

- $\theta_\mathrm{i} = \theta_\mathrm{r}$
- $\theta_\mathrm{i} \neq \theta_\mathrm{t} \Rightarrow$ refraction happens!
- furthermore, $\frac{c_1}{c_2} = \frac{\sin\theta_\mathrm{i}}{\sin\theta_\mathrm{t}}$

3 Impedance in front of a Boundary

For arbitrary angles, the impedance at distance *d* from the boundary can be given as (compare to Eq. (4))

$$z = \left(\frac{z_{\mathrm{c}1}}{\cos\theta_{\mathrm{i}}}\right) \frac{\left(\frac{z_{\mathrm{c}2}}{\cos\theta_{\mathrm{t}}}\right) + i\left(\frac{z_{\mathrm{c}1}}{\cos\theta_{\mathrm{i}}}\right)\tan(k_1 d\cos\theta_{\mathrm{i}})}{\left(\frac{z_{\mathrm{c}1}}{\cos\theta_{\mathrm{i}}}\right) + i\left(\frac{z_{\mathrm{c}2}}{\cos\theta_{\mathrm{t}}}\right)\tan(k_1 d\cos\theta_{\mathrm{i}})} \tag{5}$$

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Reflection coefficient in this case is

$$R = \frac{\frac{z_{\mathrm{c}2}}{\cos\theta_{\mathrm{t}}} - \frac{z_{\mathrm{c}1}}{\cos\theta_{\mathrm{i}}}}{\frac{z_{\mathrm{c}2}}{\cos\theta_{\mathrm{t}}} + \frac{z_{\mathrm{c}1}}{\cos\theta_{\mathrm{i}}}} \tag{6}$$

3 Total Reflection

If $c_1 < c_2$, there is a **critical angle**

$$\theta_{\mathbf{cr}} = \sin^{-1}\left(\frac{c_1}{c_2}\right), \tag{7}$$

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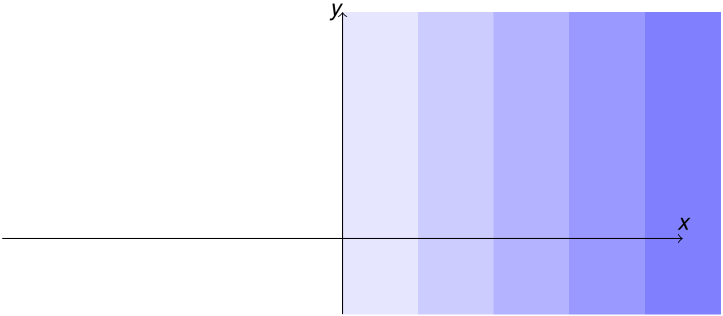
3 Snell’s Law in Multilayer or Gradually Changing Media



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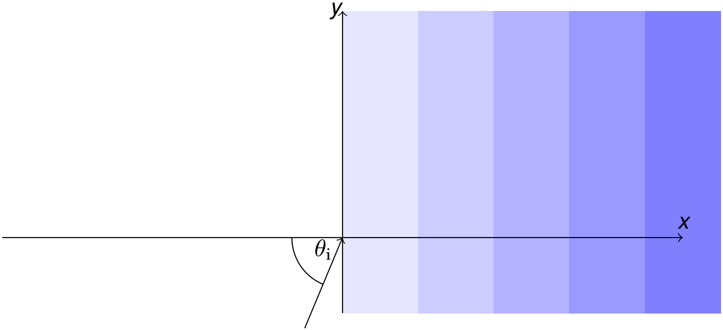

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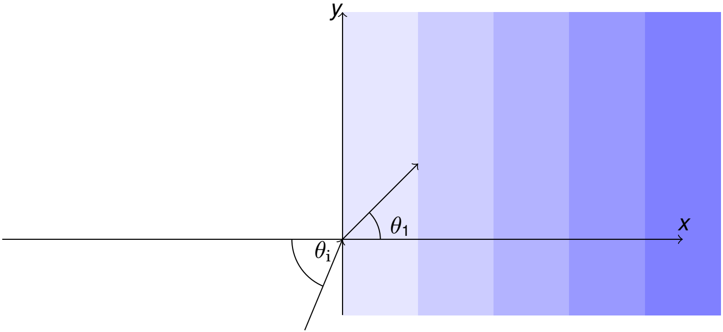

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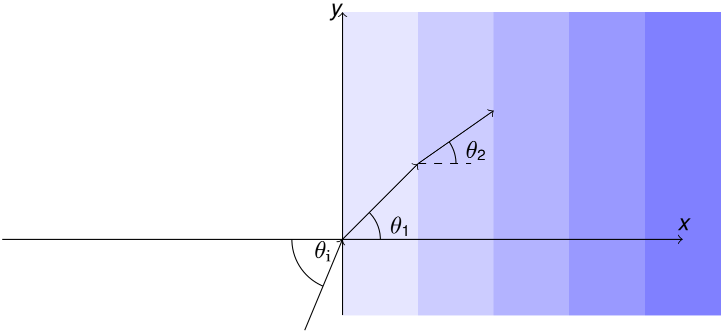

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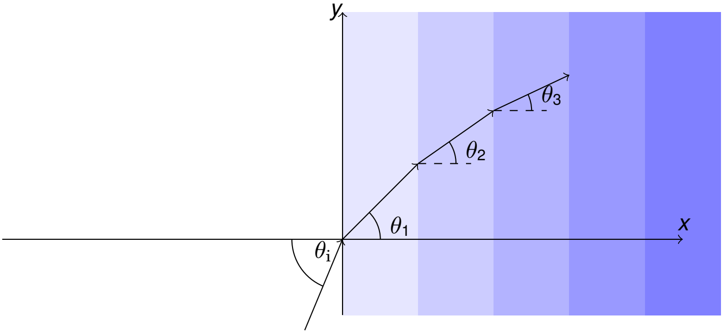

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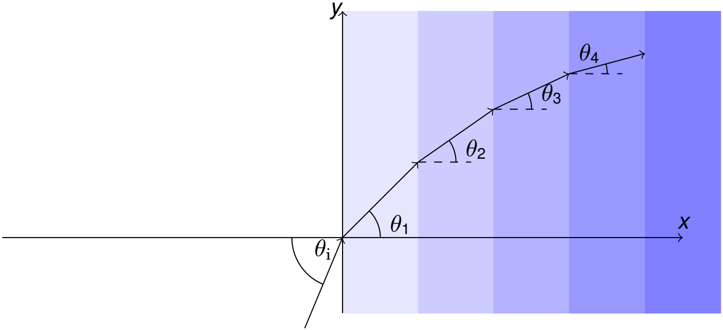

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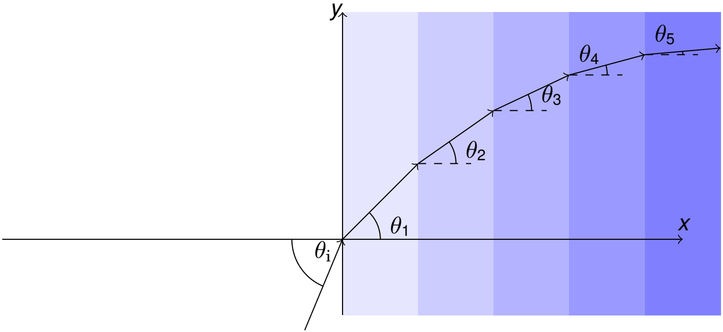

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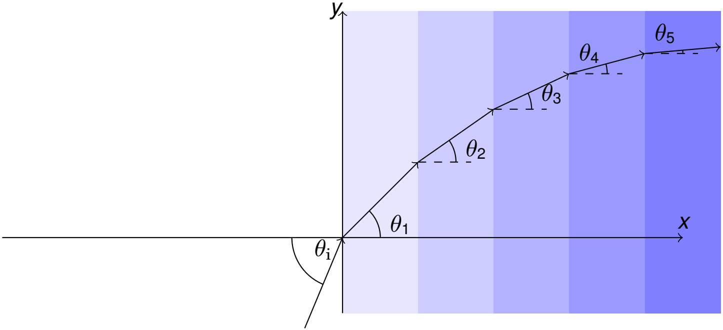

Bending of sound due to temperature gradients!

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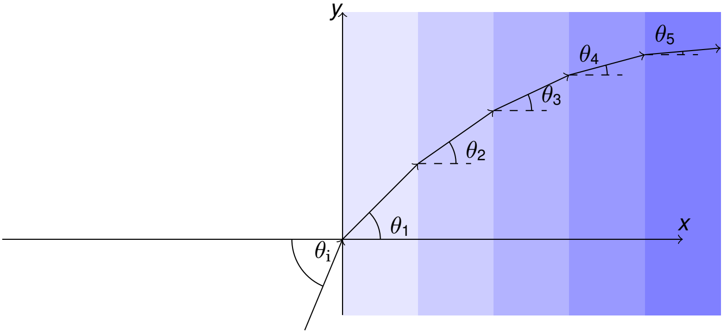

Bending of sound due to temperature gradients!

See also demo at `http://www.falstad.com/ripple/`

3 Locally Reacting Surface

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Thus, evaluation of the surface impedance becomes complicated. However, the surface wave may be neglected if

- the surface is anisotropic so that it is hard for the surface wave to propagate
- $c_1 \gg c_2$
- medium 2 is highly dissipative, so that the surface wave attenuates quickly

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Examples of locally reacting media (when air is medium 1):

- most acoustic absorption materials, such as mineral wools
- perforated materials
- soft soil

3 Locally Reacting Surface III

Example of a locally reacting surface: perforated material.