Aalto University

Acoustic Transmission Lines (part 1)

ELEC-E5610 Acoustics and the Physics of Sound, Lecture 6

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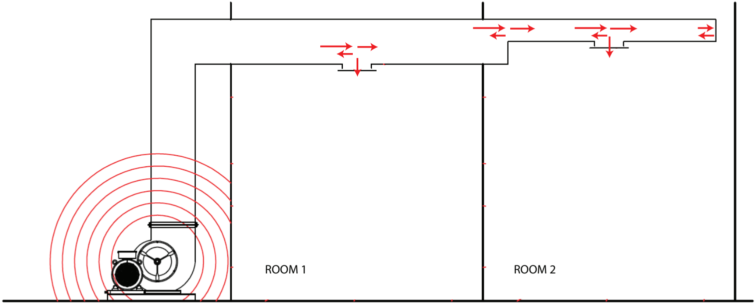
Pipes

1 Example: ventilation systems



- Noise of the motor propagates through the pipes to rooms

1 Acoustic Waveguides

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- $d < 0.586\lambda$ (for a round pipe)

In practice, viscous- and thermal losses near the pipe edges distort the plane wave somewhat, making this an approximation of a real-world case.

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- $q = uA$, where A is the cross-section area of the pipe

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- e. g. with interconnected ducts, there is a continuity rule for volume velocity
- $q = uA$, where A is the cross-section area of the pipe

The ratio between pressure and volume velocity is called **acoustic impedance**

$$z_{\mathrm{a}} = \frac{p}{q} = \frac{p}{uA} = \frac{z_{\mathrm{c}}}{A}, \; z_{\mathrm{c}} = \rho c$$

1 Transmission Line Representation

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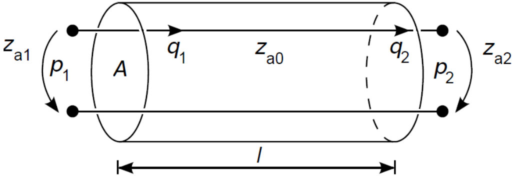

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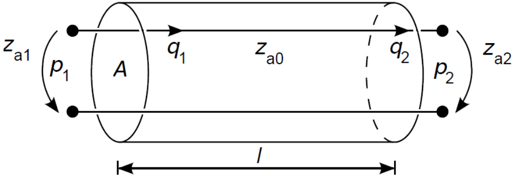

with the pressures, volume velocities and impedances as marked in the figure above.

1 Transmission Line Representation II

The relation between pressures and volume velocities on each side can be given by the transmission line equation

$$\begin{bmatrix} p_1 \\ q_1 \end{bmatrix} = \underbrace{\begin{bmatrix} \cos kl & iz_\mathrm{a} \sin kl \\ \frac{i}{z_\mathrm{a}} \sin kl & \cos kl \end{bmatrix}}_{T} \begin{bmatrix} p_2 \\ q_2 \end{bmatrix} \tag{1}$$

Variable $k = 2\pi/\lambda = \omega/c$ is the **wave number**, corresponding to the spatial frequency.

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- frequency defines k, smaller values for higher frequencies
- kl represents the *phase difference* btw the ends of the tube

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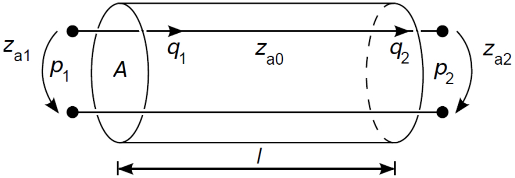
where T is called the transmission matrix. The combined effect of several cascaded pipes can be obtained by multiplying the T-matrices:

$$\begin{bmatrix} p_1 \\ q_1 \end{bmatrix} = [T_1] \begin{bmatrix} p_2 \\ q_2 \end{bmatrix}, \begin{bmatrix} p_2 \\ q_2 \end{bmatrix} = [T_2] \begin{bmatrix} p_3 \\ q_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} p_1 \\ q_1 \end{bmatrix} = [T_1][T_2] \begin{bmatrix} p_3 \\ q_3 \end{bmatrix}$$

1 The pipe we are talking about



1 Acoustic Impedance of a Pipe

The acoustic impedance of the pipe discussed above, seen to the right from the side 1 can be given as

$$z_{\mathrm{a1}} = \frac{z_0}{A} \frac{z_{\mathrm{a2}} A \cos(kl) + i z_0 \sin(kl)}{i z_{\mathrm{a2}} A \sin(kl) + z_0 \cos(kl)},$$

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where $z_0 = z_{\mathrm{c}} = \rho c$.

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- note that both impedances are periodical in frequency!

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Do these impedance forms look familiar?

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the air in the open pipe acts as a mass!

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the air in the closed pipe acts as a spring!

1 End Correction

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Why is the sound of wind instruments strictly harmonic, then? The continuous excitation together with nonlinear mechanisms forces a phase-locked oscillation to take place.

1 End Correction II

End correction for a round pipe (R&F:Fig.8.9):

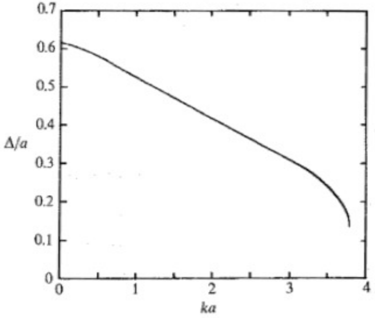

Fig. 8.9. The calculated end correction Δ for a cylindrical pipe of radius a, plotted as Δ/a, as a function of the frequency parameter ka, (after Levine and Schwinger, 1948).

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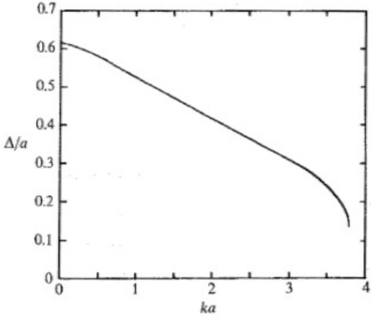

Fig. 8.9. The calculated end correction Δ for a cylindrical pipe of radius *a*, plotted as Δ/*a*, as a function of the frequency parameter *ka*, (after Levine and Schwinger, 1948).

Approximates 0.6 $\times$ radius for low frequencies.

1 Reflection from the End

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compare to the perpendicular reflection of a plane wave:

$$R = \frac{z_{\mathrm{c2}} - z_{\mathrm{c1}}}{z_{\mathrm{c2}} + z_{\mathrm{c1}}}$$

1 Pipe Terminated with a Characteristic Impedance

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Why doesn't this happen with simple open-ended pipes?

-The pipe end approximates an omnidirectional and point-like source for the surrounding air. However, zero-reflection can be implemented by

- absorption material
  - thickness $d \geq \lambda/4$
  - wedge-like or gradually changing density
- horns

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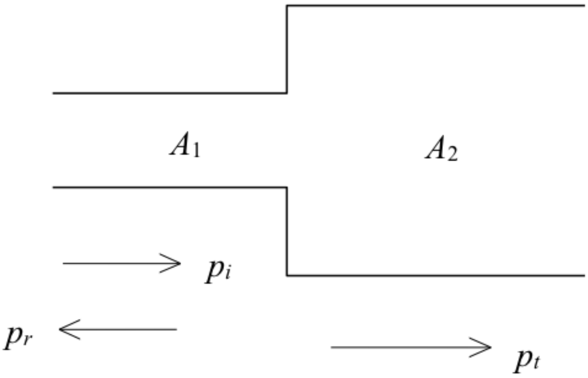
Pipes with Varying Cross-Section Area

2 Change in the Cross-Section Area



2 Change in the Cross-Section Area II

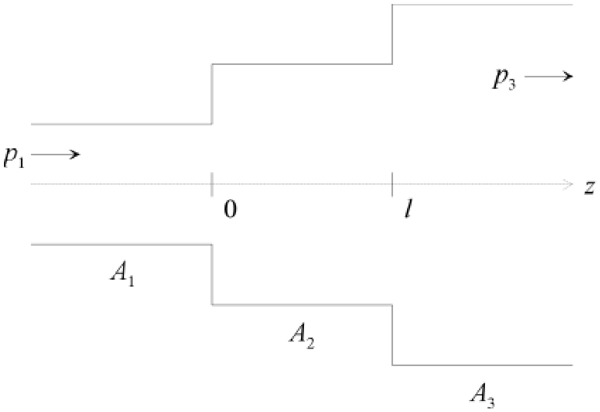
The reflection- and transmission coefficient (for pressure) calculations are identical to those in perpendicular reflection of plane waves, but this time the acoustic impedance is used instead of the characteristic impedance.

$$R = \frac{z_{\mathrm{a}2} - z_{\mathrm{a}1}}{z_{\mathrm{a}2} + z_{\mathrm{a}1}} = \frac{A_1 - A_2}{A_1 + A_2}$$

$$T = \frac{2z_{\mathrm{a}2}}{z_{\mathrm{a}2} + z_{\mathrm{a}1}} = \frac{2A_1}{A_1 + A_2}$$

$$T = R + 1$$

2 Consecutive Changes in Cross-Section Area



2 Consecutive Changes in Cross-Section Area II

The impedance seen to the right at $z = 0$ on the previous figure is given by

$$z_{\mathrm{a}0} = z_{\mathrm{a}2} \frac{z_{\mathrm{a}3} + i z_{\mathrm{a}2} \tan(kl)}{z_{\mathrm{a}2} + i z_{\mathrm{a}3} \tan(kl)} \tag{3}$$

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(note the similarity to the impedance for perpendicular plane-wave reflection from multilayer media). Evaluate R and T at $z = 0$:

$$R = \frac{z_{\mathrm{a0}} - z_{\mathrm{a1}}}{z_{\mathrm{a0}} + z_{\mathrm{a1}}} \qquad T = \frac{2 z_{\mathrm{a0}}}{z_{\mathrm{a0}} + z_{\mathrm{a1}}}$$

2 Consecutive Changes in Cross-Section Area III

Substitute Eq. (3), so that R and T become

$$R = \frac{A_2(A_1 - A_3) + i(A_1 A_3 - A_2^2)\tan(kl)}{A_2(A_1 + A_3) + i(A_1 A_3 + A_2^2)\tan(kl)}$$

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R and T **are periodical in frequency**!

- the period in frequency is $\frac{c}{2l}$
- i.e. the waves reflect and transmit the same every $\frac{c}{2l}$ Hz apart

2 Consecutive Changes in Cross-Section Area IV

Is it possible to select pipe areas A_1, A_2, A_3 and the pipe 2 length l so that there is no reflection back to pipe 1?

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- quarter-wave transformer:
  $A_2 = \sqrt{A_1 A_3}$, $l = (2n+1)\frac{\lambda}{4}$, $n = 0, 1, 2, 3, ...$

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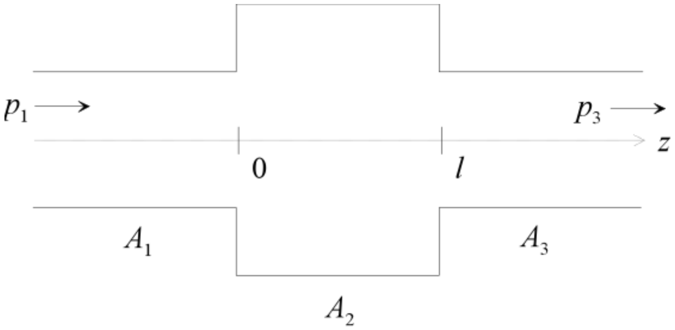
Additionally, the pipe diameters and lengths may be chosen so that maximum attenuation takes place at certain frequencies. We will consider those in the following.

2 Expansion Chambers



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$$\frac{I_\mathrm{t}}{I_\mathrm{i}} = \tau = \frac{4}{4\cos^2(kl) + (\frac{A_1}{A_2} + \frac{A_2}{A_1})^2 \sin^2(kl)} \qquad \text{(FF:(8.45a))}$$

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- for $l = n\lambda/2,\ n = 1, 2, 3, ... \Rightarrow \tau = 1$, no attenuation! For frequencies $f = n\frac{2l}{c}$ the expansion chamber is "invisible"

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For expansion chambers, $A_1 = A_3$, and A_2 and l are varied. The power transmission equation can be formulated as

$$\frac{I_{\mathrm{t}}}{I_{\mathrm{i}}} = \tau = \frac{4}{4\cos^2(kl) + (\frac{A_1}{A_2} + \frac{A_2}{A_1})^2 \sin^2(kl)} \qquad \text{(FF:(8.45a))}$$

- for $l = n\lambda/2,\ n = 1, 2, 3, ... \Rightarrow \tau = 1$, no attenuation! For frequencies $f = n\frac{2l}{c}$ the expansion chamber is "invisible"
- for $l = \lambda/4 + n\frac{\lambda}{2},\ n = 1, 2, 3, ... \Rightarrow \tau = \frac{4}{(A_1/A_2 + A_2/A_1)^2}$, maximum attenuation! The larger the area ratio A_2/A_1, the greater the attenuation.

2 Expansion Chambers III

Example transmission loss of an expansion chamber (FF:p.204).

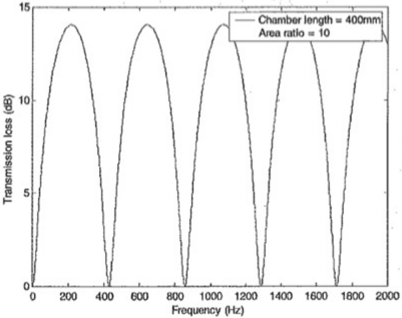

Fig. 8.12 Transmission loss produced by an expansion chamber with an area ratio of ten.

2 Expansion Chambers IV

Can you think of any practical application for the expansion chamber attenuator?

2 Expansion Chambers IV

Can you think of any practical application for the expansion chamber attenuator?

-For example the mufflers in combustion engines: