Aalto University

Acoustic Transmission Lines (part 2) + Horns

ELEC-E5610 Acoustics and the Physics of Sound, Lecture 7

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November 14, 2023

1

Transmission Lines as Equivalent Circuits

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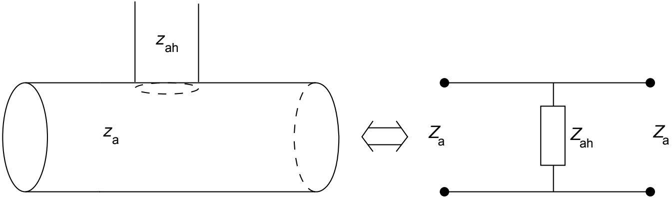
In the following, equivalent circuit representations will be given for pipe side branches.

- treating pressure similarly as voltage in electric circuits (impedance analogy)
- only round pipes will be considered

1 General Representation of Side Branches

Side branches are generally represented using a parallel impedance, since the branches share the same pressure.



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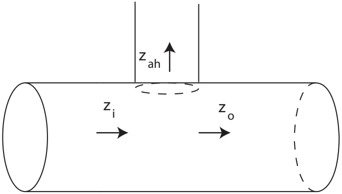

Impedance seen by a wave coming from left

$$z_\mathrm{i} = \frac{z_\mathrm{o} z_\mathrm{ah}}{z_\mathrm{o} + z_\mathrm{ah}}$$

An impedance mismatch will occur, if the values of z_ah and $z_o$are not carefully selected.
(Fahy chapter 8.6.4)

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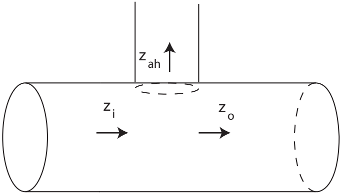

Sound power reflection coefficient α

$$\alpha = \left| \frac{z_{\mathrm{i}}' - 1}{z_{\mathrm{i}}' + 1} \right|^2$$

where $z_{\mathrm{i}}' = z_{\mathrm{i}} A / \rho_0 c$, A is area of the main duct

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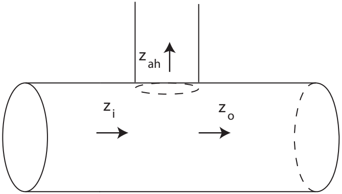

Sound power transmission coefficient for right branch

$$\tau_o = 4\left|\frac{z'_{\mathrm{ah}} z'_{\mathrm{o}}}{z'_{\mathrm{ah}} z'_{\mathrm{o}} + z'_{\mathrm{ah}} + z'_{\mathrm{o}}}\right|^2 \mathrm{Re}(1/z^*_{ah})$$

1 Open Pipe as a Side Branch

If the side branch ends with a simple opening to the surrounding air, z_{ah} seen from the beginning of the side branch towards the open end is given by Eq. 1

$$z_{\mathrm{ah}} = \frac{z_0}{A} \frac{z_{\mathrm{a2}} A \cos(kl) + i z_0 \sin(kl)}{i z_{\mathrm{a2}} A \sin(kl) + z_0 \cos(kl)} \tag{1}$$

as $z_{\mathrm{a2}} = 0$

$$z_{\mathrm{ah}} = i \frac{\rho c}{A} \tan(k(l + \triangle)),$$

where $\triangle = \begin{cases} 0.61R & \text{when } l > 0 \\ 0.85R & \text{when } l = 0 \end{cases}$ is the end correction.

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For frequencies with $f = n\frac{c}{2(l+\triangle)}$, $n = 1, 2, 3, \ldots$ the incoming wave sees a zero impedance $\Rightarrow R = -1$, no wave propagation , **infinite attenuation!**

1 Closed Pipe as a Side Branch

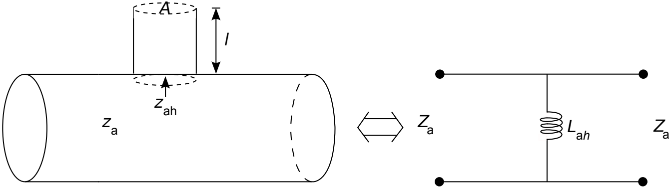
Similarly for a closed pipe as a side branch, Eq. (1) gives

$$z_{\mathrm{ah}} = -i\frac{\rho c}{A}\cot(k(l + \triangle)),$$

where $\triangle = 0.43R$. For those waves for which $(l + \triangle) = \frac{\lambda}{4} + n\frac{\lambda}{2} \Rightarrow R = -1$ and the attenuation is infinite.

1 Short Open Side Branch

For short open-ended side branches, the air in the side branch acts as a mass (or an inductor)



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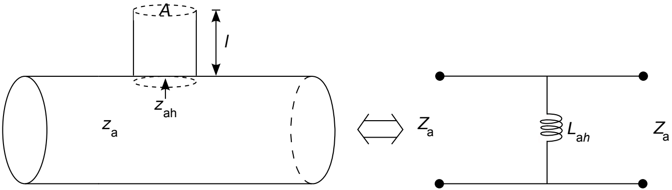

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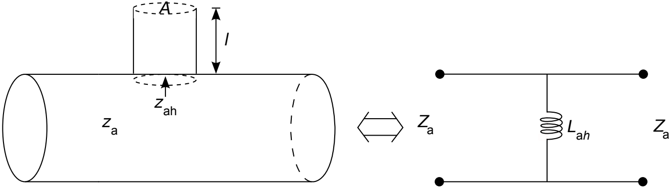

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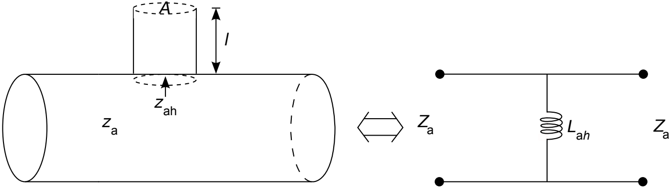

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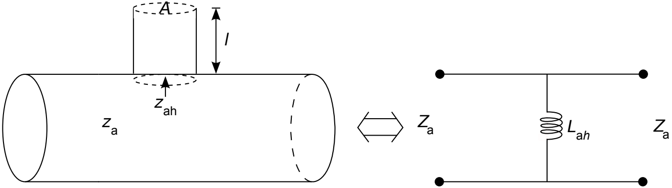

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1 Short Open Side Branch

Transmission loss in main duct due to side branch

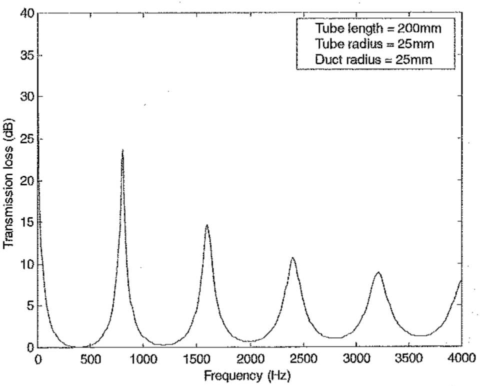
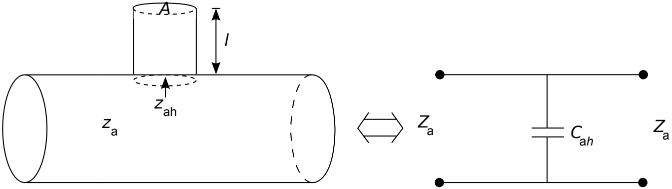


Fig. 8.15 Transmission loss produced by an open end side tube.

1 Short Closed Side Branch

For a short closed side branch, the small cavity acts as an acoustic spring (capacitor)



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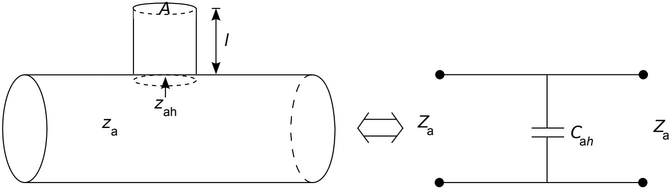

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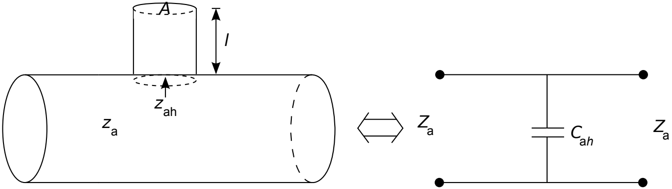


$z_{\mathrm{ah}} \approx \frac{1}{i\omega C_{\mathrm{a}}}$, $C_{\mathrm{a}} = \frac{A(l+\triangle)}{K}$. Acts as an acoustic low-pass filter (in the low-frequency range).

N.B.: $K = \rho c^2$ is the bulk modulus of air.

1 Short Open Side Branch

Transmission loss in main duct due to side branch

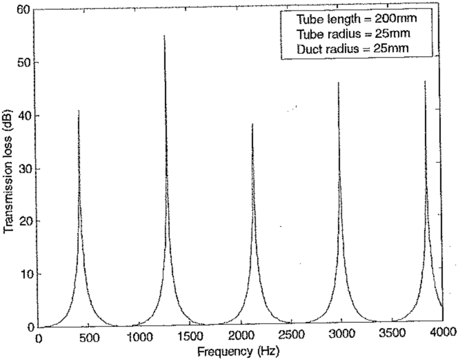
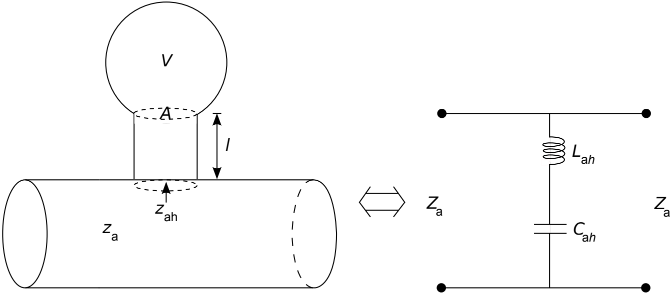


Fig. 8.14 Transmission loss produced by a closed end side tube.

1 Helmholtz Resonator as a Side Branch

A Helmholtz resonator (B&X:chap.2.7) corresponds to a series LC circuit. The cavity acts as a spring, the air inside the neck acts as a mass.



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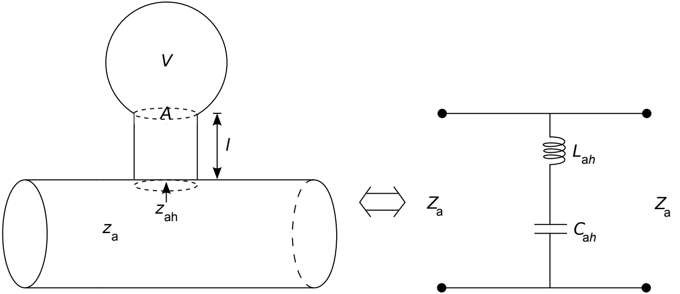

$$z_{\mathrm{ah}} = i\omega L_{\mathrm{ah}} + \frac{1}{i\omega C_{\mathrm{ah}}}$$

1 Helmholtz Resonator as a Side Branch II

For the component values:

- $C_{\mathrm{ah}} = V/K$, where K is the bulk modulus of air
- $L_{\mathrm{ah}} = \rho(l + \triangle)/A$
  - $\triangle = \begin{cases} 0.61R & \text{when } l > 0 \\ 0.85R & \text{when } l = 0 \end{cases}$ is the end correction

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- infinite attenuation
- acts as a bandstop filter
  - efficient bandwidth quite narrow
  - stopband width may be increased with absorption material, but then also maximum attenuation decreases (FF:p.211)

1 Helmholtz Resonator as a Side Branch III

Transmission loss in main duct due to side branch with resonator

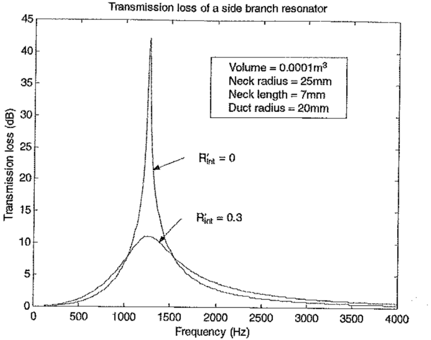

Fig. 8.16 Transmission loss produced by a side branch resonator.

2

Horns

2 General

Acoustic horns:

- pipe with a varying cross-section area

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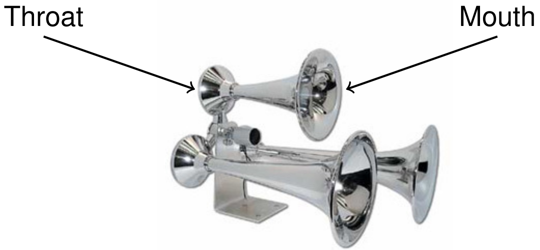

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  - in addition to the impedance matching, the horn forms the bore (resonator) of the instrument
- loudspeakers
  - also for tuning the directivity (especially for short, rapidly flaring horns)
- megaphones
- ultrasound tools
  - also horns made from solid non-hollow materials (dimensions get smaller towards the edge)
  - concentration of force and increase in velocity (compare to an axe)

2 Webster's Horn Equation

The pressure inside the horn can be stated using the Webster's horn equation:

$$p''(x,t) + \left[\frac{A'(x)}{A(x)}\right] p'(x,t) = \frac{1}{c^2}\ddot{p}(x,t), \qquad \text{(B\&X:(8.7))}$$

where $A(x)$ is the **area function**, giving the cross-section area of the horn as a function of the axial distance from the throat

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- Eq. (B&X:(8.7)) holds for (nearly) all kinds of horns

2 Solutions of Webster's Horn Equation

Those horn profiles (area functions) which have an analytical solution in Eq. (B&X:(8.7)), have been given specific names: Salmon horns:

- conical
- exponential
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Bessel horns:

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We will consider only the first two of these more thoroughly in what follows.

2 Conical Horns (R&F:chap.8.7,B&X:chap.8.2)

For conical horns, the area function is of the form

$$A(x) = A_0 \left(\frac{x}{x_0}\right)^2 \qquad \text{(B\&X:8.8)}$$

where A_0 is the area of the throat at distance of x_0.

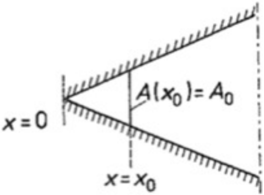

Fig. 8.3. Conical horn

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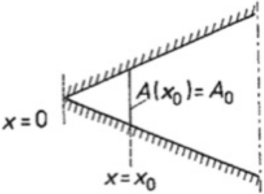

Fig. 8.3. Conical horn

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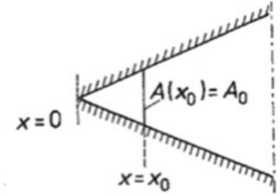

Fig. 8.3. Conical horn

- spherical wave field inside the cone!
- characteristic impedance of a spherical wave applies

2 Exponential Horns

For exponential horns, the area function is of the form

$$A(x) = A_0 e^{2\epsilon x} \qquad \text{(B\&X:8.19)}$$

where A_0 is the area of the throat and ϵ is the **flare constant**, which tells "how quickly the horn opens"

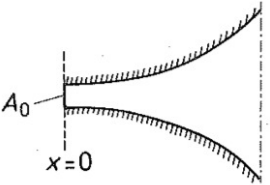

Fig. 8.5. Exponential horn

2 Exponential Horns II

For infinitely long exponential horns, we have (B&X:(8.19-28)) $\epsilon = \omega_\mathrm{l}/c$, where ω_l is a **limit frequency**:

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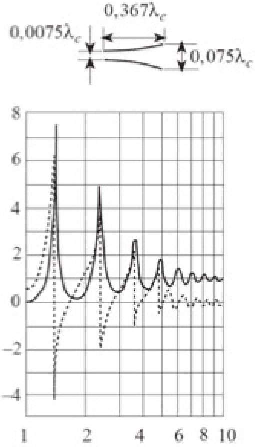
- for frequencies $< \omega_\mathrm{l}$, no sound energy escapes from the horn
- the load impedance seen by the source is purely reactive
- wave propagation velocity depends on ϵ and ω

2 Normalized Throat Impedances for Different Exponential Horns (H.F. Olson)

Solid line: resistive part
Dashed line: reactive part
Horizontal axis: normalized frequency
Vertical axis: normalized impedance (w.r.t. ρc)



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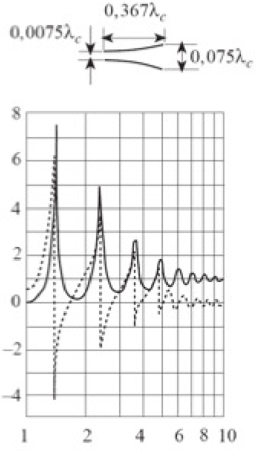

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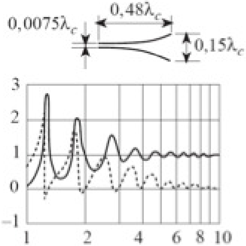
- peaks seem to get wider



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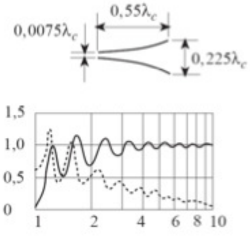

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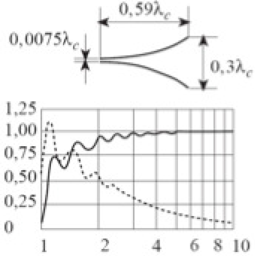
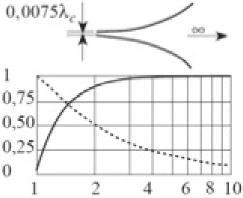


2 Normalized Throat Impedances for Different Exponential Horns (H.F. Olson) V



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Horizontal axis: normalized frequency
Vertical axis: normalized impedance (w.r.t. ρc)

- Let's see how the impedance changes when the horn is made longer:
- peaks seem to get wider... until they disappear for infinitely long horns. Plane wave at high frequencies!

2 Impedances of different horn shapes

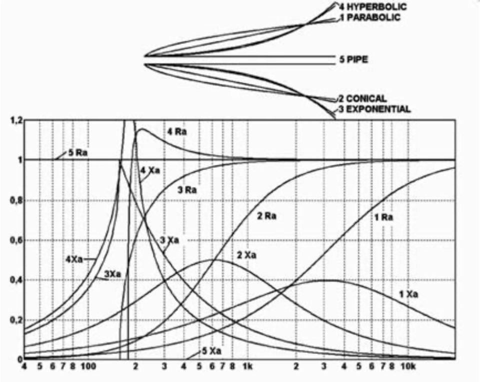

FIGURE 2: Throat acoustical resistance r_A and reactance x_A as a function of frequency for different horn types.

Kolbrek, Bjørn. "Horn theory: An introduction, part 1." Audio Xpress 1 (2008): 8-8.

2 Some Examples of Horns used with Subwoofers

Principle:

- `http://www.cowanaudio.com/hornsubjr.html`

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The largest subwoofer in the world:

- `https://makezine.com/2008/07/15/worlds-largest-subwoofer/`