Aalto University

Impedance, Equivalent Circuits

ELEC-E5610 Acoustics and the Physics of Sound, Lecture 8

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Impedance

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Example: velocity / force $=$ mechanical admittance

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For waves or signals:

input impedance (e. g. how much current the device takes with certain voltage and frequency)

output impedance (e. g. how much current the device can feed with a certain voltage and frequency)

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- Impedance matching devices: hearing horn, buffer circuit ...

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- e.g. at the bridge of a violin or at the mouth of a wind instrument

1 Example: the Violin

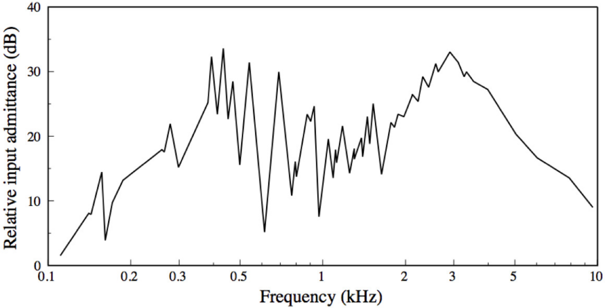

Figure: The driving point admittance at the bridge of a violin. Adapted from Fletcher, N. H., & Rossing, T. D. The physics of musical instruments.

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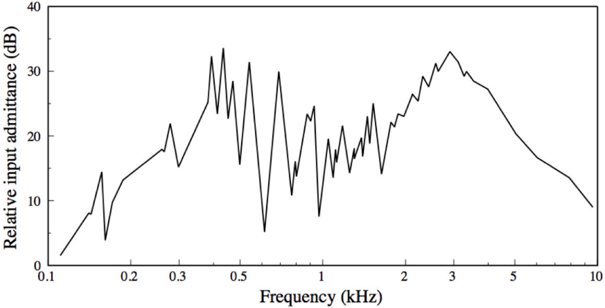

Figure: The driving point admittance at the bridge of a violin. Adapted from Fletcher, N. H., & Rossing, T. D. The physics of musical instruments.

Complicated system; impossible to design admittance (difficult to replace Stradivaris!)

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vs.

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- ⇒ electric guitar has higher bridge impedance!

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Note: it is easier for the string to move the acoustic guitar bridge ⇒ better energy transfer to the body ⇒ louder sound and shorter sustain for the acoustic guitar!

1 Usage in Calculations

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- impedance representation is a “black-box model” of the system
  - simplifies especially the estimation of the interaction between subsystems (impedance summing)
- used in acoustics with lumped systems
  - valid, if dimensions of the system $<$ wavelength

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- be careful not to mix different impedance types!

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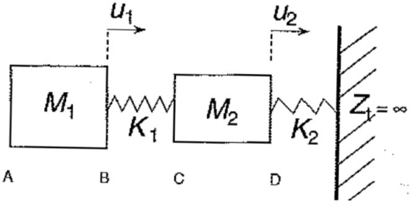
Let's calculate the mechanical impedance of a 2DOF mass-spring oscillator at location A



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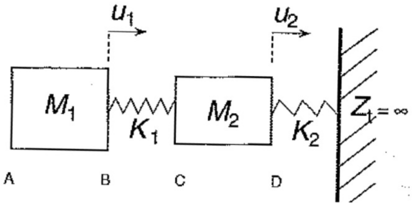

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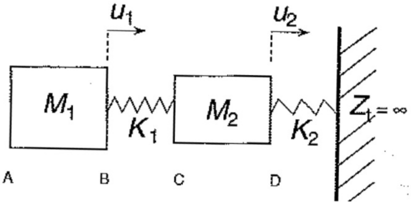

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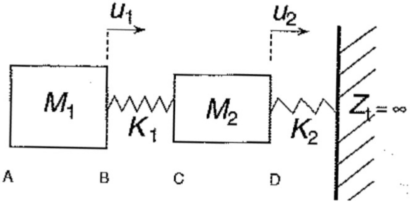

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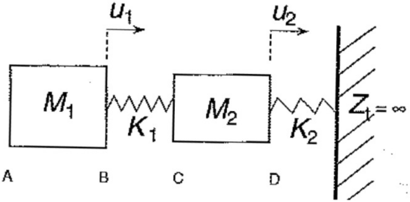

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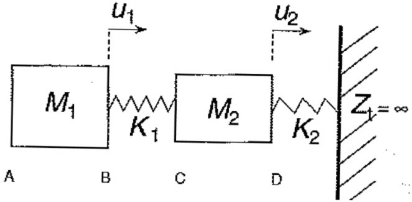

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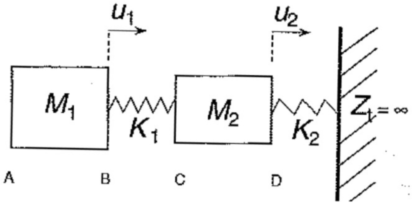

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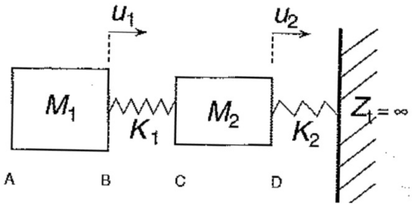

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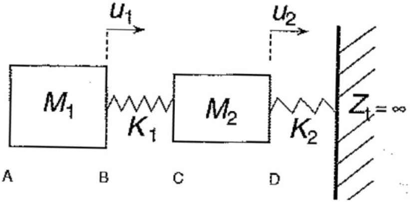

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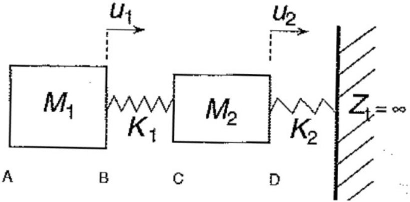

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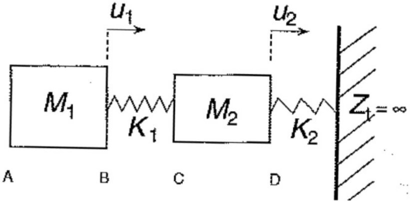

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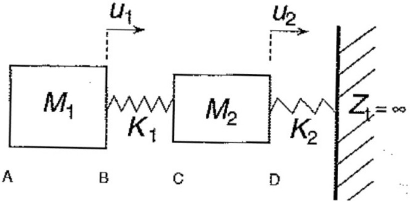

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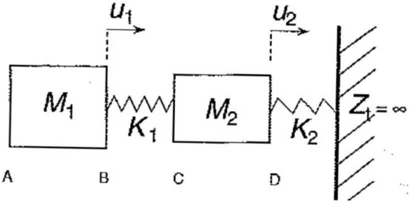

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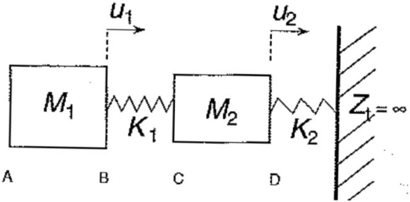

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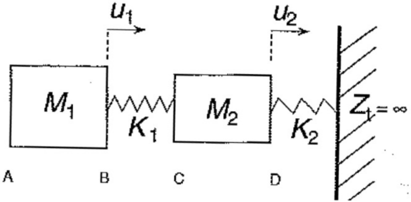

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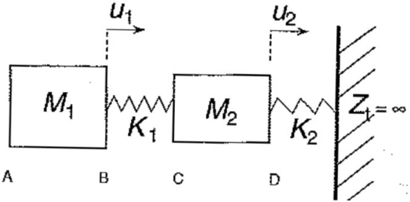

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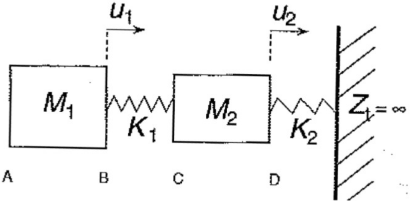

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- $Z_\mathrm{C} =? \, Z_\mathrm{M2}$ and Z_D in series or parallel? M_2 is rigid $\Rightarrow$ points C and D move together $\Rightarrow$ flux variable is constant $\Rightarrow$ Z_M2 and Z_D in series!

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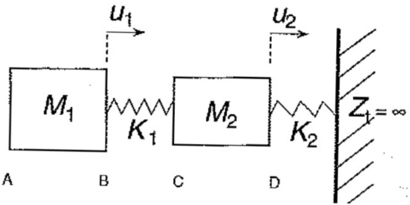

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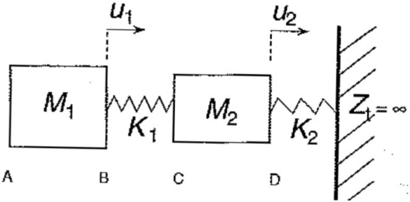
- $Z_\mathrm{C} = ?$ Z_M2 and Z_D in series or parallel? M_2 is rigid ⇒ points C and D move together ⇒ flux variable is constant ⇒ Z_M2 and Z_D in series! $Z_\mathrm{C} = Z_\mathrm{M2} + Z_\mathrm{D}$

1 Example: 2DOF Oscillator III

Let's calculate the mechanical impedance of a 2DOF mass-spring oscillator at location A



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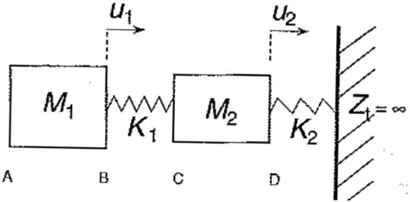

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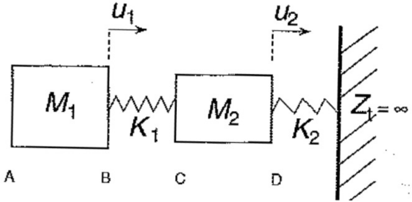

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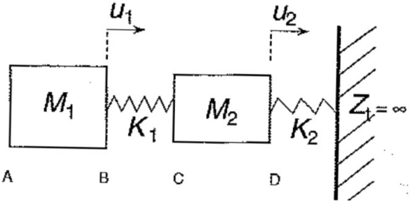

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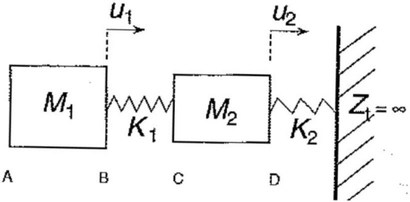

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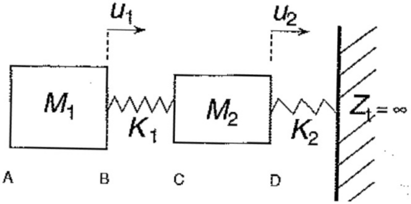

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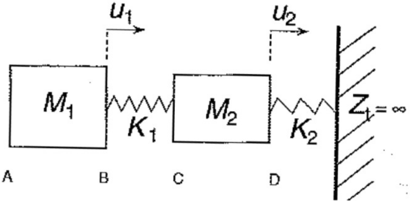
1 Example: 2DOF Oscillator IV

Let's calculate the mechanical impedance of a 2DOF mass-spring oscillator at location A



- Finally(!) : $Z_{\mathrm{A}} = Z_{\mathrm{M1}} + \frac{(Z_{\mathrm{M2}}+Z_{\mathrm{K2}})Z_{\mathrm{K1}}}{Z_{\mathrm{M2}}+Z_{\mathrm{K2}}+Z_{\mathrm{K1}}}$

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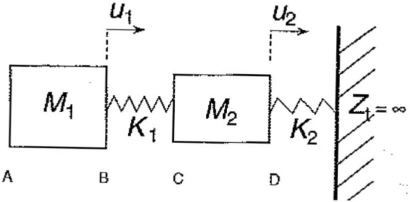

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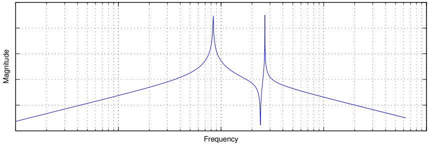

Admittance of the system with some example parameter values

1 Impedance of components & rules

Electrical impedance:

Inductor: $Z = i\omega L$
Resistor: $Z = R$
Capacitor: $Z = \frac{1}{Ci\omega}$

- 2 impedances in series:
  $Z_{\text{eq}} = Z_1 + Z_2$
- 2 impedances in parallel:
  $\frac{1}{Z_{\text{eq}}} = \frac{1}{Z_1} + \frac{1}{Z_2}$

Mechanical impedance:

Mass: $Z = i\omega m$
Resistance: $Z = R$
Spring: $Z = \frac{K}{i\omega}$

- 2 impedances in series:
  $\frac{1}{Z_{\text{eq}}} = \frac{1}{Z_1} + \frac{1}{Z_2}$
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General Principles

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Analogies are two or more systems that produce similar mathematical representations (apart from constant coefficients and units)

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In other words, acoustical and mechanical systems can be represented with electric circuits (especially when considering low frequencies).
What is the advantage of this? There are well-known methods for solving electric circuits ⇒ calculations are typically easier

2 Lumped Elements

If the dimensions of an object are small compared to the wavelength, the wave variable can be considered constant throughout the object.

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Remember: it depends on the frequency range of interest, whether the lumped representation is valid!

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- electrical representation for a mechanical or acoustical system
- mechanical representation for an acoustical system
- linear representation for a rotational system
- unified representation of a multi-domain system

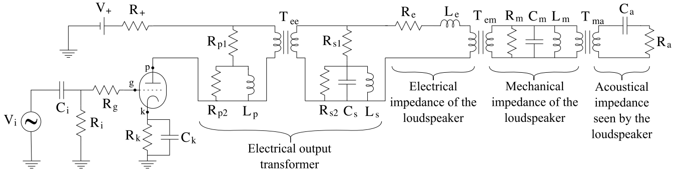

Figure: Equivalent circuit for a SET power amp output chain.

3

Analogies Between Domains

3 Mass and Inductance

Mass and Inductance
Spring and Capacitance
Resistance
Other Components
Type of Analogy
Interconnection of Components
Analogy Table

The inertial components relate the potential variable to the time derivative of the flux variable:

Mechanical domain	Electrical domain	Acoustical domain
$F = m\dot{v}$	$U = L\dot{I}$	$p = L_\mathrm{a}\dot{Q}$
where:	where:	where:
F - force	U - voltage	p - pressure
v - velocity	I - current	Q - vol. velocity
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See the analogy?

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Also, potential energy storage components relate the potential variable to the time integral of the flux variable:

Mechanical domain	Electrical domain	Acoustical domain
$F = K \int v dt$	$U = \frac{1}{C} \int I dt$	$p = \frac{1}{C_\mathrm{a}} \int Q dt$
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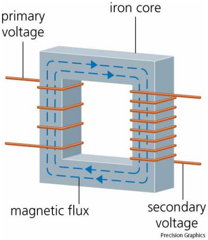
3 Other Components

Also other electric components (but not all) have mechanical or acoustical analogies

- e. g. an ideal transformer corresponds to a massless ideal lever (no acoustic counterpart, though!)



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The suitable analogy type depends on the system.

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- if there is the same force (or pressure) affecting the ends two components, the components are in
  - parallel according to the impedance analogy
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- if the ends of two components move at the same velocity (or there is a common volume flow through them), they are in
  - series according to the impedance analogy
  - parallel according to the admittance analogy

3 Analogy Table

Table: Component types and connections

el. impedance	**el. admittance**	**mechanical**	**acoustical**
voltage U	current I	force F	pressure p
current I	voltage U	velocity v	vol. velocity Q
impedance Z	admittance Y	mech. imped. Z_{M}	ac. imped. Z_{A}
resistance R	conductance G	mech. res. R_{M}	ac. res. R_{A}
inductance L	capacitance C	mass m	ac. ind. L_{A}
capacitance C	inductance L	compliance $\frac{1}{K}$	ac. cap. C_{A}
series conn.	parallel conn.	common velocity	common vol. vel.
parallel conn.	series conn.	common force	common pressure

4

How to Make Equivalent Circuits

4 Outline of the Procedure

In the following, we will learn how to make electrical equivalent circuits for mechanical systems using the

- impedance analogy
- admittance analogy

and also how to transfer an impedance analogy into an admittance analogy.

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Comparing these approaches would make a good brain exercise!

4 Impedance Analogy

The algorithm for going from mechanical to electrical representation is as follows:

1. make a circuit loop for each mass
2. make a circuit loop for each generator not connected to a mass
3. into each loop: add the electrical version of the directly involved components in series
4. connect different loops by combining their shared elements

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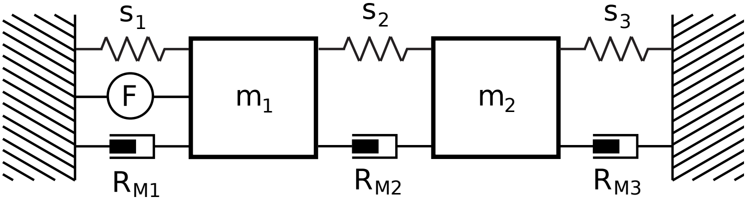
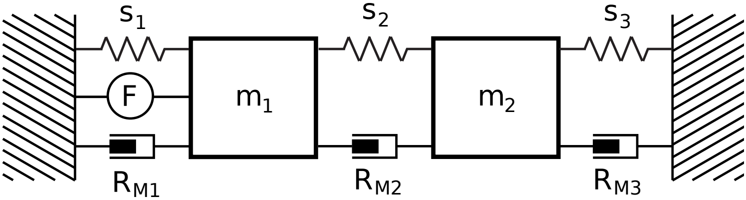
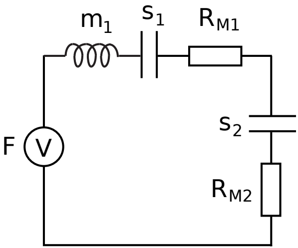
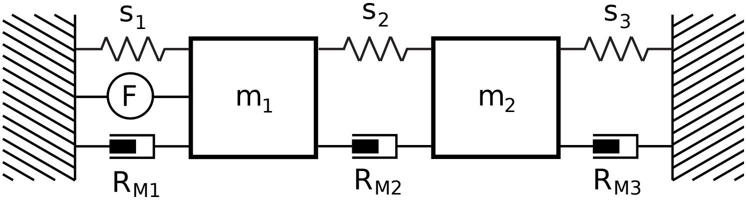
Let's make the equivalent circuit for a 2DOF mass-spring oscillator system...


s_1
s_2
s_3
F
m_1
m_2
R_{M1}
R_{M2}
R_{M3}

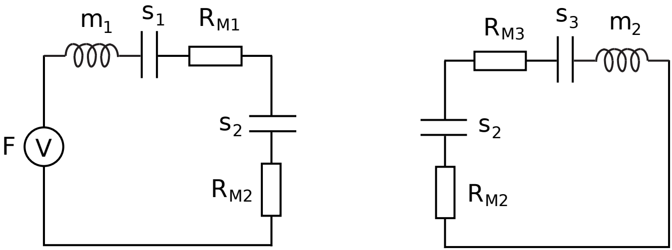


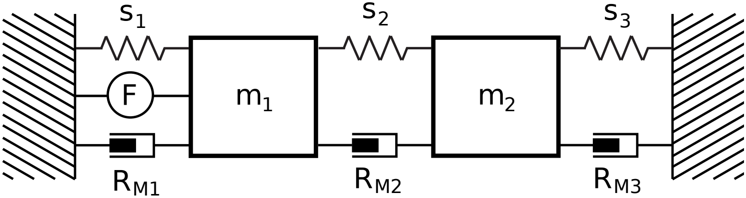
Make loop for m_1



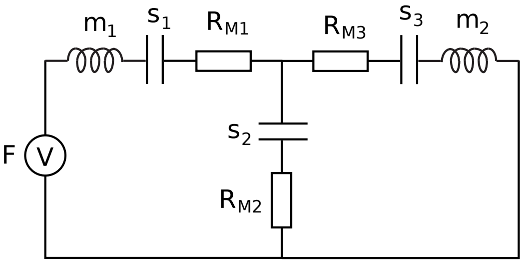


Make loop for m_1 , make loop for m_2





Make loop for m_1 , make loop for m_2 , and connect the loops.



4 Admittance Analogy

Alternatively, the admittance analogy may be used. The algorithm for this is as follows:

1. for each mass, make circuit node with a grounded capacitor
2. make a grounded node for each generator not connected to a mass, and a node grounded for a rigid body
3. into each node: add the electrical version of the directly involved components in parallel
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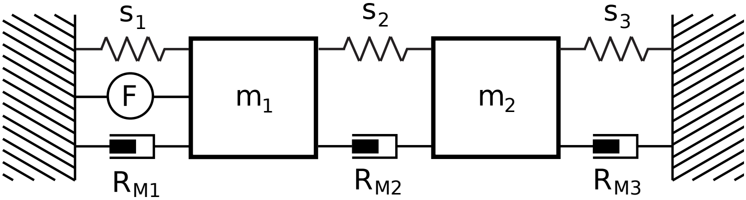
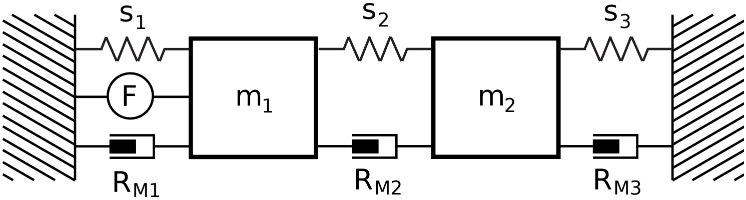
Let's re-make the equivalent circuit using the admittance analogy...


s_1
s_2
s_3
F
m_1
m_2
R_{M1}
R_{M2}
R_{M3}



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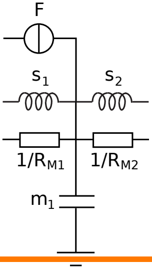
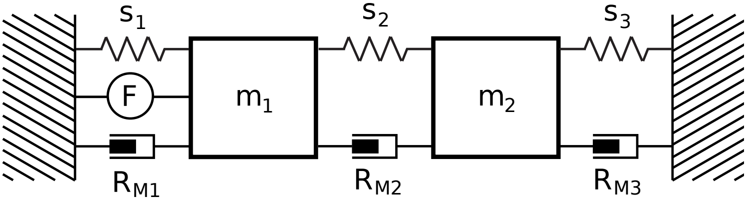
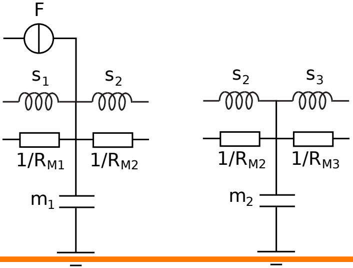
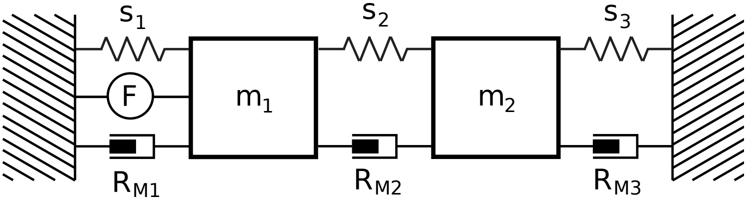


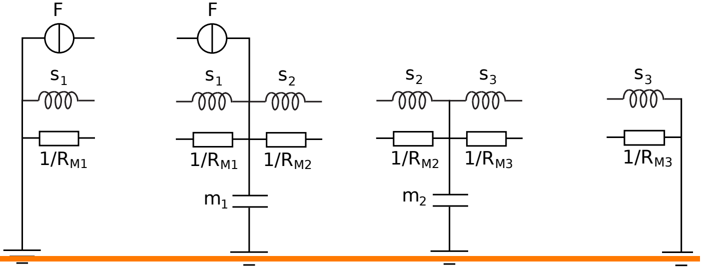


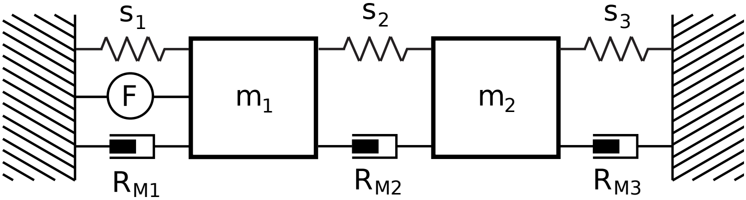
Make node for m_1 , make node for m_2



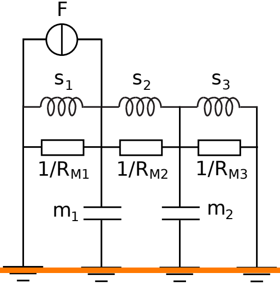


Make node for m_1 , make node for m_2 , make nodes for rigid bodies





Make node for m_1 , make node for m_2 , make nodes for rigid bodies , and connect the nodes.



4 Star-to-Triangle Transform

How to go from impedance analogy to admittance analogy?

1. inside each loop, insert a reference point
2. insert also a reference point outside the circuit
3. connect the points with lines
4. re-draw the connected pattern next to the circuit
5. for each line, change the intersecting series connection to parallel and switch the components into their dual versions

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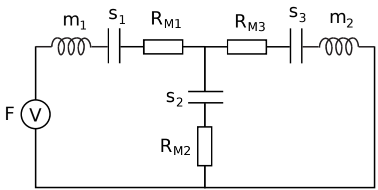
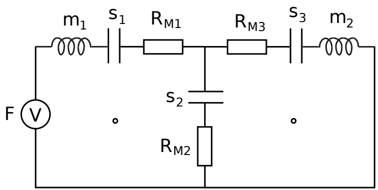
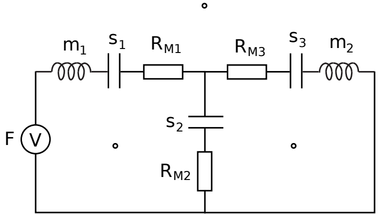
Let's re-make the equivalent circuit using the admittance analogy...

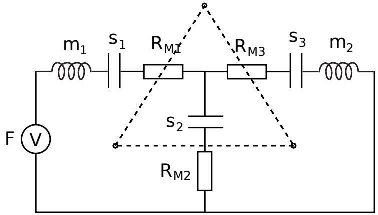

m1
s1
RM1
RM3
s3
m2
s2
F
V
RM2



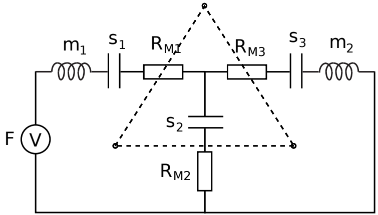
Insert points in loops



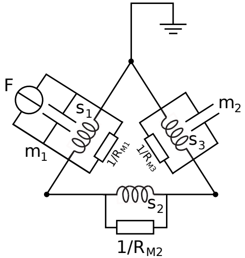
Insert points in loops, and one outside,

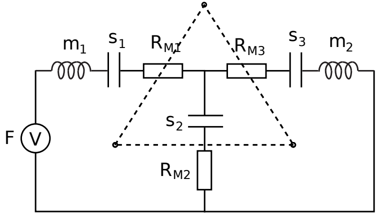


Insert points in loops, and one outside, connect the points

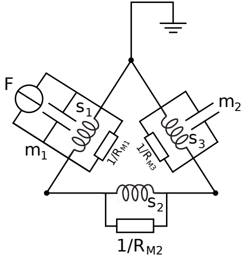


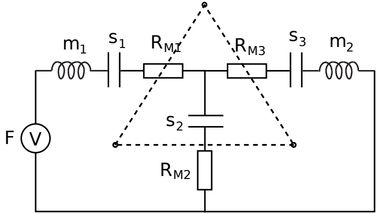
Insert points in loops, and one outside, connect the points, re-draw the shape



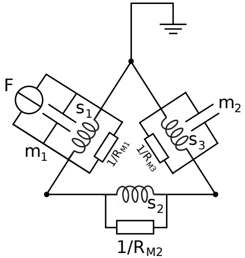


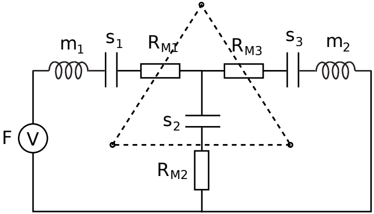
Insert points in loops, and one outside, connect the points, re-draw the shape, insert branches



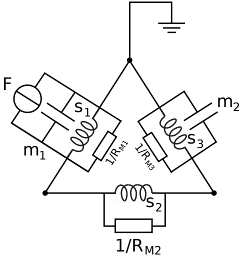


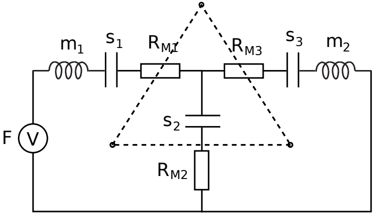
Insert points in loops, and one outside, connect the points, re-draw the shape, insert branches.



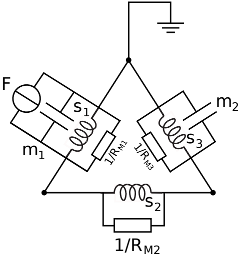


Insert points in loops, and one outside, connect the points, re-draw the shape, insert branches..





Insert points in loops, and one outside, connect the points, re-draw the shape, insert branches... and ground.



4 Suggested reading

Beranek & Mellow, Acoustics, Chapter 3.