



Aalto University

# Sound Radiation from Vibrating Objects

ELEC-E5610 Acoustics and the Physics of Sound, Lecture 10

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# 1

# General



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- vibrational distribution on the surface
  - amplitude
  - phase
  - frequency

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  - amplitude
  - phase
  - frequency
- surrounding environment

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For finite vibrating objects, the sound field created by the vibration is affected by

- the shape and dimensions of the surface
- vibrational distribution on the surface
  - amplitude
  - phase
  - frequency
- surrounding environment
- the interaction of all sides of the vibrating surface



# 2

## Circular Piston

## 2 Circular Piston

### Circular Piston

Pressure Field

Intensity Field

Scalability

Directivity  
Approximations

Radiation  
Impedance

Consider a circular piston with radius  $R$  as a sound source



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Consider a circular piston with radius  $R$  as a sound source

- vibrates as a single unit with velocity  $\tilde{u}_n$ 
  - constant amplitude, phase, and frequency



## 2 Circular Piston

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Intensity Field

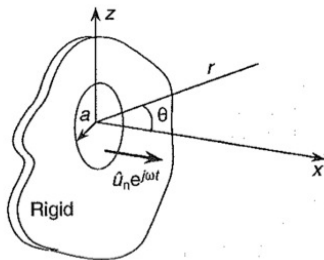
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Consider a circular piston with radius  $R$  as a sound source

- vibrates as a single unit with velocity  $\tilde{u}_n$ 
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- placed on an infinite rigid plane
  - no interaction from the other side of the plane



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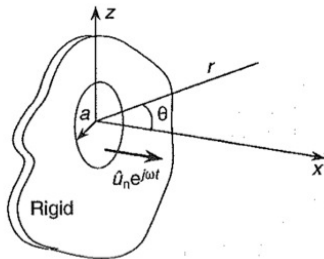
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- vibrates as a single unit with velocity  $\tilde{u}_n$ 
  - constant amplitude, phase, and frequency
- placed on an infinite rigid plane
  - no interaction from the other side of the plane



Approximation of a baffled loudspeaker! Let's study the radiation pattern more thoroughly...

## 2 Pressure Field Generated by a Circular Piston

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The far-field sound pressure created by the piston source is

$$\tilde{p}(r, \theta) = \frac{i\rho\omega R^2 \tilde{u}_n}{2r} \left[ \frac{2J_1(kR \sin \theta)}{kR \sin \theta} \right] e^{-ikr}$$

(FF:6.53, R&F:7.30)

where  $J_1$  is the 1st-order Bessel function.



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where  $J_1$  is the 1st-order Bessel function. Compare to the far-field sound pressure of the monopole ( $q_0 = 4\pi R^2 \tilde{u}_n$ ):

$$\tilde{p}(r) = \frac{i\omega\rho R^2 \tilde{u}_n}{r} e^{-ikr} \quad (\text{FF:6.20, R\&F:7.4})$$

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$$\tilde{p}(r) = \frac{i\omega\rho R^2 \tilde{u}_n}{r} e^{-ikr} \quad (\text{FF:6.20, R\&F:7.4})$$

They are the same, except for scaling by  $\frac{1}{2}$  and a multiplication term, called the **directivity function**.

## 2 Pressure Field Generated by a Circular Piston II

In other words, pressure field = pressure field of a monopole  
× directivity function  $f(\theta) = \left[ \frac{2J_1(kR \sin \theta)}{kR \sin \theta} \right]$ .

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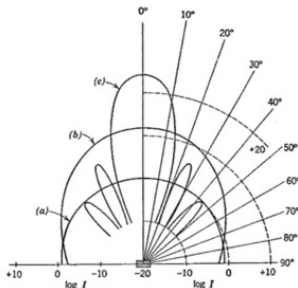


Fig. 6.17 Radial sound intensity distribution in the far field of a circular piston in a rigid baffle at (a)  $ka = \pi/4$ ; (b)  $ka = \pi$ ; (c)  $ka = 4\pi$ . Reproduced with permission from Kinsler, L. E., Frey, A. R., Coppens, A. B. and Sanders, J. V. (1982) *Fundamentals of Acoustics*, 3rd edn. John Wiley & Sons, New York.

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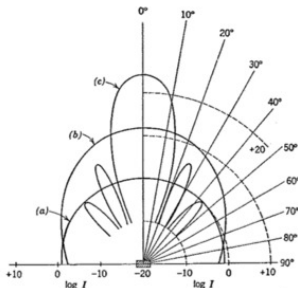


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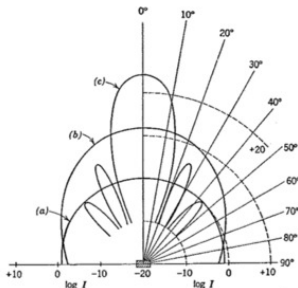


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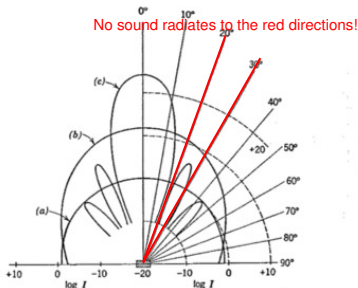


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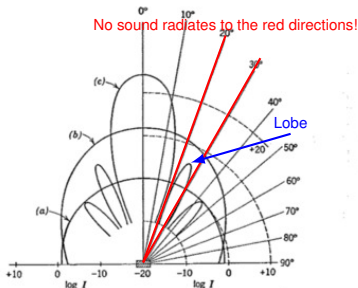


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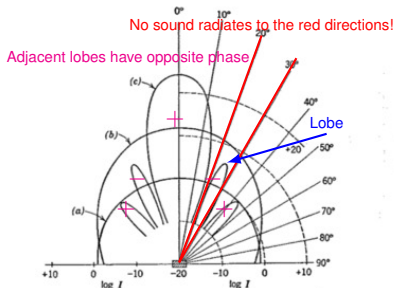


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## 2 Intensity Field Generated by a Circular Piston

Circular Piston

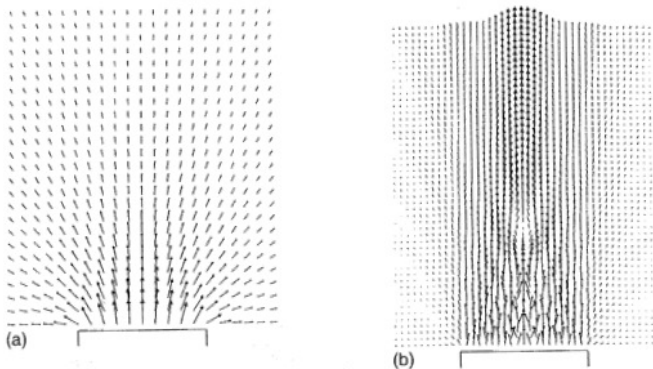
Pressure Field

Intensity Field

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**Figure:** Active intensity for (a) low and (b) high frequencies (FF:p.129)



## 2 Scalability

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As in the piston case, the directivity patterns of sound sources in general are not dependent on absolute dimensions, but rather on  $kR$ , called the **Helmholtz number**.



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- sound field behaves similarly when the wavelength changes proportionally to the object dimensions
- $\Rightarrow$  enables the use of acoustic scale models!



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- sound field behaves similarly when the wavelength changes proportionally to the object dimensions
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However, scale models should be used carefully if significant losses are present, for example

- with very high frequencies
- small ducts, porous materials, etc.



## 2 Scalability II

Circular Piston

Pressure Field

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**Figure:** Scale model of a concert hall

## 2 Directivity Approximations

Circular Piston

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Intensity Field

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When increasing  $kR \sin \theta$  from zero, we find out that the  $f(\theta) = \left[ \frac{2J_1(kR \sin \theta)}{kR \sin \theta} \right]$  has its first zero when  $kR \sin \theta \approx 3.83$ .



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- $\Rightarrow$  low frequencies radiate to all directions!



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In fact, for  $kR \ll 1$  (i.e. if the dimensions of the piston are very small compared to wavelength)

$\Rightarrow f(\theta) = \left[ \frac{2J_1(kR \sin \theta)}{kR \sin \theta} \right] \approx 1$ , so that the piston source approximates an elementary monopole.

## 2 Radiation Impedance of a Circular Piston

Circular Piston

Pressure Field

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The mechanical radiation impedance of a circular piston (relation between piston velocity and the resulting force exerted by the fluid to the piston) is

$$Z_{\text{mrad}} \approx \rho c \pi R^2 \left[ \frac{(kR)^2}{2} + i \frac{8kR}{3\pi} \right] \quad (\text{FF:(6.55)})$$

when  $kR \ll 1$ .



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- on the surface of the piston: piston velocity = particle velocity of air

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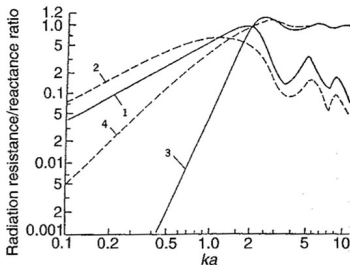
when  $kR \ll 1$ .

- on the surface of the piston: piston velocity = particle velocity of air
- Eq. (FF:(6.55)) is a low  $kR$  approximation. Let's see what the exact  $\frac{Z_{\text{mrad}}}{\rho c \pi R^2}$  looks like...

## 2 Radiation Impedance of a Circular Piston II

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Foundations of Engineering Acoustics



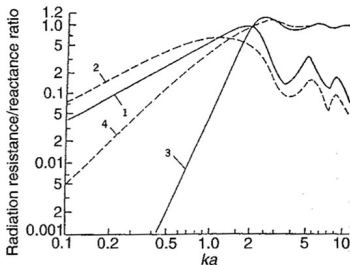
**Fig. 6.19** Radiation impedance ratios of baffled and unbaffled pistons. Radiation impedances as follows: (1)  $X_{rad}/\pi a^2 \rho_0 c$  for an unbaffled piston; (2)  $X_{rad}/\pi a^2 \rho_0 c$  for a baffled piston; (3)  $R_{rad}/\pi a^2 \rho_0 c$  for an unbaffled piston; (4)  $R_{rad}/\pi a^2 \rho_0 c$  for a baffled piston. Reproduced with permission from Rschevkin, S. N. (1963) *Lectures on the Theory of Sound*. Pergamon Press, Oxford.

**Figure:** Real  $R$  and imaginary  $X$  components of rad. impedance

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Foundations of Engineering Acoustics



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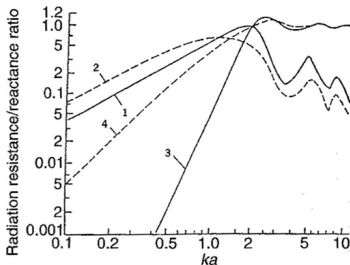
**Figure:** Real  $R$  and imaginary  $X$  components of rad. impedance

What can you say about the low-frequency-response of the baffled vs. unbaffled piston?

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**Figure:** Real  $R$  and imaginary  $X$  components of rad. impedance

What can you say about the low-frequency-response of the baffled vs. unbaffled piston? What about high frequencies?

## 2 Radiation Impedance of a Circular Piston III

- Circular Piston
- Pressure Field
- Intensity Field
- Scalability
- Directivity
- Approximations

### Radiation Impedance

Let's take a look at the reactive part of Eq. (FF:(6.55)):

$$\text{Im}[Z_{\text{mrad}}] \approx \rho c \pi R^2 \left[ i \frac{8kR}{3\pi} \right] = ick \frac{8\rho R^3}{3}$$



## 2 Radiation Impedance of a Circular Piston III

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How would you interpret the reactive radiation impedance for the piston?



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remember  $k = \omega/c \Rightarrow$

$$\text{Im}[z_{\text{mrad}}] \approx i\omega \frac{8\rho R^3}{3}.$$

How would you interpret the reactive radiation impedance for the piston? It represents an attached mass of  $\frac{8\rho R^3}{3}$  kg.

## 2 Radiation Impedance of a Circular Piston IV

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Since FF:Fig.6.19 showed that the bracketed resistive part of  $Z_{\text{mrad}} \approx 1$  for high frequencies, it can be concluded that

$$\text{Re}[Z_{\text{mrad}}] \approx \pi R^4 \rho c$$



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How would you interpret this?



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How would you interpret this? Remember the characteristic impedance of a plane wave  $z_c = \rho c$ . This means that for high frequencies, the piston behaves as a plane wave source.



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See also <https://www.acs.psu.edu/drussell/Demos/BaffledPiston/BaffledPiston.html>.

# 3

## Radiator Groups

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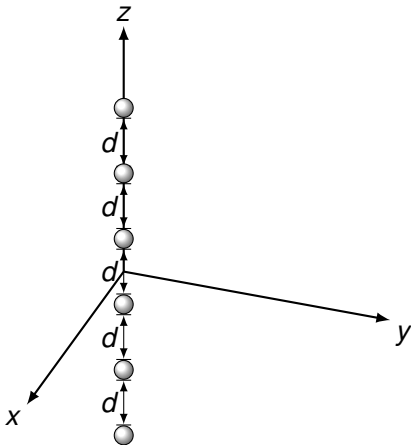
#### Radiator Groups

Sound Pressure

Directivity Patterns

Wavefield Synthesis

Consider a case where a group of equal-phase monopoles are placed on the  $z$ -axis,  $d$  meters apart.



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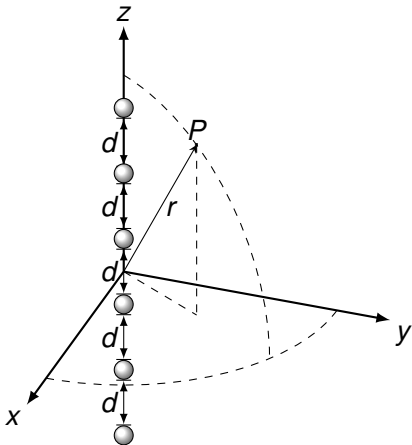
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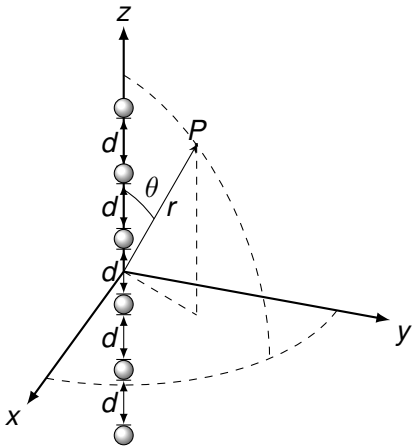
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- $P$  is the observation point at a distance  $r$  from the group midpoint
- $\theta$  is the angle between  $P$ , group midpoint, and  $z$ -axis



# 3 Sound Pressure

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The sound pressure at  $P$  caused by  $N$  monopoles can be given as

$$\tilde{p}(\theta, r) \approx \left( \frac{i\omega\rho q_0}{4\pi r} \right) e^{-ikr} \left[ \frac{\sin\left(\frac{N\pi d}{\lambda} \cos\theta\right)}{\sin\left(\frac{\pi d}{\lambda} \cos\theta\right)} \right] \quad (\text{R\&F:(7.17)})$$

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- **directivity function**

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(note that book version of R&F:7.17 uses a number of  $2N$  monopoles)

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- Figure: directivity patterns created by 7 point sources (illustrated with red circles)

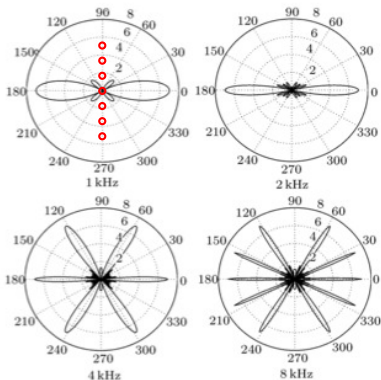


Fig. 2. Simulated far-field polar response of a 7-point-source uniform array.

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- Figure: directivity patterns created by 7 point sources (illustrated with red circles)
- lobe number increases with frequency!

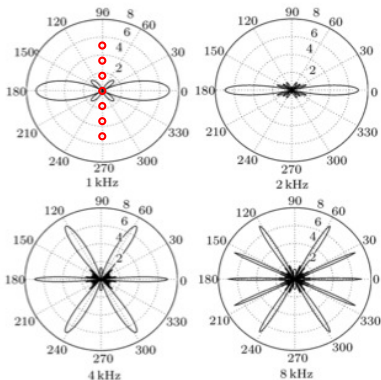


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The same principle can be used also for microphone arrays!

- acoustic beamforming by summing the mic signals

