Aalto university
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Exercise sheet 3
Complex Analysis, MS-C1300.
Hand in exercise 1 and 2 in separate files for grading. Deadline Monday 30.10 at 23:59. The exercises should be uploaded to the correct folder on MyCourses as pdf-files with name and student number in the file name. **Submission via MyCourses is the only accepted way.** Done during class Tuesday 31.10 or Wednesday 1.11.

(1) (a) Prove the reverse triangle inequality
$$\big||z| - |w|\big| \leq |z - w|$$
for all $z, w \in \mathbb{C}$. (*Hint*: $z = (z - w) + w$.) (3p)

(b) Let $z_0 \in \mathbb{C}$ and $r \geq 0$. Show that the set
$$\overline{\Delta(z_0, r)} = \{z \in \mathbb{C}; |z - z_0| \leq r\}$$
is closed. (*Hint:* The reverse triangle inequality is useful to show that the complement is open.) (3p)

(2) Compute
$$\lim_{n\to\infty} z_n$$
when:
(a) $z_n = i^{n!} + 2^{-n}$ (2p)
(b) $z_n = 2^{-n+i\sqrt{n}}$ (2p)
(c) $z_n = \sqrt[n]{z}$, for $z \in \mathbb{C}$ (2p)

(3) A complex sequence is defined recursively by $z_1 = 0$, $z_2 = i$, and
$$z_n = \frac{z_{n-1} + z_{n-2}}{2}$$
for $n \geq 3$. Show that
$$z_n = \frac{2i}{3}\left(1 - \left(-\frac{1}{2}\right)^{n-1}\right)$$
when $n \geq 2$. Calculate
$$\lim_{n\to\infty} z_n.$$

(4) Compute:
(a) $\lim_{z\to i} \frac{z^4+1}{z+i}$
(b) $\lim_{z\to -i} \frac{z^4-1}{z+i}$

(c) $\lim_{z\to 2i} \frac{z^2-iz+2}{z^2+4}$.
(WARNING: You cannot use l'Hospital's rule (yet)! You don't know how to differentiate at this point in the development of the theory.)