Aalto university
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Exercise sheet 4
Complex Analysis, MS-C1300.
Hand in exercise 1 and 2 in separate files for grading. Deadline Wednesday 1.11 at 23:59. The exercises should be uploaded to the correct folder on MyCourses as pdf-files with name and student number in the file name. **Submission via MyCourses is the only accepted way.** Done during class Thursday 2.11 or Friday 3.11.

(1) For each of the following functions determine the largest open set in which it is analytic and calculate it's derivative:

(a)
$$f(z) = z^3(1+z)^6$$
(2p)

(b)
$$g(z) = \frac{z+1}{z-2i}$$
(2p)

(c)
$$h(z) = \left(\frac{z-1}{z^3-27}\right)^4$$
(2p)

(2) Put $z = x + iy$ and let
$$f(z) = 1 - y^2 + i(2xy - y^2).$$
Locate all points z at which f is complex differentiable, and determine $f'(z)$ for each such point. (6p)

(3) Let $U \subseteq \mathbb{C}$ be an open set. Assume that $h\colon U \to \mathbb{R}$ has continuous partial derivatives of first and second order and satisfy Laplace's equation
$$h_{xx} + h_{yy} = 0$$
in U. Show that
$$f(z) = h_x(x, y) - ih_y(x, y)$$
is analytic in U. (A function solving Laplace's equation is called harmonic.)

(4) Assume that $U \subseteq \mathbb{C}$ is an open set, and that $f\colon U \to \mathbb{C}$ is analytic. Let $u = \mathrm{Re}(f)$ and $v = \mathrm{Im}(f)$ and assume that u and v has continuous partial derivatives of first and second order. (We will later see that these assumptions on u and v are automatically true.) Show that u and v are harmonic (that is solve Laplace's equation (see Exercise 3)).