Aalto university
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Exercise sheet 5
Complex Analysis, MS-C1300.
Hand in exercise 1 and 2 in separate files for grading. Deadline Monday 6.11 at 23:59. The exercises should be uploaded to the correct folder on MyCourses as pdf-files with name and student number in the file name. **Submission via MyCourses is the only accepted way.** Done during class Tuesday 7.11 or Wednesday 8.11.

(1) Let $f = u + iv$ be analytic in a domain D. Assume that
$$v(z) = u(z)^2$$
for all $z \in D$. Prove that f is a constant function. (*Hint*: Differentiate $u(z)^2 - v(z)$ with respect to x and y.) (6p)

(2) Let $f = u+iv$ where $u(z) = x^3+axy^2$ and $v(z) = bx^2y+cy^3+1$. Determine the values of the real numbers a, b, and c that make f an entire function. Then express f as a polynomial function of z. (*Hint*: Once you have determined a, b, and c start thinking about how $z^n = (x + iy)^n$ looks. Here the binomial theorem is useful.) (6p)

(3) For each of the following functions determine the largest open in which it is analytic, and compute its derivative in that set:
(a)
$$f(z) = \frac{e^z - 1}{e^z + 1}$$
(b)
$$g(z) = \frac{\sin(\sqrt{iz + 1})}{z^2 + 1}$$

(4) If a function f is analytic in a domain D and satisfies the differential equation
$$f'(z) - \alpha f(z) = 0$$
in D for a constant $\alpha \in \mathbb{C}$ show that f takes the form $f(z) = Ae^{\alpha z}$ for some constant $A \in \mathbb{C}$. (*Hint*: Study the function $g(z) = e^{-\alpha z} f(z)$.)