Aalto university
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Exercise sheet 6
Complex Analysis, MS-C1300.
Hand in exercise 1 and 2 in separate files for grading. Deadline Wednesday 8.11 at 23:59. The exercises should be uploaded to the correct folder on MyCourses as pdf-files with name and student number in the file name. **Submission via MyCourses is the only accepted way.** Done during class Thursday 9.11 or Friday 10.11.

(1) Let $\gamma(t) = te^{it}$ for $0 \le t \le \pi$ and calculate:
(a)
$$\int_\gamma \bar{z}\, dz$$
(2p)

(b)
$$\int_\gamma |z|\, |dz|$$
(2p)

(c)
$$\int_\gamma z\, dz$$
(2p)

(2) Let $\gamma(t) = -2e^{it}$ for $0 \le t \le 2\pi$. Evaluate
$$\int_\gamma \frac{1}{z^2 - 1}\, dz.$$
(*Hint:* Partial fractions simplifies calculations.) (6p)

(3) Let a and b be real numbers satisfying $a < b$, and let $I(c)$ be defined for any real number c by
$$I(c) = \int_{\gamma_{a,b}(c)} e^{-z^2}\, dz$$
where $\gamma_{a,b}(c)$ is the straight line with initial point $c+ia$ and terminal point $c+ib$. Show that $\lim_{c\to\infty} |I(c)| = 0$ and $\lim_{c\to-\infty} |I(c)| = 0$

(4) Evaluate the integrals (where $\gamma(t) = e^{it}, 0 \le t \le 2\pi$):

(a)

$$\int_\gamma \frac{1}{(z-2)^2}\ dz$$

(b)

$$\int_\gamma \frac{1}{z^2-4}\ dz$$

(c)

$$\int_\gamma \left(z+\frac{1}{z}\right)^n\ dz$$

where $n = 1, 2, 3, \ldots$.