

Galilein muunnos ja Newtonin 2.

$$m\ddot{x} \rightarrow \text{mitä kun } \begin{aligned} x' &= x - vt \\ t' &= t \end{aligned}$$

$$\dot{x} = \frac{dx}{dt} = \frac{dt'}{dt} \frac{d}{dt'}(x' + vt') = \frac{dx'}{dt'} - v$$

$$\ddot{x} = \frac{d^2x}{dt^2} = \frac{d^2x'}{dt'^2}, \text{ koska } v = \text{vakio}$$

Jos voima riippuu vain suhteellisesta koordinaatista, $F(\bar{x} - \bar{Q})$

↑
nollako-
paikka

↑
voiman lähtee-
paikka

$$\Rightarrow F(\hat{x} - \hat{Q}) = F(\bar{x}' - \bar{Q}'), \text{ koska}$$

"vt" termi on sama \bar{x}' :ssä ja \bar{Q}' :ssä

\Rightarrow on invariantti \Rightarrow kiihtyvyyden on sama kaikissa inertiaalikoordinaatistoissa.

Galilein muunnos: nopeuksien yhteenlasku
esimerkki

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} 1 & -v \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix}$$

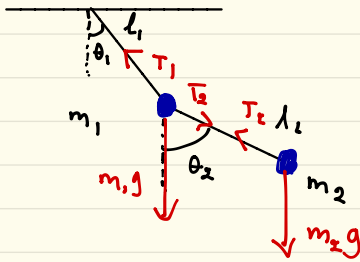
Tehdään 2 peräkkäistä muunnosta
koordinaatistoihin jotka liikkuvat nopeuksilla
 v_1 ja v_2 .

$$\Rightarrow \begin{pmatrix} x'' \\ t'' \end{pmatrix} = \begin{pmatrix} 1 & -v_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -v_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix}$$
$$\begin{pmatrix} 1 - v_2 \cdot 0 & -v_2 - v_1 \\ 0 \cdot 1 + 1 \cdot 0 & -v_1 \cdot 0 + 1 \cdot 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -v_1 - v_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix}$$

Ei: Galilein muunnos missä nopeus
 $v_1 + v_2$!!

Double pendulum: equations of motion



$$\vec{r}_1 = l_1 (\sin \theta_1, \cos \theta_1, 0)$$

$$\vec{r}_2 = \vec{r}_1 + l_2 (\sin \theta_2, \cos \theta_2, 0)$$

$$\dot{\vec{r}}_1 = \vec{v}_1 = l_1 \dot{\theta}_1 (\cos \theta_1, -\sin \theta_1)$$

$$\ddot{\vec{r}}_1 = \vec{a}_1 = l_1 \ddot{\theta}_1 (\cos \theta_1, -\sin \theta_1) - l_1 \dot{\theta}_1^2 (\sin \theta_1, \cos \theta_1)$$

$$\dot{\vec{r}}_2 = \vec{v}_2 = \dot{\vec{r}}_1 + l_2 \dot{\theta}_2 (\cos \theta_2, -\sin \theta_2)$$

$$\ddot{\vec{r}}_2 = \vec{a}_2 = \ddot{\vec{r}}_1 + l_2 \ddot{\theta}_2 (\cos \theta_2, -\sin \theta_2) - l_2 \dot{\theta}_2^2 (\sin \theta_2, \cos \theta_2)$$

$$\text{Forces on 1: } \vec{F}_1 = T_1 \frac{-\vec{r}_1}{|\vec{r}_1|} + T_2 \frac{(\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|} + m_1 \vec{g}$$

$$= -\frac{T_1}{l_1} \vec{r}_1 + \frac{T_2}{l_2} (\vec{r}_2 - \vec{r}_1) + m_1 \vec{g}$$

$$\text{Forces on 2: } -\frac{T_2}{l_2} (\vec{r}_2 - \vec{r}_1) + m_2 \vec{g}$$

$$\text{Eqs: } m_1 \ddot{\vec{r}}_1 = \vec{F}_1 \Rightarrow \text{4 eqs, 4 unknowns } \theta_1, \theta_2, T_1, T_2$$

$$m_2 \ddot{\vec{r}}_2 = \vec{F}_2 \Rightarrow \text{should work.}$$

$$m_1 l_1 (\ddot{\theta}_1 \cos \theta_1 - \dot{\theta}_1^2 \sin \theta_1) = -T_1 \sin \theta_1 + T_2 \sin \theta_2 \quad \text{x-comp for mass 1}$$

$$-m_1 l_1 (\dot{\theta}_1^2 \sin \theta_1 + \ddot{\theta}_1 \cos \theta_1) = -T_1 \cos \theta_1 + T_2 \cos \theta_2 + m_1 g \quad \text{y-comp}$$

$$m_2 (l_1 \ddot{\theta}_1 \cos \theta_1 - l_1 \dot{\theta}_1^2 \sin \theta_1 + l_2 \ddot{\theta}_2 \cos \theta_2 - l_2 \dot{\theta}_2^2 \sin \theta_2) = -T_2 \sin \theta_2 \quad (3)$$

$$-m_2 (l_1 \ddot{\theta}_1 \sin \theta_1 + l_1 \dot{\theta}_1^2 \cos \theta_1 + l_2 \ddot{\theta}_2 \sin \theta_2 + l_2 \dot{\theta}_2^2 \cos \theta_2) = -T_2 \cos \theta_2 + m_2 g \quad (4)$$

use $\sin^2 \theta + \cos^2 \theta = 1$ and $\sin \theta_2 \cos \theta_1 - \cos \theta_2 \sin \theta_1 = \sin(\theta_2 - \theta_1)$

$$(1) \cos \theta_1 - (2) \sin \theta_1$$

$$\Rightarrow m l_1 \ddot{\theta}_1 = T_2 \sin \theta_2 \cos \theta_1 - T_2 \cos \theta_2 \sin \theta_1 - m_1 g \sin \theta_1$$

$$= T_2 \sin(\theta_2 - \theta_1) - m_1 g \sin \theta_1$$

(ok)

$$(1) \sin \theta_1 + (2) \cos \theta_1$$

$$\Rightarrow -m l_1 \dot{\theta}_1^2 = -T_1 + T_2 (\underbrace{\sin \theta_1 \sin \theta_2 + \cos \theta_1 \cos \theta_2}_{\cos(\theta_2 - \theta_1)}) + m_1 g \cos \theta_1$$

$$m l_1 \dot{\theta}_1^2 = T_1 + T_2 \cos(\theta_2 - \theta_1) + m_1 g \cos \theta_1 \quad (\text{ok})$$

$$(3) \cos \theta_2 - (4) \sin \theta_2$$

$$\Rightarrow m_2 l_2 \ddot{\theta}_2 + m_2 l_1 \ddot{\theta}_1 \cos(\theta_2 - \theta_1) + m_2 l_1 \dot{\theta}_1^2 \sin(\theta_2 - \theta_1) = -m_2 g \sin \theta_2$$

(ok)

$$(3) \sin \theta_2 + (4) \cos \theta_2$$

$$\Rightarrow -m_2 l_2 \dot{\theta}_2^2 + m_2 l_1 \dot{\theta}_1^2 \sin(\theta_2 - \theta_1) - m_2 l_1 \dot{\theta}_1^2 \cos(\theta_2 - \theta_1) = -T_2 + m_2 g \cos \theta_2$$

(ok)

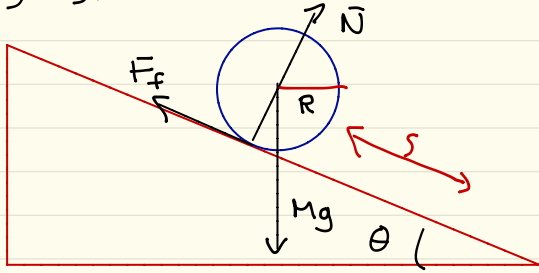
(ok) $\Rightarrow T_2$ sijoitus (ok) antaa $\ddot{\theta}_1$ in

T_1 in voi ratkaista (x) :stä, mutta
itseasiassa likeyhtälöitä

(x) ja (x) voidaan ratkaista
ilman sifakia.

Rolling cylinder:

Typically we assume only center of mass physics and ignore the rest. As an example, let us discuss rolling cylinder on an incline



Rotating cylinder has kinetic energy

$$E_R = \frac{1}{2} I \omega^2, \quad I = \text{moment of inertia} = \frac{1}{2} M R^2$$

$\omega = \text{angular velocity}$

Method 1: using conservation of energy

$$\frac{1}{2} M v^2 + \frac{1}{2} I \omega^2 - Mgh = 0 \quad h = \text{how much dropped}$$

no sliding just rolling $\Rightarrow v = \omega R$

$$\Rightarrow v = \sqrt{\frac{4}{3} gh} \quad \text{which is less than } \sqrt{2gh} \text{ for center of mass only.}$$

Average velocity $v_{\text{avg}} = \frac{v}{2} = \frac{s}{\Delta t} \Rightarrow \Delta t = \frac{2s}{v}$

acceleration $a = \frac{v}{\Delta t} = \frac{v^2}{2s} = \frac{2g}{3} \sin \theta, \quad \sin \theta = \frac{h}{s}$

Method 2: torque

If cylinder rolls angular momentum came from somewhere. Only friction F_f can cause torque others go through center of mass.

What is F_f ? If ω is somehow too large so that static friction no longer works, cylinder slides.

But here we could assume this doesn't happen.

along the incline: $Mg \sin \theta - F_f = Ma$ (*)

torque $\tau = F_f R = I \alpha$ ($\alpha =$ angular acceleration)

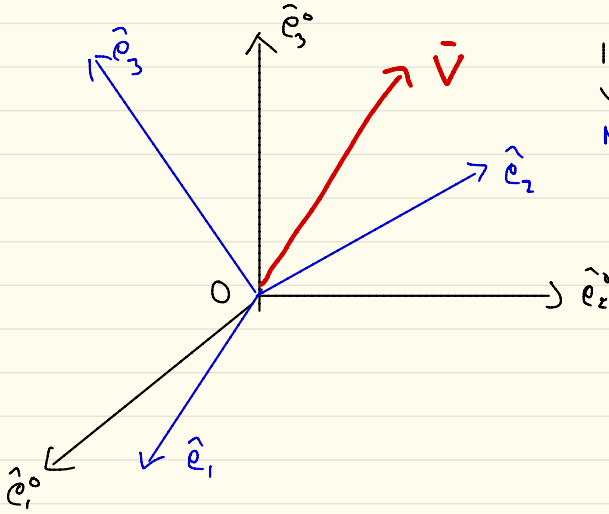
no slipping $\rightarrow \alpha = a/R$

$$\Rightarrow \frac{1}{2} MR^2 \frac{a}{R} = F_f R \Rightarrow F_f = \frac{1}{2} Ma$$

$$(*) \Rightarrow Mg \sin \theta = \frac{3}{2} Ma \rightarrow a = \frac{2}{3} g \sin \theta$$

2. Accelerated coordinate systems:

For example: motion as observed from a lab fixed on rotating earth.



Inertial frame \hat{e}_i^0
 with origin O ($i = \{1, 2, 3\}$)
 Moving orthonormal axes
 \hat{e}_i , same origin.
 Same vector \vec{V} .

$$\vec{V} = \sum_{i=1}^3 v_i^0 \hat{e}_i^0 \quad \text{or} \quad \vec{V} = \sum_{i=1}^3 v_i \hat{e}_i$$

$$\text{In inertial frame: } \left(\frac{d\vec{V}}{dt} \right)_{\text{inertial}} = \sum_{i=1}^3 \frac{dv_i^0}{dt} \hat{e}_i^0$$

$$\text{or } \left(\frac{d\vec{V}}{dt} \right)_{\text{inertial}} = \sum_{i=1}^3 \frac{dv_i}{dt} \hat{e}_i + \sum_{i=1}^3 v_i \frac{d\hat{e}_i}{dt}, \text{ where first term}$$

is the rate of change in the body-fixed frame

$$\left(\frac{d\vec{V}}{dt} \right)_{\text{body}} = \sum_{i=1}^3 \frac{dv_i}{dt} \hat{e}_i \Rightarrow \left(\frac{d\vec{V}}{dt} \right)_{\text{inertial}} = \left(\frac{d\vec{V}}{dt} \right)_{\text{body}} + \sum_{i=1}^3 v_i \frac{d\hat{e}_i}{dt}$$

3. Infinitesimal rotations

Consider small dt : $\hat{e}_i(t+dt) = \hat{e}_i(t) + d\hat{e}_i$

Orthogonal basis: $\hat{e}_i \cdot \hat{e}_j = \delta_{ij}$ (δ_{ij} = Kronecker delta) (*)

\Rightarrow to lowest order in dt : $\hat{e}_i \cdot d\hat{e}_i = 0$ (**)

and $d\hat{e}_i$ is perpendicular to \hat{e}_i (to lowest order in dt)

(can be expanded so: $d\hat{e}_i = \sum_{j=1}^3 d\Omega_{ij} \hat{e}_j$ with some coeff. $d\Omega_{ij}$)

(**) $\Rightarrow d\Omega_{ii} = 0$ ($\& d\hat{e}_i \cdot \hat{e}_j = d\Omega_{ij}$)

(*) $\Rightarrow d\hat{e}_i \cdot \hat{e}_j + \hat{e}_i \cdot d\hat{e}_j = 0 \Rightarrow d\Omega_{ij} = -d\Omega_{ji}$

and we only have 3 independent elements

$$d\Omega_{12} = d\Omega_{23}, \quad d\Omega_{23} = d\Omega_1, \quad d\Omega_{31} = d\Omega_2$$

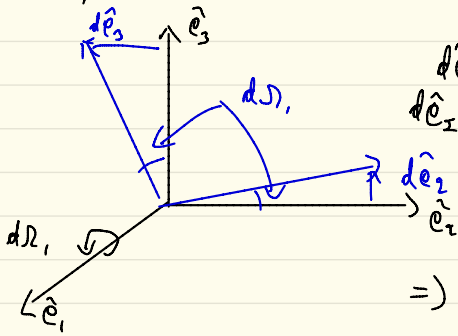
So that for example $d\hat{e}_1 = d\Omega_3 \hat{e}_2 - d\Omega_2 \hat{e}_3 = d\bar{\Omega} \times \hat{e}_1$

where $d\bar{\Omega} = d\Omega_1 \hat{e}_1 + d\Omega_2 \hat{e}_2 + d\Omega_3 \hat{e}_3$

... more generally $d\hat{e}_i = d\bar{\Omega} \times \hat{e}_i$

$d\bar{\Omega}$ interpretation: combination of 3 separate rotations around each axis.

4. Right hand convention: $d\bar{\mathbf{r}}_i = d\Omega_i \hat{\mathbf{e}}_i$



$$d\hat{\mathbf{e}}_i = 0$$

$$d\hat{\mathbf{e}}_1 = d\Omega_1 \hat{\mathbf{e}}_3 = d\bar{\Omega}_1 \times \hat{\mathbf{e}}_2$$

$$d\hat{\mathbf{e}}_3 = -d\Omega_1 \hat{\mathbf{e}}_2 = d\bar{\Omega}_1 \times \hat{\mathbf{e}}_3$$

=> formula OK.

On the other hand $d\hat{\mathbf{e}}_i = \frac{d\hat{\mathbf{e}}_i}{dt} dt = \frac{d\bar{\Omega}}{dt} \times \hat{\mathbf{e}}_i dt = \bar{\omega} \times \hat{\mathbf{e}}_i dt$

$\bar{\omega}$ = instantaneous angular velocity vector of the rotating frame as seen in the inertial frame $\bar{\omega} = \frac{d\bar{\Omega}}{dt}$

Then $\sum_{i=1}^3 v_i \frac{d\hat{\mathbf{e}}_i}{dt} = \sum_{i=1}^3 v_i \bar{\omega} \times \hat{\mathbf{e}}_i = \bar{\omega} \times \bar{\mathbf{v}}$ since $\bar{\mathbf{v}} = \sum_{i=1}^3 v_i \hat{\mathbf{e}}_i$

=> $\left(\frac{d\bar{\mathbf{v}}}{dt}\right)_{\text{inertial}} = \left(\frac{d\bar{\mathbf{v}}}{dt}\right)_{\text{body}} + \bar{\omega} \times \bar{\mathbf{v}}$ Applies to any vector $\bar{\mathbf{v}}$.

Example: $\bar{\mathbf{v}} = \bar{\mathbf{r}}$, $d\bar{\mathbf{r}} = d\theta \hat{\mathbf{z}}$ (rotation around $\hat{\mathbf{z}}$, $\bar{\mathbf{r}}$ = position vector fixed in moving frame)

$$d\bar{\mathbf{r}} = d\bar{\Omega} \times \bar{\mathbf{r}}$$

2 consecutive rotations $d\bar{\Omega}_1$ & $d\bar{\Omega}_2 \Rightarrow \bar{\mathbf{r}}_2 = \bar{\mathbf{r}}_1 + d\bar{\Omega}_2 \times \bar{\mathbf{r}}_1$
 $= \bar{\mathbf{r}} + d\bar{\Omega}_1 \times \bar{\mathbf{r}} + d\bar{\Omega}_2 \times (\bar{\mathbf{r}} + d\bar{\Omega}_1 \times \bar{\mathbf{r}}) \approx \bar{\mathbf{r}} + (d\bar{\Omega}_1 + d\bar{\Omega}_2) \times \bar{\mathbf{r}} + O(d\Omega^2)$

Note: infinitesimal rotations commute (order doesn't matter)

However, finite rotations do not commute

=> $\left(\frac{d\bar{\mathbf{r}}}{dt}\right)_{\text{inertial}} = \frac{d\bar{\Omega}}{dt} \times \bar{\mathbf{r}} = \bar{\omega} \times \bar{\mathbf{r}}$ $\left(\left(\frac{d\bar{\mathbf{r}}}{dt}\right)_{\text{body}} = 0 \text{ for } \bar{\mathbf{r}}\right)$

5. $d\vec{r} = \text{vector} \Rightarrow$ so is $\dot{\vec{w}} \Rightarrow \left(\frac{d\dot{\vec{w}}}{dt}\right)_{\text{inertial}} = \left(\frac{d\dot{\vec{w}}}{dt}\right)_{\text{body}}$

(since $\vec{\omega} \times \vec{\omega} = 0$)

observers in inertial and rotating frames agree on the rate of change of $\dot{\vec{w}}$.

Now take a coordinate of a moving particle \vec{r}

$\Rightarrow \left(\frac{d\vec{r}}{dt}\right)_{\text{inertial}} = \left(\frac{d\vec{r}}{dt}\right)_{\text{body}} + \vec{\omega} \times \vec{r}$

For dynamics we need accelerations which are vectors as well.

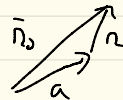
" $\left(\frac{d}{dt}\right)_{\text{inertial}} = \left(\frac{d}{dt}\right)_{\text{body}} + \vec{\omega} \times$ "

twice $\Rightarrow \left(\frac{d^2\vec{r}}{dt^2}\right)_{\text{inertial}} = \left[\left(\frac{d}{dt}\right)_{\text{body}} + \vec{\omega} \times\right] \left[\left(\frac{d\vec{r}}{dt}\right)_{\text{body}} + \vec{\omega} \times \vec{r}\right]$
 $= \left(\frac{d^2\vec{r}}{dt^2}\right)_{\text{body}} + 2\vec{\omega} \times \left(\frac{d\vec{r}}{dt}\right)_{\text{body}} + \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$

since $\frac{d\vec{\omega}}{dt}$ was independent of the frame.

\Rightarrow Acceleration in rotating frame different from inertial frame

Add also translations? $\vec{r}_0 = \vec{r} + \vec{a}$



$\Rightarrow \left(\frac{d^2\vec{r}_0}{dt^2}\right)_{\text{inertial}} = \left(\frac{d^2\vec{a}}{dt^2}\right)_{\text{inertial}} + \left(\frac{d^2\vec{r}}{dt^2}\right)_{\text{inertial}}$

$= \left(\frac{d^2\vec{a}}{dt^2}\right)_{\text{inertial}} + \left(\frac{d^2\vec{r}}{dt^2}\right)_{\text{body}} + 2\vec{\omega} \times \left(\frac{d\vec{r}}{dt}\right)_{\text{body}} + \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$

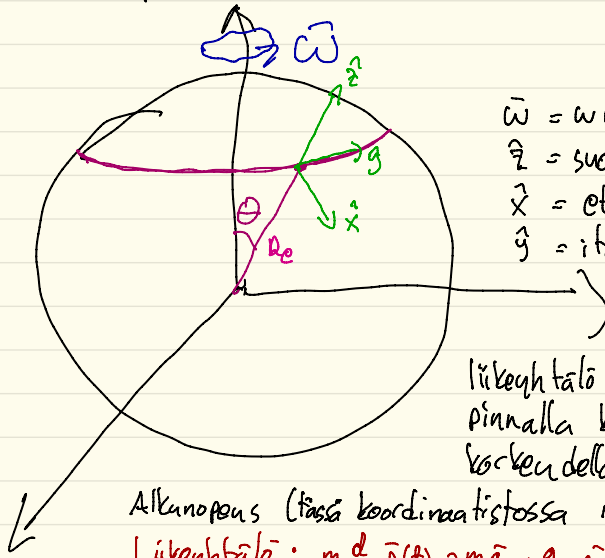
6. Newton's laws in accelerated system.

$$m \left(\frac{d^2 \vec{r}}{dt^2} \right)_{\text{inertial}} = \vec{F}_e$$

$$\Rightarrow m \left(\frac{d^2 \vec{r}}{dt^2} \right)_{\text{body}} = \vec{F}_e - m \left(\frac{d^2 \vec{a}}{dt^2} \right)_{\text{inertial}} - \underbrace{2m \vec{\omega} \times \left(\frac{d\vec{r}}{dt} \right)_{\text{body}}}_{\text{Coriolis}}$$

$$\underbrace{-m \vec{\omega} \times (\vec{\omega} \times \vec{r})}_{\text{Centrifugal}} - \underbrace{m \frac{d\vec{\omega}}{dt} \times \vec{r}}_{\text{Euler Force}}$$

Putoava kappale esimerkki:



$$\bar{\omega} = \omega \cos\theta \hat{z} - \omega \sin\theta \hat{x}$$

\hat{z} = suoraan ylös
 \hat{x} = etelään
 \hat{y} = itään

liikettä koordinaatistossa maan pinnalla kappaleen alla. Kappale alussa korkeudella h eli $\vec{r}(0) = (0, 0, h) = h\hat{z}$

Alkunopeus (tässä koordinaatistossa) $\vec{v}(0) = 0$

Liikkeenälö: $m \frac{d}{dt} \vec{r}(t) = m\vec{g} - 2m\bar{\omega} \times \vec{v}$ ← Coriolis-voima

$$\vec{g} = -g_0 \hat{z}$$

Haetaan ratkaisun suoraan putoavan ratkaisun läheltä eli

$$\vec{r}(t) = \vec{r}_0(t) + \vec{r}_1(t), \text{ missä } \vec{r}_0(t) = \left(-\frac{1}{2}g_0 t^2 + h\right) \hat{z}, \vec{v}(t) = -g_0 t \hat{z} + \underbrace{\frac{d}{dt} \vec{r}_1(t)}_{\vec{v}_1(t)}$$

Sijoitus liikkeenälöön:

$$m \left(-g_0 \hat{z} + m \frac{d^2}{dt^2} \vec{r}_1(t) \right) = -mg_0 \hat{z} - 2m\bar{\omega} \times \left(-g_0 t \hat{z} + \vec{v}_1(t) \right)$$

$$= m \frac{d^2}{dt^2} \vec{r}_1(t) \approx 2mg_0 t \bar{\omega} \times \hat{z}, \text{ koska } \bar{\omega} \times \vec{v}_1 \sim \text{pieni} \times \text{pieni} \sim \text{tosi pieni}$$

$$\bar{\omega} \times \hat{z} = \omega \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -\sin\theta & 0 & \cos\theta \\ 0 & 0 & 1 \end{vmatrix} = +\sin\theta \hat{y}$$

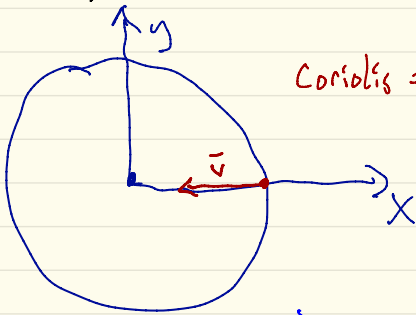
$$\Rightarrow \vec{r}_1(t) = +\frac{1}{3} m g_0 t^3 \sin\theta \hat{y} \Rightarrow \text{poikkeaa itään kun } \theta < \pi/2. \text{ Miksi?}$$

$$h = 100 \text{ m} \Rightarrow t = \sqrt{2h/g_0} \approx 4.5 \text{ s}$$

Poikkeama itään 2.2 cm

Mihin suuntaan ilma kiertää matalapaineen ympäri

- Pohjoisessa maan pyöriminen suunnilloen \hat{z} suuntaan
- Ilmaan vaikuttaa painegradientti matalapaineen suuntaan (pöykii tähtämään reian)
- Coriolis $-2m\vec{\omega} \times \vec{v}$



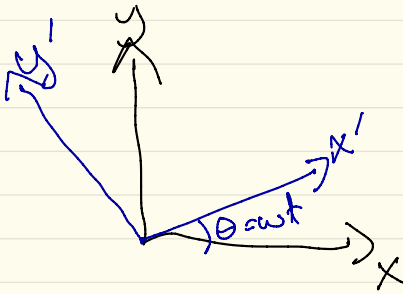
$$\text{Coriolis} = -2m\omega \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & 1 \\ -v & 0 & 0 \end{vmatrix}$$

$$= +2m\omega v \hat{y}$$

\Rightarrow LÄHTEE POIKKEAMAAN Y-SUUNTAAN

\Rightarrow VASTAPÄIVÄIN PÖHJOISESSÄ

Koordinaattimuunnos 2D:ssä pyörivään koordinaatistoon?



$$\begin{cases} x' = x \cos \theta + y \sin \theta \\ y' = y \cos \theta - x \sin \theta \end{cases} \quad (*)$$

Inertiaalikoordinaatistossa: $m\ddot{x} = F_x$, $\vec{F} = (F_x, F_y)$
 $m\ddot{y} = F_y$

mutta koska nyt $\theta = \omega t$, yhtälö x' :n ja y' :n avulla on varmaan erilainen.

Käännetään (*)

$$\Rightarrow x' \cos \theta = x \cos^2 \theta + y \sin \theta$$

$$y' \sin \theta = y \sin \theta \cos \theta - x \sin^2 \theta$$

$$\Rightarrow x = x' \cos \theta - y' \sin \theta, \text{ koska } \sin^2 \theta + \cos^2 \theta = 1$$
$$y = y' \cos \theta + x' \sin \theta$$

Nyt voidaan laskea aikaderivaatat. Huomaa $\dot{\theta} = \frac{d\theta}{dt} = \omega$

$$\dot{x} = \dot{x}' \cos \theta - \dot{y}' \sin \theta - x' \omega \sin \theta - y' \omega \cos \theta$$

$$\ddot{x} = \ddot{x}' \cos \theta - \ddot{y}' \sin \theta - \dot{y}' \sin \theta - \dot{y}' \omega \cos \theta - \dot{x}' \omega \sin \theta - x' \omega^2 \cos \theta - \dot{y}' \omega \cos \theta + y' \omega^2 \sin \theta$$

$$(1) = \ddot{x}' \cos \theta - \ddot{y}' \sin \theta - 2\dot{x}' \omega \sin \theta - 2\dot{y}' \omega \cos \theta - x' \omega^2 \cos \theta + y' \omega^2 \sin \theta$$

$$\dot{y} = \dot{y}' \cos \theta - y' \omega \sin \theta + \dot{x}' \sin \theta + x' \omega \cos \theta$$

$$\ddot{y} = \ddot{y}' \cos \theta + \ddot{x}' \sin \theta - 2 \dot{y}' \omega \sin \theta + 2 \dot{x}' \omega \cos \theta - y' \omega^2 \cos \theta + x' \omega^2 \sin \theta \quad (2)$$

$$(1) \cdot \cos \theta + (2) \cdot \sin \theta = \ddot{x}' + x' \omega^2 - 2 \dot{y}' \omega$$

$$(1) \cdot (-\sin \theta) + (2) \cdot \cos \theta = \ddot{y}' + y' \omega^2 + 2 \dot{x}' \omega$$

$$\vec{v}' = (\dot{x}', \dot{y}'), \quad \vec{\omega} = \omega \hat{e}_z$$

$$\Rightarrow m \ddot{x}' = -m \omega^2 x' + 2m \omega \dot{y}' + \underbrace{F_x'}_{\text{voiman komponentti } x' \text{ suunnassa}}$$

$$m \ddot{y}' = -m \omega^2 y' - 2m \omega \dot{x}' + F_y'$$

$$-\vec{\omega} \times \vec{v}' = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ 0 & 0 & -\omega \\ v_x' & v_y' & 0 \end{vmatrix} = \omega v_y' \hat{e}_x - \omega v_x' \hat{e}_y$$

Keskipakotusvoima $-m \omega^2 \vec{r}$ ilmestyy

Samoin Coriolis-voima $-2m \vec{\omega} \times \vec{v}$