Aalto University
School of Engineering

Power Electronics

ELEC-E8412 Power Electronics, 5 ECTS

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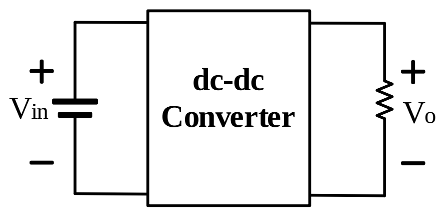
At the end of this course, you will be able to:

analyze the operation of the basic converter topologies (**buck, boost, voltage source inverters**) using the switching power pole as the building block.

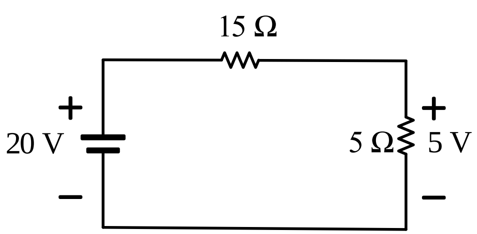
dc-dc Converters

- dc-dc converters are power electronic circuits that convert a dc voltage to a different dc voltage level, often providing a regulated output.
- The circuits described in this chapter are classified as **switched-mode dc-dc converters**, also called **switching power supplies** or **switchers**.

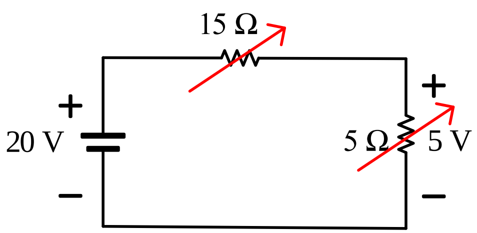


$$\text{dc-dc converters} \begin{cases} \text{step down} \quad V_{in} > V_o \quad \text{(Buck)} \\ \text{step up} \quad V_{in} < V_o \quad \text{(Boost)} \\ \text{both} \quad \text{(Buck-Boost)} \end{cases}$$

$$\text{dc-dc converters} \begin{cases} \text{non-isolated} \quad \text{(Buck, Boost, ...)} \\ \text{isolated} \quad \text{(Flyback, Forward, ...)} \end{cases}$$



$\eta = 25\%$ (only 25% of energy can be delivered from the source to the load and 75% will be lossed on R=15Ω).



For variable loads we need to build a variable resistor instead of R=15Ω, which is challenging.

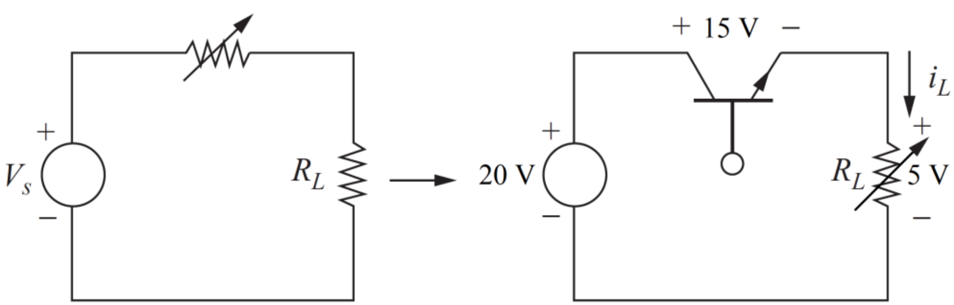


Figure 5-1: linear regulator.

Solution would be using transistor instead of R=15Ω. By adjusting the injected current to the base, we can adjust the amount of current flows through the loop, and set the operating point; therefore, even though we have variable load, we can always see 5 V over the terminal of this load, and the remain 15 V appears on transistor.

A Basic Switching Converter

An efficient alternative to the linear regulator is the switching converter. In a switching converter circuit, the transistor operates as an electronic switch by being completely ON or completely OFF. This circuit is also known as a **dc chopper**.

- The output is the same as the input when the switch is closed, and the output is zero when the switch is open.
- Periodic opening and closing of the switch results in the pulse output shown in Fig. 5-2c.
- The average or dc component of the output voltage is:

$$V_o = \frac{1}{T}\int_0^T v_o(t)dt = \frac{1}{T}\int_0^{DT} V_s dt = V_s D$$

- The dc component of the output voltage is controlled by adjusting the duty ratio D (duty cycle), which is the fraction of the switching period that the switch is closed.

$$D = \frac{t_{on}}{t_{on} + t_{off}} = \frac{t_{on}}{T} = t_{on} f$$

where f is the switching frequency.

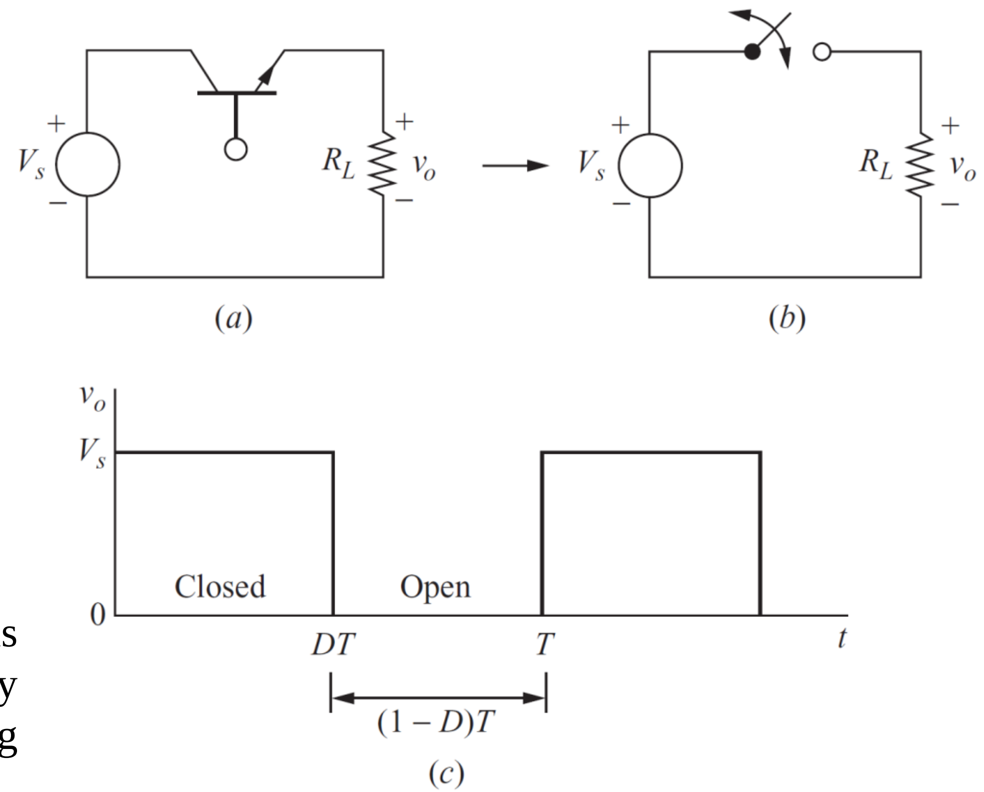


Figure 5-2 (*a*) A basic dc-dc switching converter; (*b*) Switching equivalent; (*c*) Output voltage.

The BUCK (Step-Down) Converter

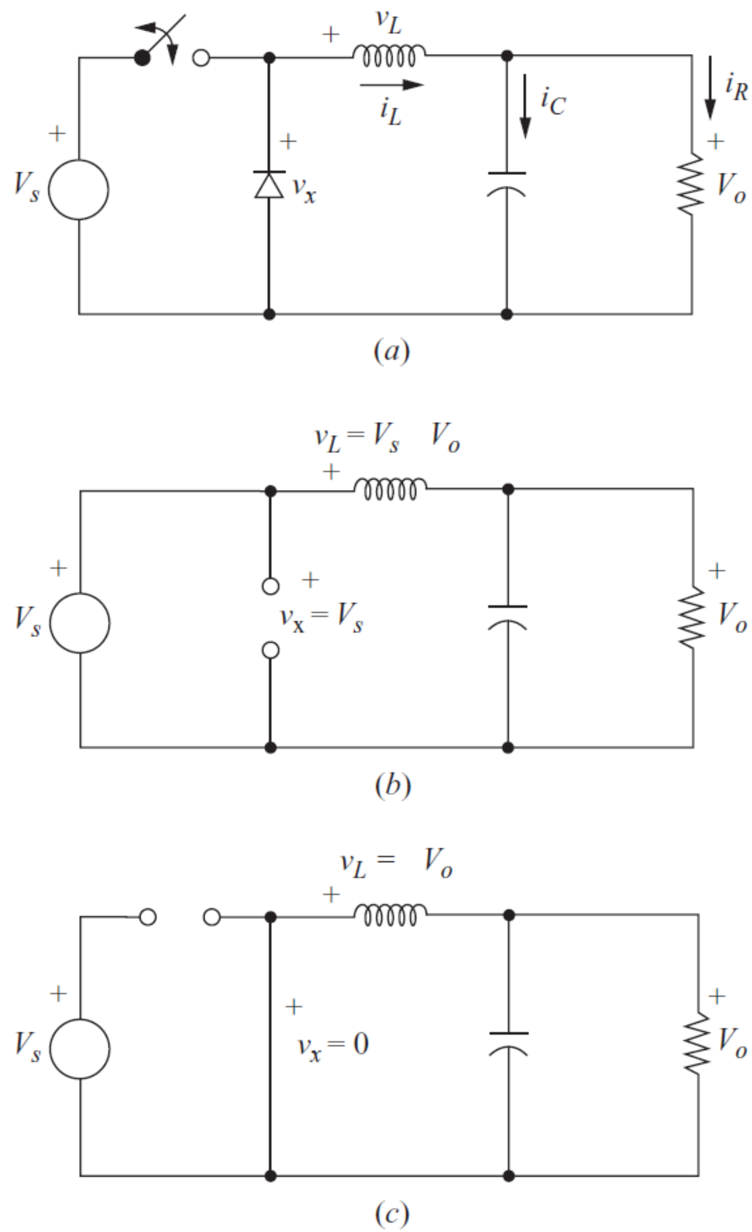
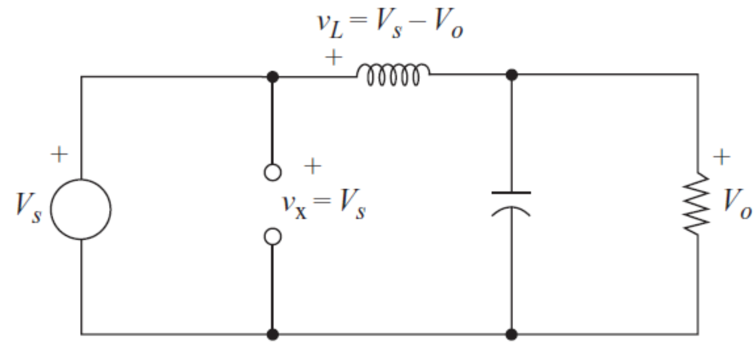
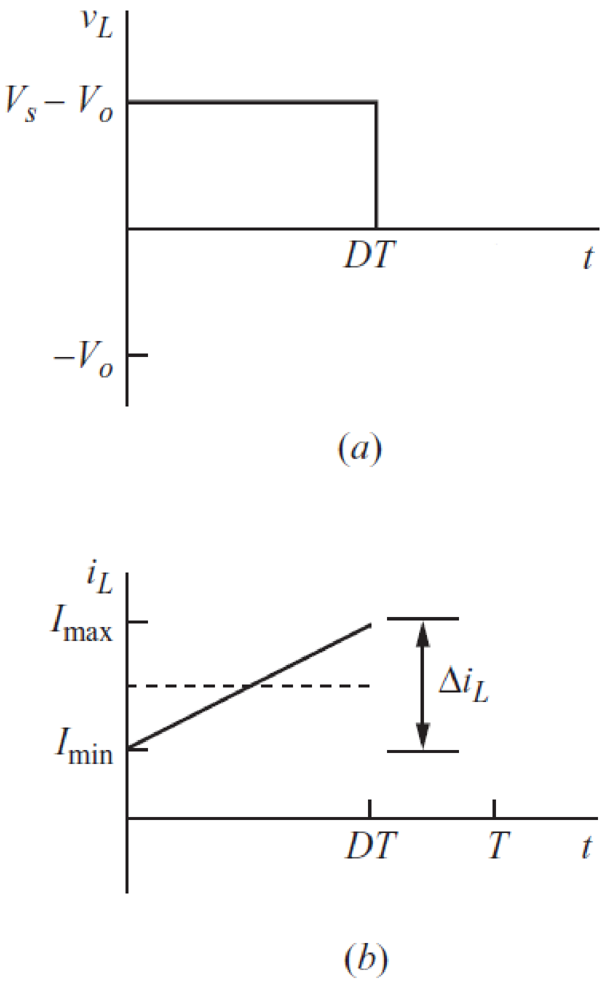


Figure 5-3 (*a*) Buck dc-dc converter; (*b*) Equivalent circuit for the switch closed; (*c*) Equivalent circuit for the switch open.

Analysis for the Switch Closed:

When the switch is closed (ON) in the buck converter circuit of Fig. 5-3a, the diode is reverse-biased and Fig. 5-3b is an equivalent circuit. The voltage across the inductor is:





Mode I: S: ON 0<t<DT

v_x=-Vs<0 → diode is in reverse bias and is OFF (open circuit)

$$v_L = V_s - V_o = L\frac{di_L}{dt} \rightarrow \frac{di_L}{dt} = \frac{V_s - V_o}{L}$$

Since the derivative of the current is a positive constant, the current increases linearly as shown in Fig. 5-4 (b). The change in current while the switch is closed is computed by modifying the preceding equation.

$$\frac{di_L}{dt} = \frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_s - V_o}{L} \Rightarrow (\Delta i_L)_{closed} = \left(\frac{V_s - V_o}{L}\right)DT$$

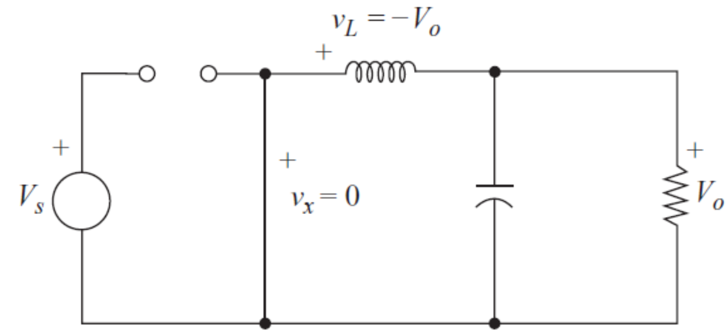
$$i_L(t) = \mathrm{I}_{min} + \frac{V_s - V_o}{L}t \qquad (1)$$

$$\mathrm{I}_{max} = i_L(DT) = \mathrm{I}_{min} + \frac{V_s - V_o}{L}DT \qquad (2)$$

$$\Delta i_L = \mathrm{I}_{max} - \mathrm{I}_{min} = \frac{V_s - V_o}{L}DT \qquad (3)$$

Figure 5-4 Buck converter waveforms: (a) Inductor voltage; (b) Inductor current.

Analysis for the Switch Open:



When the switch is open (OFF), the diode becomes forward-biased to carry the inductor current and the equivalent circuit of Fig. 5-3 (c) applies. The voltage across the inductor when the switch is open is:

Mode II: S: OFF DT<t<T

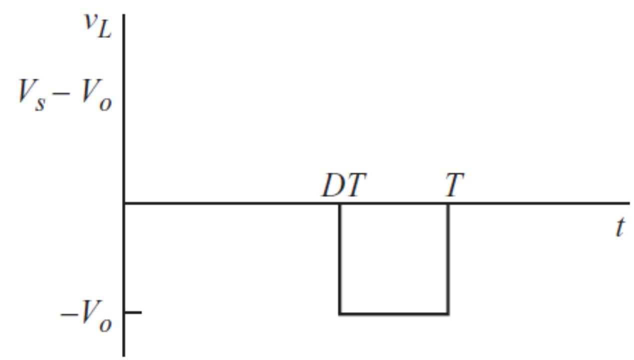
$i_L(DT) > 0 \Rightarrow L$ forces diode to turn ON

$$-v_x + v_L + V_o = 0 \Rightarrow v_L = -V_o = L\frac{di}{dt} = L\frac{\Delta i_L}{\Delta t} \rightarrow \frac{-V_o}{L} = \frac{\Delta i_L}{\Delta t}$$

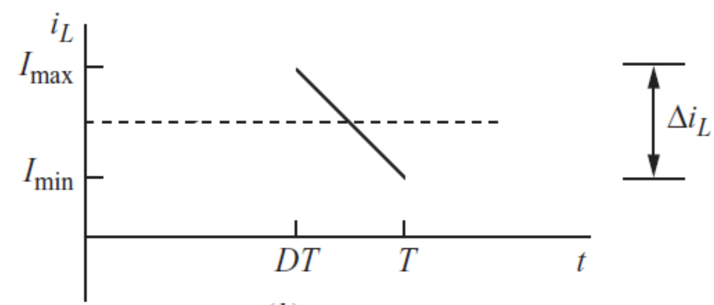
$$i_L(t) - \mathrm{I}_{max} = \frac{\Delta i_L}{\Delta t}(t - DT)$$

$$\frac{\Delta i_L}{\Delta t} = -\frac{V_o}{L} \rightarrow i_L(t) = \mathrm{I}_{max} - \frac{V_o}{L}(t - DT) \qquad (4)$$

$$\Delta i_L = \mathrm{I}_{max} - \mathrm{I}_{min} = \frac{V_o}{L}(1 - D)T \qquad (5)$$



(a)



(b)

Steady-State Condition:

$\langle v_L \rangle = 0 \qquad i_L(t=0) = i_L(t=T)$

Mode I: $v_L = V_s - V_o$ 0<t<DT

Mode II: $v_L = -V_o$ DT<t<T

$$(V_s - V_o) \times DT + (-V_o) \times (T - DT) = 0$$

$\frac{V_o}{V_s} = D$

Buck

→ $0 < D < 1 \Rightarrow 0 < V_o < V_s$

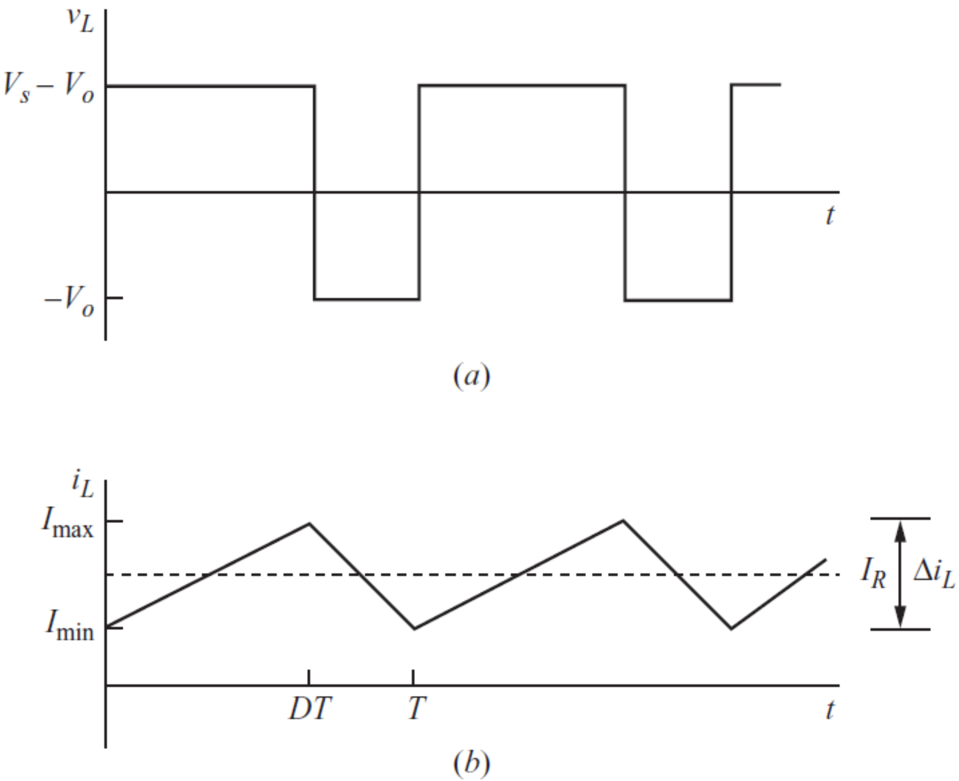


Figure 5-5 Buck converter waveforms: (a) Inductor voltage; (b) Inductor current.

For periodic currents the average voltage across an inductor is zero.

i_L is periodic; therefore, $\langle v_L \rangle = 0$

$$KCL: \quad i_L(t) = i_c(t) + i_R(t)$$

$$\langle i_L(t) \rangle = \langle i_c(t) \rangle + \langle i_R(t) \rangle$$

The average inductor current must be the same as the average current in the load resistor, since the average capacitor current must be zero (periodic voltages) for steady-state operation:

$$\left. \begin{array}{r} i_R(t) = \dfrac{V_o(t)}{R} \\ V_o(t) = V_o \end{array} \right\} \Rightarrow i_L(t) = i_R(t) = I_R = \frac{V_o}{R}$$

The maximum and minimum values of the inductor current are computed as:

$$\mathrm{I}_{max} = I_L + \frac{\Delta i_L}{2} = \frac{V_o}{R} + \frac{1}{2}\left[\frac{V_o}{R}(1-D)T\right] = V_o\left(\frac{1}{R} + \frac{1-D}{2Lf}\right)$$

$$\mathrm{I}_{min} = I_L - \frac{\Delta i_L}{2} = \frac{V_o}{R} - \frac{1}{2}\left[\frac{V_o}{R}(1-D)T\right] = V_o\left(\frac{1}{R} - \frac{1-D}{2Lf}\right)$$

where $f = 1/T$ is the switching frequency.

Example: The buck dc-dc converter of Fig. 5-3a has the following parameters:

$$V_s = 50V,\ D = 0.4\ \ \&\ \ 0.8,\ L = 400\mu H,\ C = 100\mu F,\ f = 20kHz,\ and\ R = 20\Omega$$

Assuming ideal components, calculate (a) the output voltage Vo, and (b) the maximum and minimum inductor current.

Solution:

(a):

$$V_o = DV_s = \begin{cases} I: D = 0.4 \rightarrow V_o = 0.4\times 50 = 20V \\ II: D = 0.8 \rightarrow V_o = 0.8\times 50 = 40V \end{cases}$$

(b):

$$\mathrm{I}_{max} = I_L + \frac{\Delta i_L}{2} = \frac{V_o}{R} + \frac{1}{2}\left[\frac{V_o}{R}(1-D)T\right] = V_o\left(\frac{1}{R} + \frac{1-D}{2Lf}\right) \rightarrow \begin{cases} \mathrm{I}: \mathrm{I}_{max} = 20\left(\frac{1}{20} + \frac{1-0.4}{2\times 400\times 10^{-6}\times 20\times 10^{3}}\right) = 1.5(A) \\ \mathrm{II}: \mathrm{I}_{max} = 40\left(\frac{1}{20} + \frac{1-0.8}{2\times 400\times 10^{-6}\times 20\times 10^{3}}\right) = 2.5(A) \end{cases}$$

$$\mathrm{I}_{min} = I_L - \frac{\Delta i_L}{2} = \frac{V_o}{R} - \frac{1}{2}\left[\frac{V_o}{R}(1-D)T\right] = V_o\left(\frac{1}{R} - \frac{1-D}{2Lf}\right) \rightarrow \begin{cases} \mathrm{I}: \mathrm{I}_{min} = 20\left(\frac{1}{20} - \frac{1-0.4}{2\times 400\times 10^{-6}\times 20\times 10^{3}}\right) = 0.25(A) > 0\ \mathrm{CCM} \\ \mathrm{II}: \mathrm{I}_{min} = 40\left(\frac{1}{20} - \frac{1-0.8}{2\times 400\times 10^{-6}\times 20\times 10^{3}}\right) = 1.5(A) > 0\ \mathrm{CCM} \end{cases}$$

The Boost (Step-Up) Converter

- The boost converter is another switching converter that operates by periodically opening and closing an electronic switch.
- It is called a boost converter because the output voltage is larger than the input.

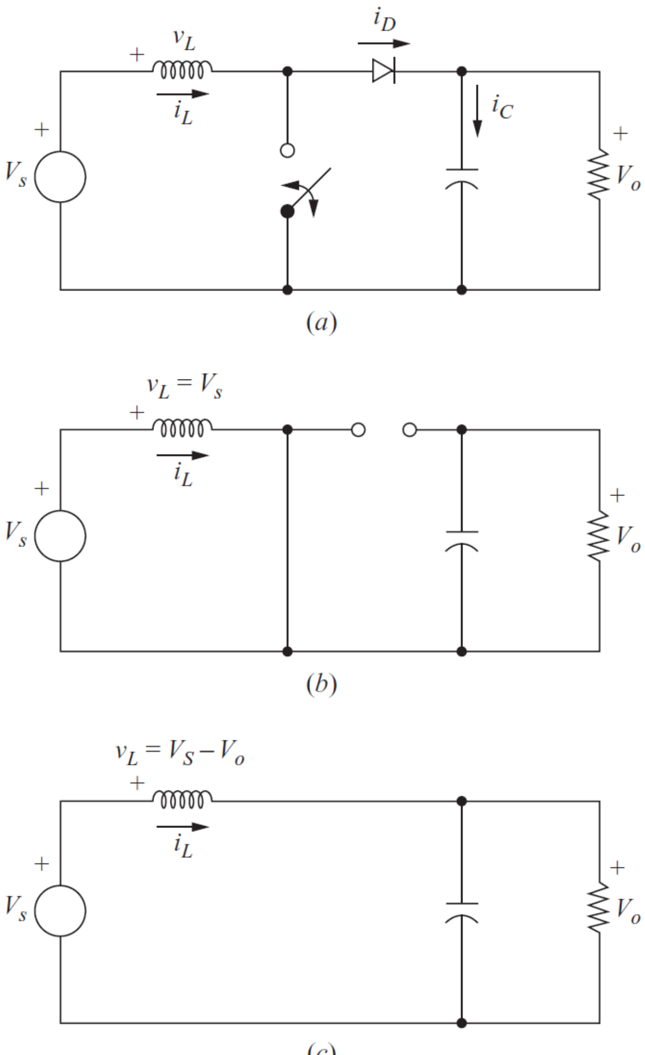


Figure 5-6 The boost converter. (a) Circuit; (b) Equivalent circuit for the switch closed; (c) Equivalent circuit for the switch open.

Voltage and Current Relationships:

1. The converter is operating under steady-state operating condition ($\langle V_L \rangle = 0$, $\langle i_C \rangle = 0$, $V_C(T) = V_C(0)$, and $i_L(T) = i_L(0)$)

2. The converter is operating at continuous conduction mode (CCM, $I_{min} > 0$ & $I_{min} \neq 0$)

3. The capacitor is very large (we can neglect the ripple of the output voltage), and the output voltage is held constant at voltage V_o ($V_o(t) = V_o$) (the instantaneous value of the output voltage is almost the same as its average value).

4. The components are ideal ($P_{in} = P_{out}$)

5. Switching transients are neglected

$$Switching\ period = T$$

$$Switching\ frequency = f = \frac{1}{T}$$

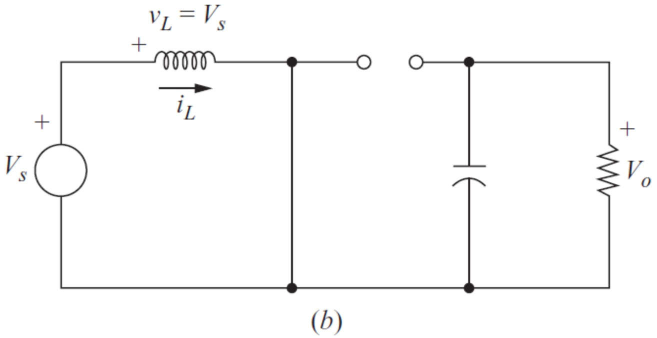
$$Switch\ status \begin{cases} ON & 0 < t < DT \\ OFF & DT < t < T \end{cases}$$

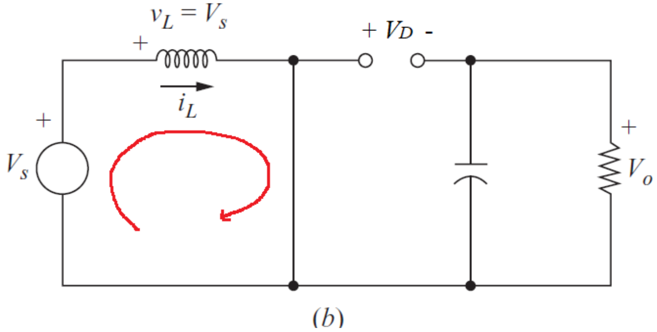
$$D(duty\ cycle) = \frac{ON\ time\ of\ the\ Switch}{T}$$

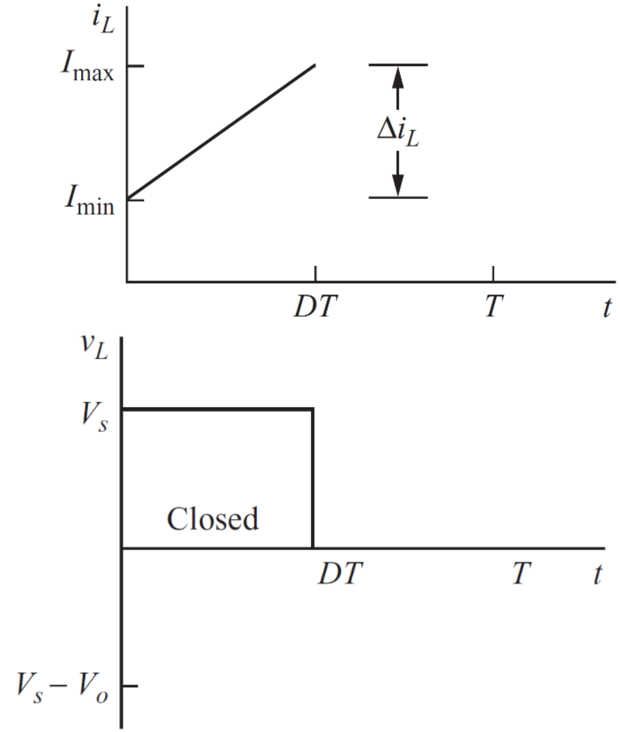
Mode I: Analysis for the Switch Closed (ON)

When the switch is closed, the diode is reverse biased. Kirchhoff's voltage law around the path containing the source, inductor, and closed switch is:

$$V_S = V_L = L\frac{di_L}{dt} \Rightarrow \frac{di_L}{dt} = \frac{V_S}{L}$$







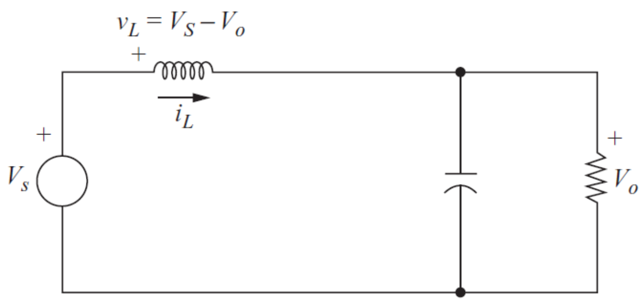
$$\left.\begin{aligned} V_L &= V_S > 0 \\ V_L &= L\frac{di_L}{dt} \end{aligned}\right\} \Rightarrow i_L(t) = \mathrm{I}_{min} + \frac{V_S}{L}t$$

$$\mathrm{I}_{max} = i_L(DT) = \mathrm{I}_{min} + \frac{V_S}{L}DT \qquad (1)$$

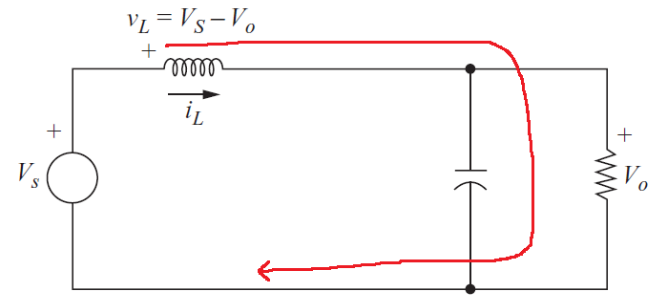
$$\Delta i_L = \mathrm{I}_{max} - \mathrm{I}_{min} = \frac{V_S}{L}DT \qquad (2)$$

Mode II: Analysis for the Switch Open (OFF)

When the switch is opened, the inductor current cannot change instantaneously, so the diode becomes forward-biased to provide a path for inductor current.



(c)



(c)

$$\left.\begin{array}{r} V_L = V_S - V_o < 0 \\ V_L = L\dfrac{di_L}{dt} \end{array}\right\} \Rightarrow i_L(t) = \mathrm{I}_{\max} + \frac{V_S - V_o}{L}(t - DT)$$

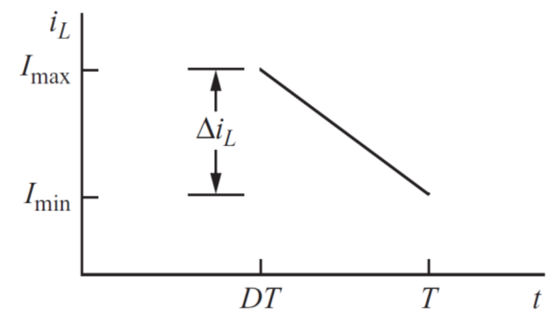
$$\mathrm{I}_{\min} = i_L(T) = \mathrm{I}_{\max} + \frac{V_S - V_o}{L}(T - DT) \qquad (3)$$

$$\Delta i_L = \mathrm{I}_{\max} - \mathrm{I}_{\min} = -\frac{V_S - V_o}{L}(T - DT) \qquad (4)$$

$$(2)\,\&\,(4) \Rightarrow \frac{V_S}{L}DT = -\frac{V_S - V_o}{L}(T - DT)$$

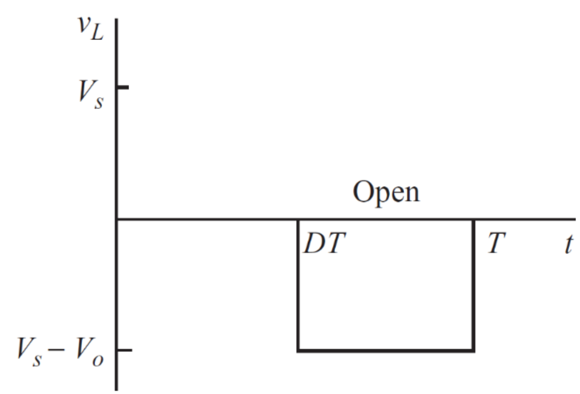
$$In\ Boost\ Converter\ (CCM) \Rightarrow \boxed{\frac{V_o}{V_S} = \frac{1}{1 - D}} \qquad (5)$$

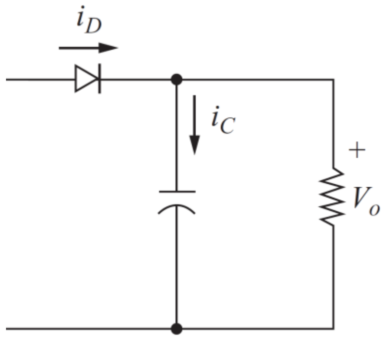
$$\boxed{0 < D < 1 \Rightarrow V_S < V_o < \infty}$$



$$\Delta i_L \neq \mathrm{I}_{min} - \mathrm{I}_{max}$$

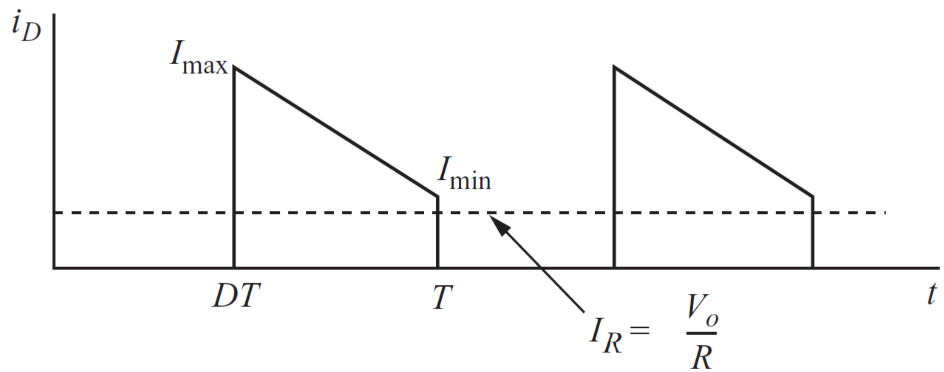
$$\Delta i_L = \mathrm{I}_{max} - \mathrm{I}_{min}$$





$$i_D(t) = i_C(t) + i_R(t)$$

$$\langle i_D(t)\rangle = \langle i_C(t)\rangle + \langle i_R(t)\rangle \Rightarrow \langle i_D(t)\rangle = \underbrace{\langle i_C(t)\rangle}_{0=\text{DC and periodic ripple}} + \langle i_R(t)\rangle \Rightarrow \langle i_D(t)\rangle = I_R$$



$$\frac{\frac{I_{max}+I_{min}}{2}\times(T-DT)}{T} = \frac{V_0}{R} \Rightarrow I_{max}+I_{min} = \frac{2V_0}{(1-D)R}$$

$$\left.\begin{aligned} I_{max}+I_{min} &= \frac{2V_0}{(1-D)R} \\ V_o &= \frac{V_S}{(1-D)} \end{aligned}\right\} \Rightarrow I_{max}+I_{min} = \frac{2V_S}{(1-D)^2 R} \qquad (6)$$

$$I_{max}-I_{min} = \frac{V_S}{L}DT \qquad (2)$$

$$(2)+(6) \Rightarrow 2I_{max} = \frac{2V_S}{(1-D)^2 R} + \frac{V_S}{L}DT \Rightarrow I_{max} = \frac{V_S}{(1-D)^2 R} + \frac{V_S}{2L}DT \qquad (7)$$

$$(7)\,\&\,(2) \Rightarrow \frac{V_S}{(1-D)^2 R} + \frac{V_S}{2L}DT - I_{min} = \frac{V_S}{L}DT \Rightarrow I_{min} = \frac{V_S}{(1-D)^2 R} - \frac{V_S}{2L}DT \qquad (8)$$

$$\langle i_L\rangle = \frac{I_{max}+I_{min}}{2} \Rightarrow \frac{Eq.(7)+Eq.(8)}{2} \Rightarrow \boxed{\langle i_L\rangle = \frac{V_S}{(1-D)^2 R}} \qquad (9)$$

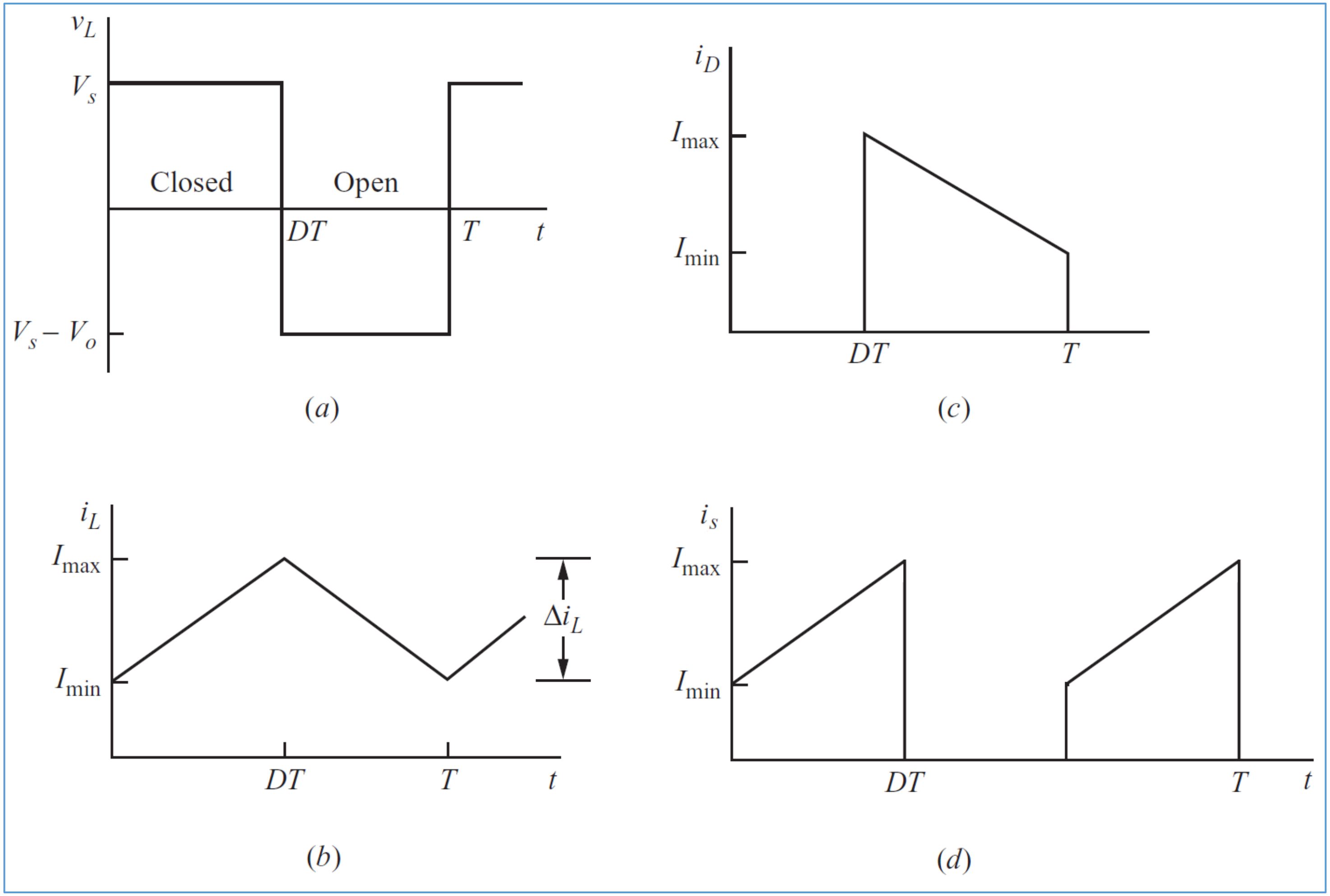


Figure 5-7 Boost converter waveforms. (a) Inductor voltage; (b) Inductor current; (c) Diode current; (d) Switch current.

- If plot the rate of output voltage to input voltage, it can be seen that for D=0, the voltage gain will be equal to one, and the output voltage will be equal to input voltage. As it can be seen in Fig. 5-8, by increasing the D value, gain is increased more, and for D=1, gain gets to infinity (Ideal Situation). Practically this doesn't happen, because they are losses in the system. So, if the D get closed to 100%, the output voltage goes down.
- It is not a good idea to keep the D between 0 and 20% (0<D<20%) or 80% and 100% (80%<D<100%). More reasonable range for D is between 20% and 80% (20%<D<80%).
- If we select the D beyond this boundary, it leads to some extreme cases such as discontinuous mode, high ripple in inductor current, or less efficiency.

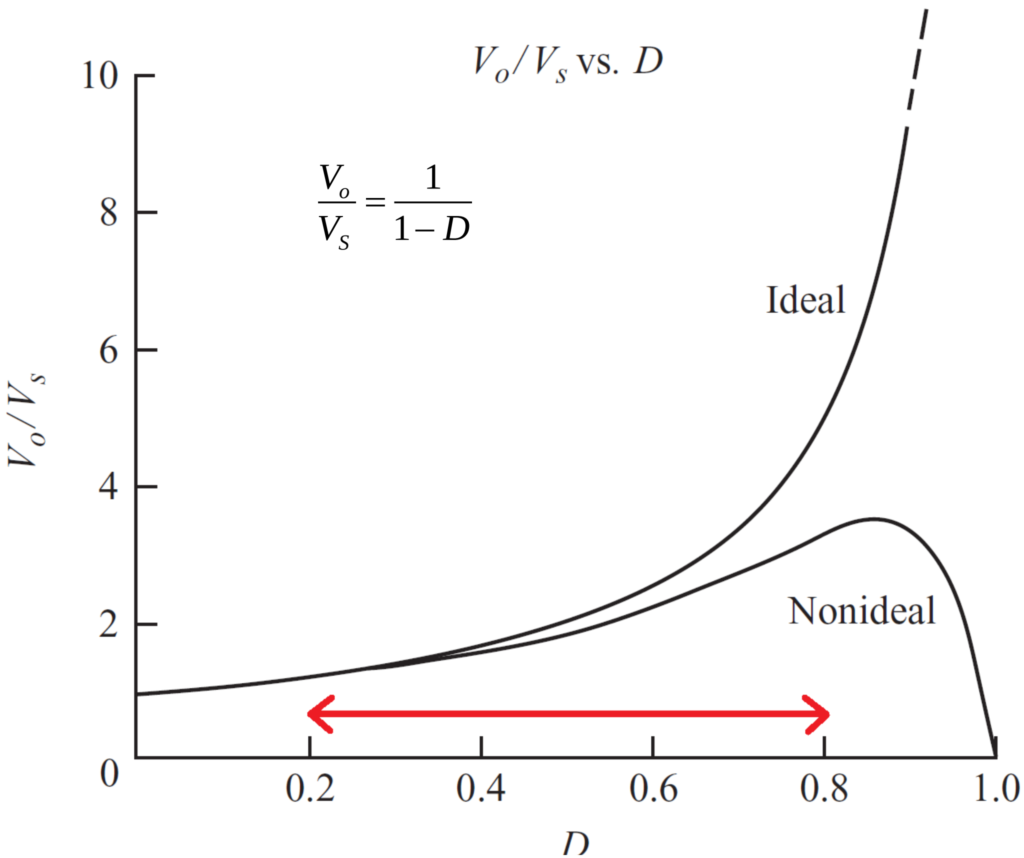


Figure 5-8 Voltage gain in a boost

The Buck-Boost Converter

Another basic switched-mode converter is the buck-boost converter shown in Fig. 5-9 (a). The output voltage of the buck boost converter can be either higher or lower than the input voltage.

Voltage and Current Relationships

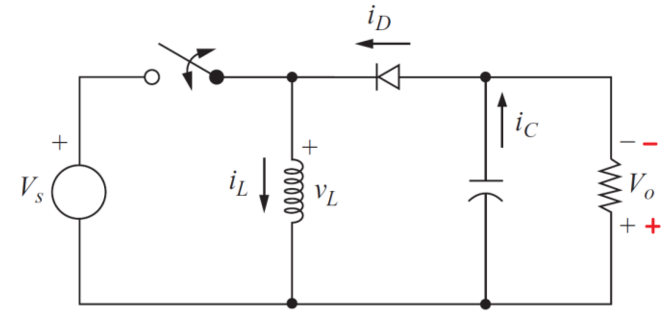
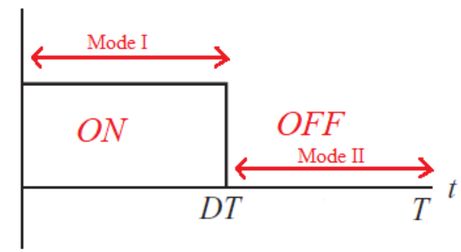


Figure 5-9 (a) Buck-boost converter circuit

Assumptions made about the operation of the converter are as follows:

1. The circuit is operating in the steady state.
2. The inductor current is continuous.
3. The capacitor is large enough to assume a constant output voltage.
4. The switch is closed for time DT and open for (1-D)T.
5. The components are ideal.



Analysis for the Switch Closed (Mode I: S:ON, 0<t<DT): When the switch is closed, the voltage across the diode is negative and it is OFF. The inductor is getting energized through the switch and the output capacitor which has been pre-charged is providing energy to the load.

$$\left.\begin{aligned} V_L &= V_S > 0 \\ V_L &= L\frac{di_L}{dt} \end{aligned}\right\} \Rightarrow i_L(t) = \mathrm{I}_{min} + \frac{V_S}{L}t \qquad (1)$$

$$\Delta i_L = \mathrm{I}_{max} - \mathrm{I}_{min} = \frac{V_S}{L}DT \qquad (2)$$

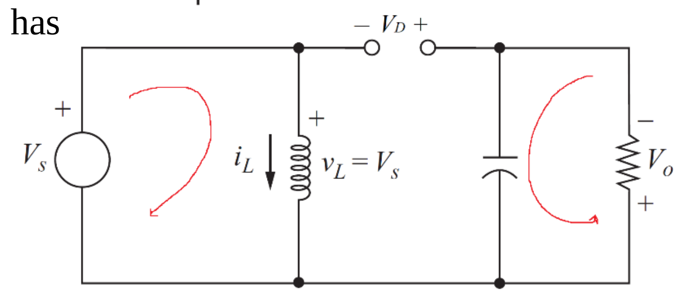


Figure 5-9(b) Equivalent circuit for the switch closed

Analysis for the Switch Open (Mode II: S:OFF, DT<t<T): When the switch getting open, the last moment in the first mode some currents passing through the inductor, which is continuous. This inductor current should find a pass to induct which force the diode to turn ON. The energy which was stored in Mode I, charging the capacitor.

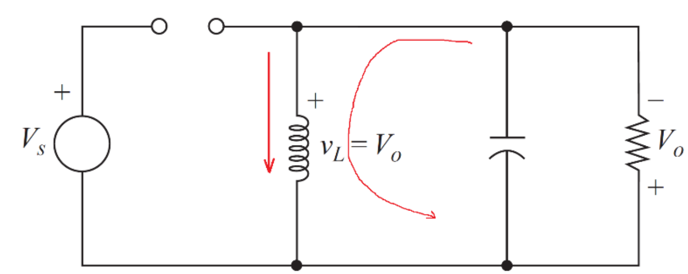


Figure 5-9 (c) Equivalent circuit for the switch open

$$V_L(t) = -V_O < 0$$

$$i_L(t) = \mathrm{I}_{\max} - \frac{V_O}{L}(t - DT)$$

$$\mathrm{I}_{\min} = i_L(T) = \mathrm{I}_{\max} - \frac{V_O}{L}(T - DT) \qquad (3)$$

$$\Delta i_L = \mathrm{I}_{\max} - \mathrm{I}_{\min} = \frac{V_O}{L}(T - DT) \qquad (4)$$

By comparing (2) & (4):

$$\frac{V_S}{L}DT = \frac{V_O}{L}(T - DT) \Rightarrow \boxed{\frac{V_O}{V_S} = \frac{D}{1-D}} \qquad (5)$$

Buck-Boost Converter
CCM Operation Mode

- For 0<D<0.5, Vo<Vs and converter act as Buck
- For 0.5<D<1, Vo>Vs and converter act as Boost

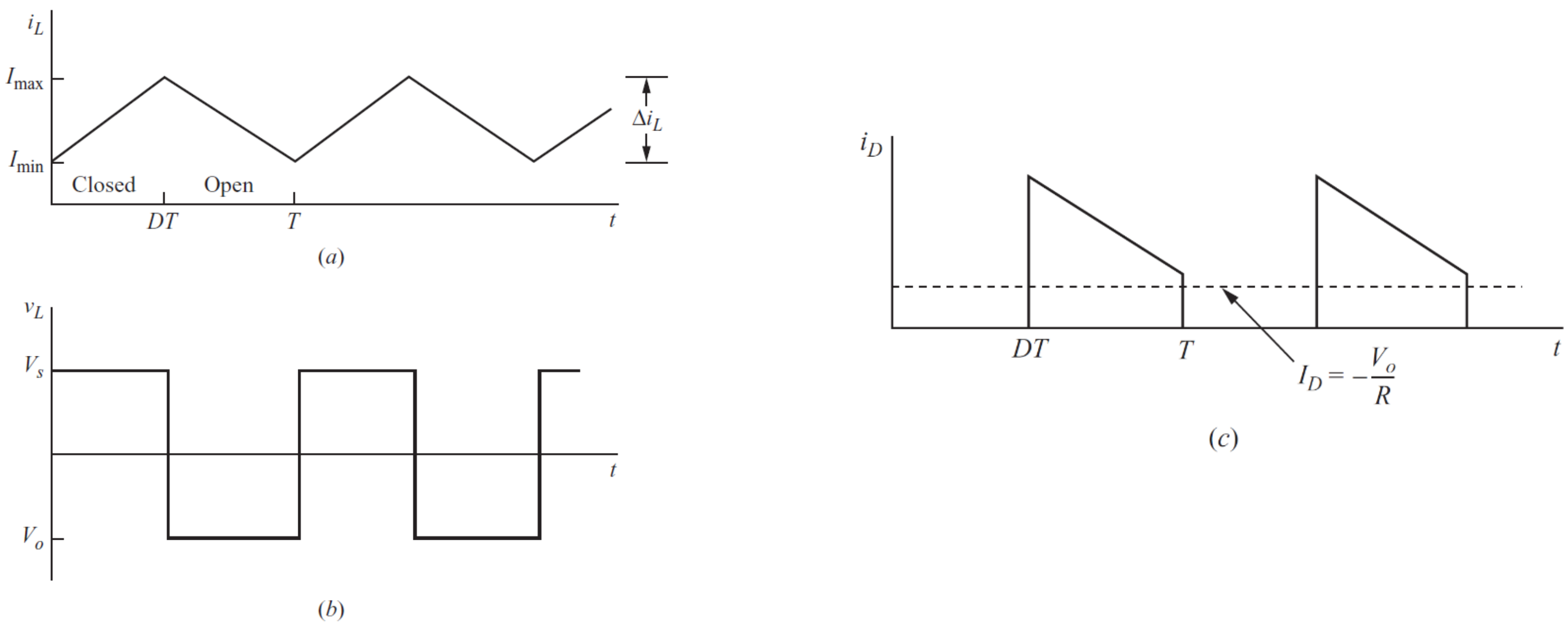


Figure 5-10 Buck-boost converter waveforms. (*a*) Inductor current; (*b*) Inductor voltage; (*c*) Diode current.

Ideal Converter: $P_{in}=P_{out}$

$$V_O = V_S \times \frac{D}{1-D}$$

$$Vs \times \langle i_S \rangle = \frac{V_O^2}{R} = V_S^2 \frac{D^2}{(1-D)^2 R} \quad \longrightarrow \quad \boxed{\langle i_S \rangle = \frac{V_S}{R}\left(\frac{D}{1-D}\right)^2}$$

$$\frac{\frac{I_{max}+I_{min}}{2}\times(DT)}{T} = \langle i_S \rangle \Rightarrow I_{max}+I_{min} = \frac{2V_S}{R}\frac{D}{(1-D)^2}$$

$$\left.\begin{aligned} & I_{max}+I_{min} = \frac{2V_S}{R}\frac{D}{(1-D)^2} \\ & \text{From (2)}\ \ I_{max}-I_{min} = \frac{V_S}{L}DT \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} I_{min} &= \frac{V_S}{R}\frac{D}{(1-D)^2} - \frac{V_S}{2L}DT \\ I_{max} &= \frac{V_S}{R}\frac{D}{(1-D)^2} + \frac{V_S}{2L}DT \end{aligned}\right. \Rightarrow \left\{\begin{aligned} I_{min} &= \overbrace{\frac{V_S}{R}\frac{D}{(1-D)^2}}^{\langle i_L \rangle} - \overbrace{\frac{V_S}{2L}}^{\frac{\Delta i_L}{2}}DT \\ I_{max} &= \frac{V_S}{R}\frac{D}{(1-D)^2} + \frac{V_S}{2L}DT \end{aligned}\right.$$

Example: The buck-boost circuit of Fig. 5-9 (a) has these parameters:

Vs =24 V, D =0.4, R =5 **Ω**, L =20 μH, C =80 μF, and f =100 kHz.

Determine the output voltage, and inductor current average, maximum and minimum values.

Solution:

$$\frac{V_O}{V_S} = \frac{D}{1-D} \Rightarrow V_O = 24 \times \frac{0.4}{1-0.4} = 16V$$

$$I_L = \frac{V_S}{R}\frac{D}{(1-D)^2} = \frac{24}{5}\frac{0.4}{(1-0.4)^2} = 5.33A$$

$$\Delta i_L = \frac{V_S}{L}DT = \frac{V_S}{Lf}D = \frac{24 \times 0.4}{20 \times 10^{-6} \times 100{,}000} = 4.8A$$

$$I_{L,\max} = I_L + \frac{\Delta i_L}{2} = 5.33 + \frac{4.8}{2} = 7.33A$$

$$I_{L,\min} = I_L - \frac{\Delta i_L}{2} = 5.33 - \frac{4.8}{2} = 2.93A$$

Questions and comments are most welcome!